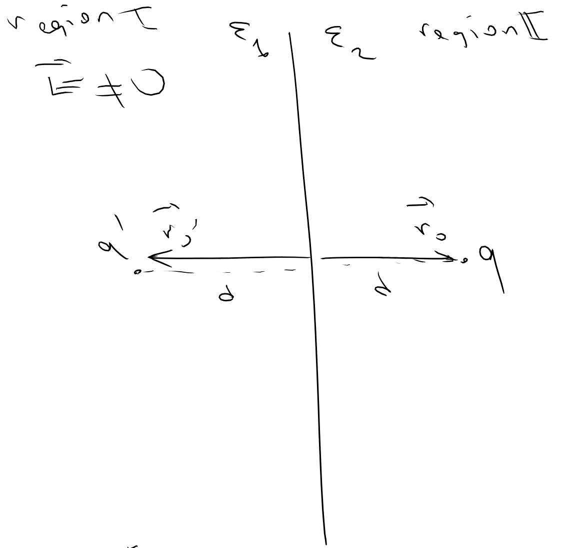
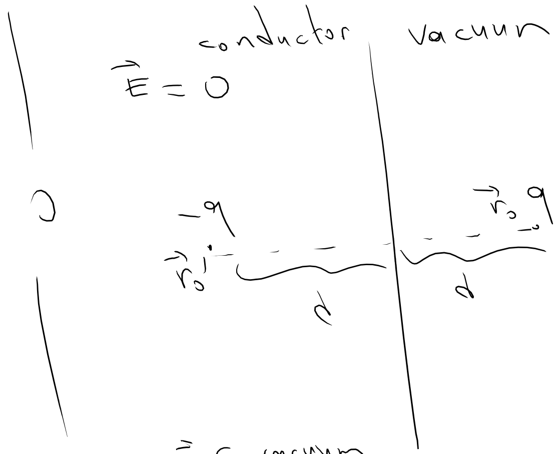




$\vec{r} \in$ vacuum

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}-\vec{r}_0|} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{|\vec{r}-\vec{r}_0'|}$$

$\vec{r} \in$ region II

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_2}\cdot\frac{q}{|\vec{r}-\vec{r}_0|} + \frac{1}{4\pi\varepsilon_2}\frac{q'}{|\vec{r}-\vec{r}_0'|}$$

$\vec{r} \in$ region I

$$q' = \frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}q \qquad q'' = \frac{2\varepsilon_1 q}{\varepsilon_1+\varepsilon_2}$$

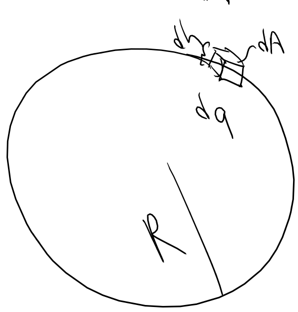
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_1}\frac{q''}{|\vec{r}-\vec{r}_0|}$$

V is continuous on the boundary $\Leftrightarrow E_{\parallel}$ is continuous on the boundary

$D_{\perp}$ is continuous on the boundary $(\sigma_f = 0)$

$dV = dA\,dh$

Ex



total charge $+Q$

inside $\rho = 0$

outside $\rho = 0$

$\rho \neq 0$ only for $r = R$

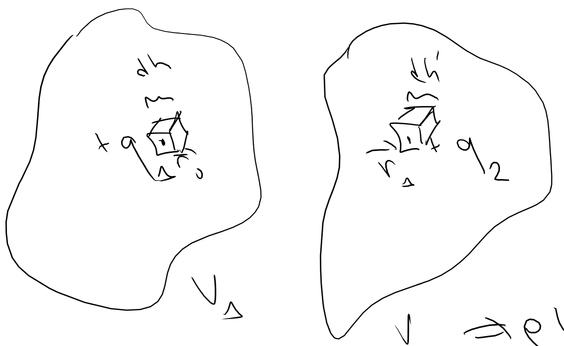
$$\rho = \lim_{\substack{dh\to 0\\ dA\to 0}}\frac{dq}{dh\,dA} = \lim_{dA\to 0}\lim_{dh\to 0}\left(\frac{dq}{dh}\right)\frac{1}{dA}$$

$\rho \to \infty$ on the surface

$$\rho(r,\theta,\phi) = \rho(\theta\phi)\,\delta(r-R) = A\,\delta(r-R)$$

$$Q = \int \rho\, dV = \int A\,\delta(r-R)\,r^2 dr\,\sin\theta\, d\theta\, d\phi = 4\pi A R^2$$

$$A = \frac{Q}{4\pi R^2} \Rightarrow \rho = \frac{Q}{4\pi R^2}\delta(r-R) = \sigma\,\delta(r-R)$$



if $\vec r \neq \vec r_0$ & $\vec r \neq \vec r_1$

$$\rho(\vec r) = 0$$

$$\rho(\vec r_0) = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} = \lim_{dh \to 0} \frac{q}{(dh)^3}$$

$$V_2 \Rightarrow \rho(\vec r_0) \to \infty$$

$$\rho(\vec r_1) = \lim_{dh' \to 0} \frac{q_2}{(dh')^3} \to \infty$$

$$\rho(\vec r) = A\,\delta^{(3)}(\vec r - \vec r_0) + B\,\delta^{(3)}(\vec r - \vec r_1)$$

$$q_1 = \int_{V_1} \rho(\vec r)\, dV = \underbrace{\int_{V_1} A\,\delta^{(3)}(\vec r - \vec r_0)\, dV}_{A} + \overbrace{\int_{V_1} B\,\delta^{(3)}(\vec r - \vec r_1)\, dV}^{0}$$

$$A = q_1$$

$$q_2 = \int_{V_2} \rho(\vec r)\, dV = B$$

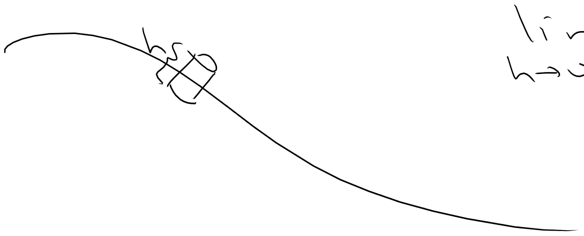
$$\rho(\vec r) = q_0\,\delta^{(3)}(\vec r - \vec r_0) + q_1\,\delta^{(3)}(\vec r - \vec r_1)$$

$$\rho(\vec r) = \sigma(\vec r)\,\delta(n - n_0)$$

$$\vec\nabla \cdot \vec B = g \Rightarrow \boxed{\left(\vec B^{out} - \vec B^{in}\right) \cdot n = \sigma_g}$$

$g \longrightarrow \sigma_g\,\delta(n - n_0)$ on the surface

$$\vec\nabla \cdot \vec B = g \Rightarrow \oint_{\partial V} \vec B \cdot d\vec S = \int_V g\, dV$$



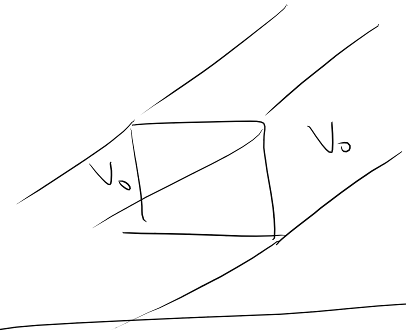
$$\lim_{h\to 0}\oint \vec{B}\cdot d\vec{S} = \left(B^{out}-B^{in}\right)\cdot\hat{n}\, dS$$

$$= \lim_{h\to 0}\int g\, dV = \sigma_g\, dS$$

$$\left.\begin{aligned}\vec{\nabla}\cdot\vec{B} &= g\\ \vec{\nabla}\times\vec{B} &= 0\end{aligned}\right\} \Rightarrow \left.\begin{aligned}&\left(\vec{B}^{out}-\vec{B}^{in}\right)\cdot\hat{n} = \sigma_g\\ &B^{out}_{\parallel} - B^{in}_{\parallel} = 0\end{aligned}\right\} \vec{B}^{out} - \vec{B}^{in} = \sigma_g \hat{n}$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f \Rightarrow \oint \vec{D}\cdot d\vec{S} = Q_f^{enc}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$



$$\nabla^2\phi = 0$$

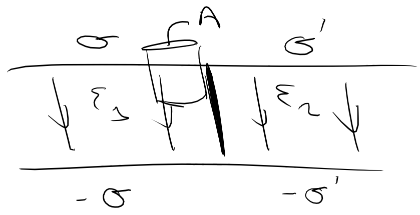
$$\boxed{E = \int \frac{1}{2}\vec{E}\cdot\vec{D}\, d^3r} = \frac{1}{2}CV^2$$

$$C = \varepsilon_r C^{(0)}$$

$$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

$$\vec{D} = \varepsilon\vec{E}$$

Ex



$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\oint \vec{D}\cdot d\vec{S} = \int_{side} \overset{0}{\overset{\shortparallel}{\vec{D}\cdot d\vec{S}}} + \int_{top} \overset{0}{\overset{\shortparallel}{\vec{D}\cdot d\vec{S}}} + \int_{botton} \vec{D}\cdot d\vec{S}$$

$$= \int_{botton} D\,dS = DA = Q_f$$

$$D_1 A = \sigma A \Rightarrow \boxed{D_1 = \sigma}$$

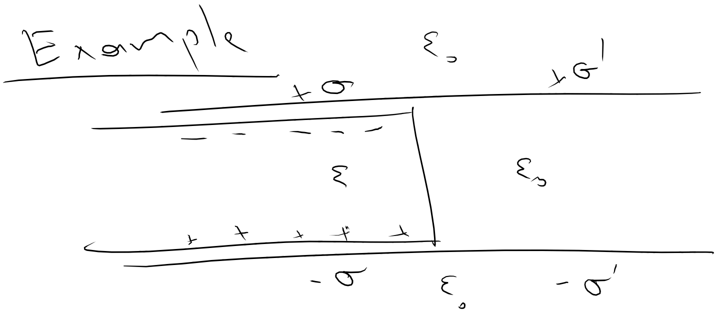
$$\boxed{\begin{aligned} D_1 &= \sigma \\ D_2 &= \sigma' \end{aligned}}$$

$$E_1 = \frac{D_1}{\varepsilon_1} = \frac{\sigma}{\varepsilon_1}$$

$$E_2 = \frac{D_2}{\varepsilon_2} = \frac{\sigma'}{\varepsilon_2}$$

$$E_1 = E_2$$

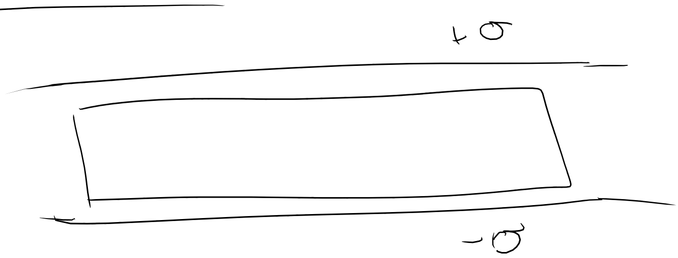
Example



$$\varepsilon = \varepsilon_0 + \varepsilon_0 x$$

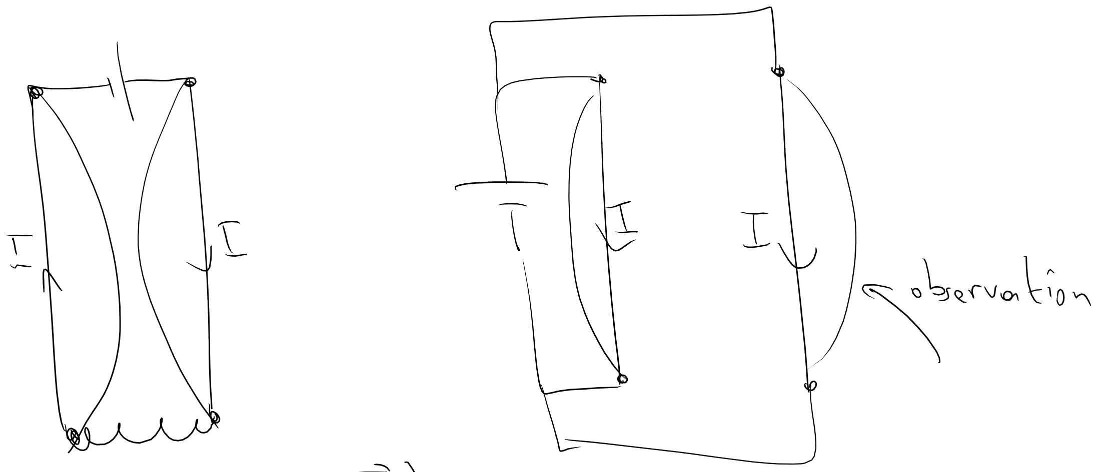
$$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

Example



Magnetostatics

$$\boxed{\frac{\partial \rho}{\partial t} = 0} \qquad \frac{\partial \vec{J}}{\partial t} = 0$$



$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$: Lorentz Force.

$\vec{B}$: magnetic induction

$\underline{\vec{E} = 0}$ work done by a magnetic field:

$$W = \int \vec{F} \cdot d\vec{l} = Q \int (\vec{v} \times \vec{B}) \cdot d\vec{l} = Q \int \left(\frac{d\vec{l}}{dt} \times \vec{B} \right) \cdot d\vec{l} = 0$$

$$W_T = \Delta(KE) \qquad \text{if } W_T = 0 \Leftrightarrow \Delta(KE) = 0$$

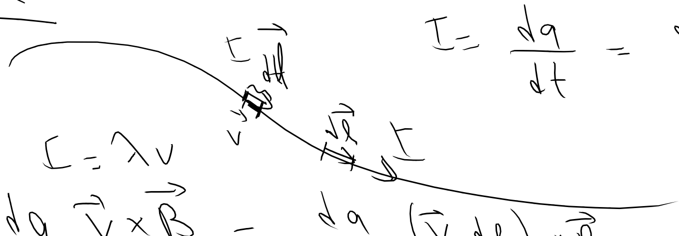
$\Downarrow$

speed is constant

Example

$\vec{B}$

$$I = \frac{dq}{dt} = \frac{dq}{dl}\frac{dl}{dt} = \lambda v$$



$I = \lambda v$

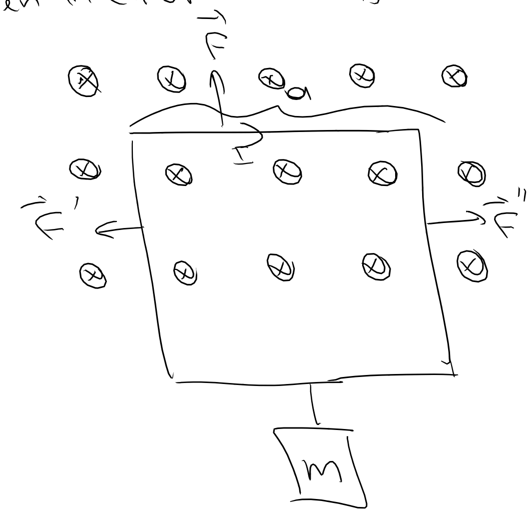
$$d\vec{F} = dq\, \vec{v} \times \vec{B} = \frac{dq}{dl} (\vec{v}\, dl) \times \vec{B} = \lambda (\vec{v}\, dl) \times \vec{B}$$

$$\vec{v}\, dl = v\, d\vec{l}$$

$$d\vec{F} = (\lambda v)\, \vec{dl}\times\vec{B} = \boxed{I\, \vec{dl}\times\vec{B} = d\vec{F}}$$

if $\vec{B}$ is uniform & wire lies on the plane perpendicular to $\vec{B}$

$$|\vec{dl}\times\vec{B}| = dl\, B\sin 90^\circ = B\, dl$$



$$\vec{F} = \int I\, \vec{dl}\times\vec{B} = I\left(\int \vec{dl}\right)\times\vec{B}$$

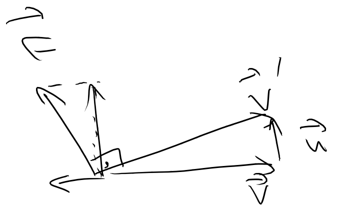
$$F = IaB$$

$$I = \lambda v$$

$$I_0 aB = mg$$

$$\boxed{I_0 = \frac{mg}{aB}}$$

For $I > I_0$



$$\vec{F} = q\,\vec{v}\times\vec{B}$$