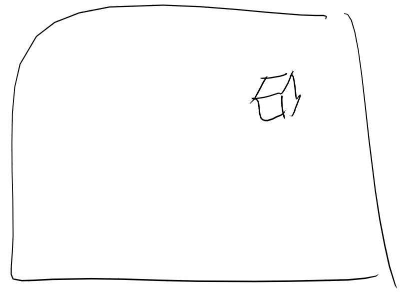
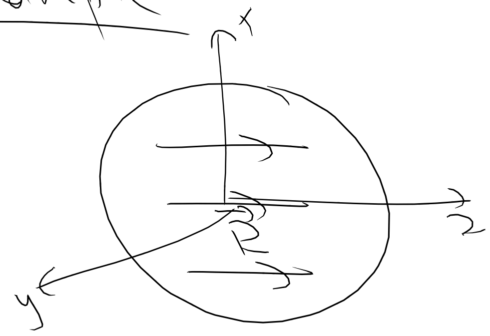
$$\vec P(\vec r) = \frac{\Delta \vec p}{\Delta V}$$

$$\rho_b = -\vec\nabla\cdot\vec P$$

$$\sigma_b = \vec P\cdot\hat n$$



Example

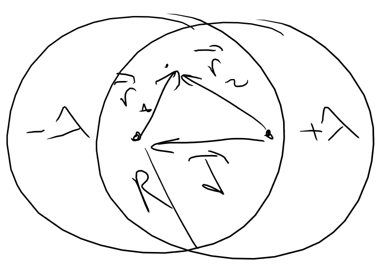


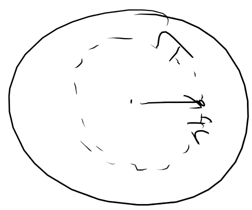
$$\vec P = P\hat z$$

$$\rho_b = -\vec\nabla\cdot\vec P = 0$$

$$\sigma_b = \vec P\cdot\hat n = \vec P\cdot\hat r = P\cos\theta$$

Example





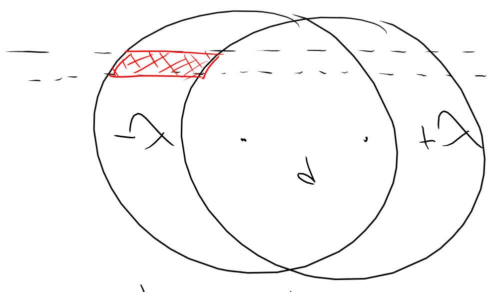
$$\vec E(\vec r) = \hat r\,\frac{1}{4\pi\varepsilon_0}\,\frac{\frac{4}{3}\pi r^{\cancel{3}}\lambda}{\cancel{r^2}}$$

$$= \frac{\lambda}{3\varepsilon_0}(r\hat r) = \boxed{\frac{\lambda}{3\varepsilon_0}\vec r = \vec E(\vec r)}$$

$$\vec E = \frac{(-\lambda)}{3\varepsilon_0}\vec r_1 + \frac{\lambda}{3\varepsilon_0}\vec r_2 = \frac{\lambda}{3\varepsilon_0}(\vec r_2 - \vec r_1)$$

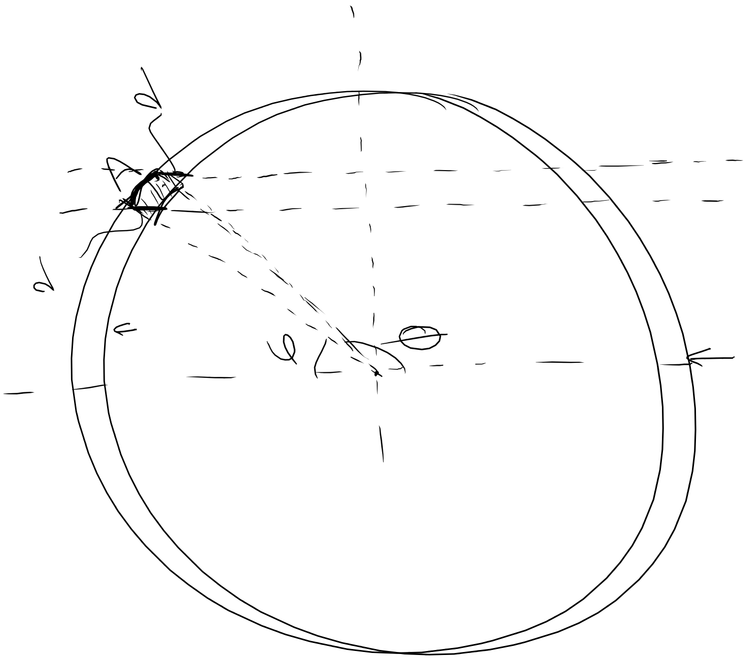
$$\boxed{\vec E = \frac{\lambda}{3\varepsilon_0}\vec d} = \frac{\frac{4}{3}\pi R^3\lambda\,\vec d}{3\varepsilon_0\,\frac{4}{3}\pi R^3} = \frac{Q\vec d}{4\pi\varepsilon_0 R^3} = \frac{-\vec p}{3\varepsilon_0\,\frac{4}{3}\pi R^3}$$

$$\boxed{\vec{E} = -\frac{\vec{P}}{3\varepsilon_0}} \qquad \vec{P} = \lambda \vec{d}$$



$$\begin{cases} d \to 0 \\ \lambda \to \infty \\ \lambda d = P \quad \text{constant} \end{cases}$$

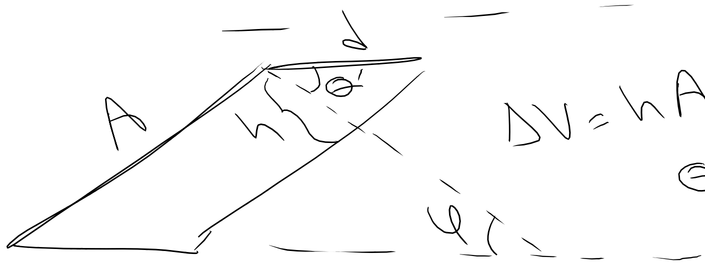
λ : volume charge density



$$\Delta q = \sigma \, \Delta A$$

$$\sigma \propto \cos\theta$$

$$\Delta q \overset{?}{=} \lambda d \cos\theta \, \Delta A$$



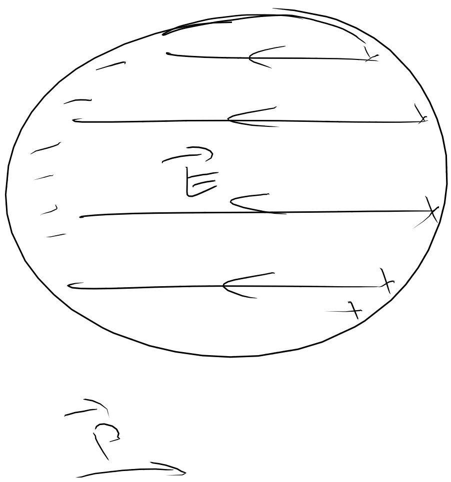
$$\Delta V = hA = A d \cos\theta'$$

$$\theta' = \varphi - \pi - \theta$$

$$\Delta V = A d \cos(\pi - \theta)$$
$$= A d |\cos\theta|$$

$$\Delta q = A\, d\, (x) \cos\theta$$

$$\sigma = \frac{\Delta q}{A} = d x \cos\theta \rightarrow P \cos\theta$$



$$\sigma = P \cos\theta$$

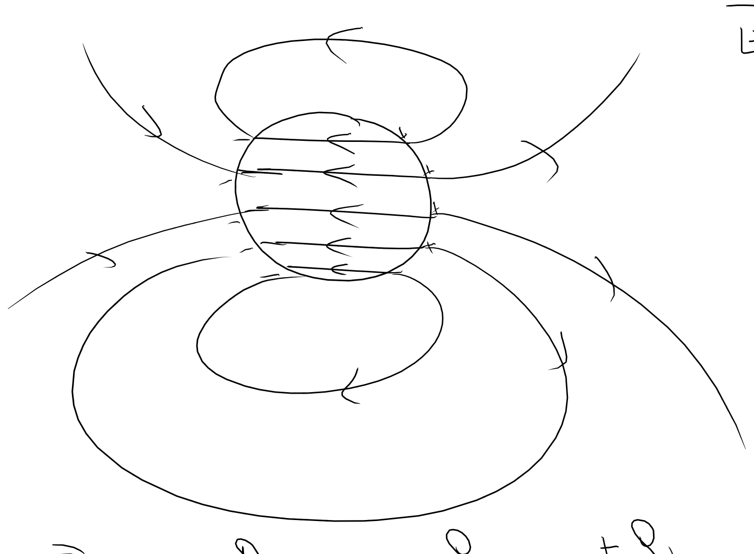
$$\vec{E} = \frac{-\vec{P}}{3\varepsilon_0} \leftarrow \text{inside}$$

outside

$$V = \frac{\left(\frac{4}{3}\pi R^3 \vec{P}\right)\cdot \hat{r}}{r^2} \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}^{\,\text{outside}} = \frac{1}{4\pi\varepsilon_0} \frac{(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}$$

$$\vec{p} = \frac{4}{3}\pi R^3 \vec{P}$$



$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} = \frac{\rho_{free} + \rho_b}{\varepsilon_0}$$

$$\vec{\nabla}\times\vec{E} = 0$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$\varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho_{free} - \vec{\nabla}\cdot\vec{P}$$

$$\vec{\nabla}\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_{free}$$

$\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$; displacement field.

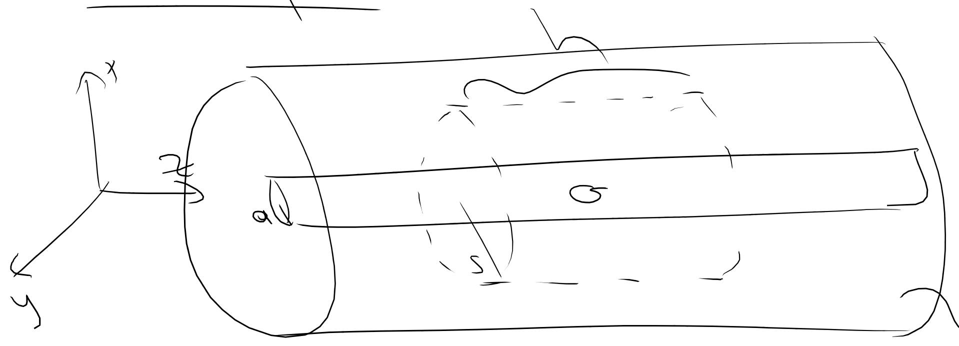
$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_{free}}$$

$$\vec{\nabla} \times \vec{E} = 0 \Longleftarrow$$

$$\int \vec{E} \cdot d\vec{s}$$

Example

$$Q^{enc}_{free} = (2\pi a h)\sigma$$



$$s = \sqrt{x^2 + y^2}$$

dielectric

$$\vec{D} = ?$$

$$\vec{\nabla} \cdot \vec{D} = \rho_{free} \Longrightarrow \boxed{\oint \vec{D} \cdot d\vec{S} = Q^{enc}_{free}}$$

$$\vec{D} = \hat{s} D(s)$$

$$\oint \vec{D} \cdot d\vec{S} = \underbrace{\int \vec{D} \cdot d\vec{S}}_{\substack{top \\ = 0}} + \underbrace{\int \vec{D} \cdot d\vec{S}}_{\substack{bottom \\ = 0}} + \underbrace{\int \vec{D} \cdot d\vec{S}}_{side}$$

$$d\vec{S} = dS\,\hat{s}$$

on the side

$$\vec{D} \cdot d\vec{S} = D(s)\,dS$$

$$\oint \vec{D} \cdot d\vec{S} = \int_{side} D\, dS = D(s)(\cancel{2\pi} s \cancel{h}) = \cancel{2\pi} a \cancel{h} \sigma$$

$$\boxed{D(s) = \frac{a}{s}\sigma}$$

$$D(s) = \frac{(2\pi a \sigma)}{2\pi s}$$

$$\boxed{\vec{D}(s) = \frac{2\pi a \sigma}{2\pi s}\,\hat{s}}$$

$$\varepsilon_0 \vec{E} + \vec{P} = \vec{D}$$

$\vec{E}$ outside the dielectric: $\qquad \vec{P} = \frac{\Delta \vec{p}}{\Delta V}$

$$\vec{P} = 0$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{1}{2\pi\varepsilon_0 s}(2\pi a \sigma)\,\hat{s} = \frac{1}{2\pi\varepsilon_0 s}\lambda\,\hat{s}$$

$\vec{E}$ inside the dielectric

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} - \frac{\vec{P}}{\varepsilon_0}$$

$Q^{enc}_{free} = (2\pi a h)\sigma$ charge in a length h of wire.

linear charge density $\lambda = \frac{Q}{h} = 2\pi a \sigma$

Example

$\vec D = D(r)\hat r$

$\oint \vec D\cdot d\vec S = \oint D(R)\, dS$

$d\vec S = \hat r\, dS$

$\vec D\cdot d\vec S = D\, dS$

$$\frac{4}{3}\pi R_0^3 \rho = \oint \vec D\cdot d\vec S = D(R)\oint dS = D(R)\,4\pi R^2$$

$$D(R) = \frac{1}{4\pi R^2} Q_{tot}$$

$$\boxed{\vec D(\vec r) = \frac{1}{4\pi r^2}Q_{tot}\,\hat r} \qquad r > R_0$$

outside the dielectric $\vec P = 0$

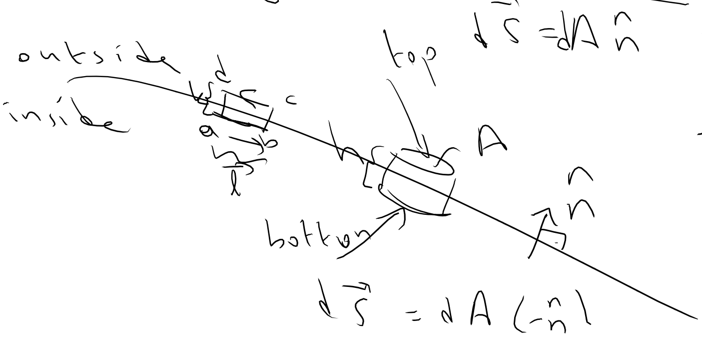
$$\vec E = \frac{\vec D}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{tot}}{r^2}\hat r$$

$$\left.\begin{array}{l}\vec\nabla\cdot\vec E = \frac{\rho}{\varepsilon_0}\\ \vec\nabla\times\vec E = 0\end{array}\right\} \longrightarrow \vec E(\vec r) = \frac{1}{4\pi\varepsilon_0}\int d\vec r'\,\rho(\vec r')\frac{(\vec r-\vec r')}{|\vec r-\vec r'|^3}$$

$$\vec\nabla\cdot\vec D = \rho_{free} \Rightarrow \vec D(\vec r) \neq \frac{1}{4\pi}\int d\vec r'\,\rho_{free}(\vec r')\frac{(\vec r-\vec r')}{|\vec r-\vec r'|^3}$$

$\vec\nabla\times\vec D \neq 0$ in general

Boundary Condition



$$\vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{D} = \rho_{free}$$

$$\int_A d\vec{A}\cdot(\vec{\nabla}\times\vec{E}) = 0$$

$$\parallel$$

$$\boxed{\oint_{\partial A} d\vec{l}\cdot\vec{E} = 0}$$

$$0 = \oint \vec{E}\cdot d\vec{l} = \int_a^b \vec{E}\cdot d\vec{l} + \underbrace{\int_b^c \vec{E}\cdot d\vec{l}}_{\text{as } h\to 0\ \to\ 0} + \int_c^d \vec{E}\cdot d\vec{l} + \underbrace{\int_d^a \vec{E}\cdot d\vec{l}}_{\text{as } h\to 0\ \to\ 0}$$

$$0 = \frac{\vec{E}^{inside}\cdot\vec{l} + \vec{E}^{outside}\cdot(-\vec{l})}{l}$$

$$0 = \left(\vec{E}^{inside} - \vec{E}^{outside}\right)\cdot\hat{l}$$

$$\Longrightarrow E^{inside}_{\parallel} - E^{outside}_{\parallel} = 0 \Longleftarrow \vec{\nabla}\times\vec{E} = 0$$

$$\oint \vec{D}\cdot d\vec{S} = Q^{free}_{enc}$$

$$\oint \vec{D}\cdot d\vec{S} = \int_{top} \vec{D}\cdot d\vec{S} + \int_{bottom} \vec{D}\cdot d\vec{S} + \underbrace{\int_{side} \vec{D}\cdot d\vec{S}}_{\text{as } h\to 0\ \to\ 0}$$

$$= \int_{top} \left(\vec{D}^{outside}\cdot\hat{n}\right) dA + \int_{bottom} \left(\vec{D}^{inside}\cdot\hat{n}\right) dA$$

$$= \left(\vec{D}^{outside} - \vec{D}^{inside}\right)\cdot\hat{n}\, A = \sigma^{free} A$$

$$\Rightarrow \boxed{\left(\vec{D}^{\text{outside}} - \vec{D}^{\text{inside}}\right) \cdot \hat{n} = \sigma^{\text{free}}}$$