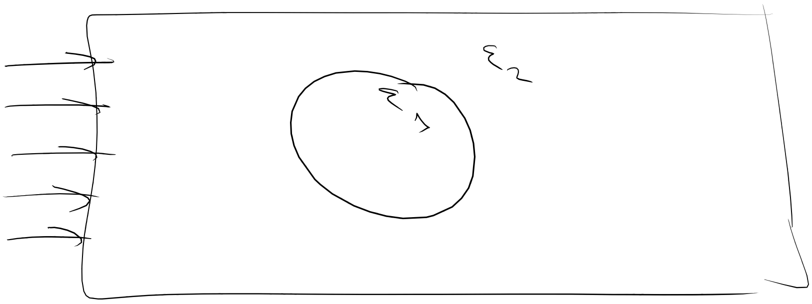


Currents

$$I = \frac{dq}{dt}$$

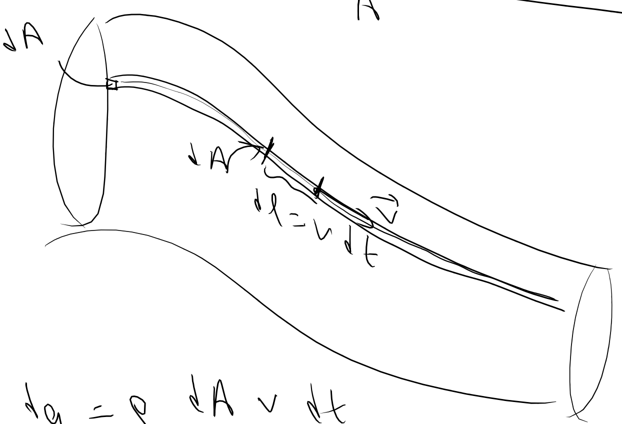
$$dl = v\,dt$$

$$dq = \lambda\,dl = \lambda v\,dt$$

$$\Rightarrow \boxed{\frac{dq}{dt} = \lambda v}$$

$$\frac{dI}{dA} = |\vec{J}|$$

$$\vec{J} \parallel \vec{v}$$

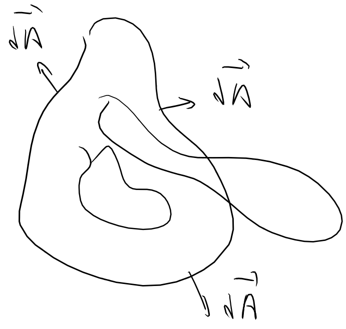


$$dq = \rho\,dA\,v\,dt$$

$$\frac{1}{dA}\left(\frac{dq}{dt}\right) = \rho v \Rightarrow \boxed{\vec{J} = \rho\vec{v}}$$

$$I = \int \vec{J}\cdot d\vec{A}$$

$$\oint_{\partial V} \vec{J}\cdot d\vec{A} = -\frac{dQ}{dt} = -\frac{d}{dt}\int_V \rho\,dV$$



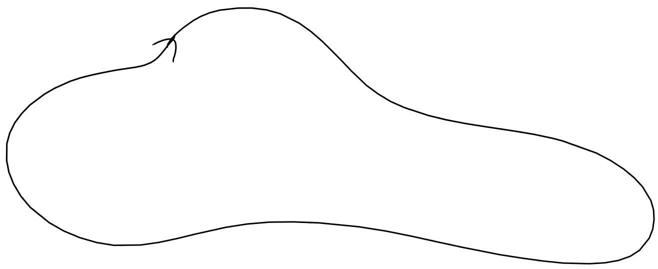
$$\oint_{\partial V} \vec{J}(\vec{r},t)\cdot d\vec{A} = -\frac{d}{dt}\int \rho(\vec{r},t)\,dV$$

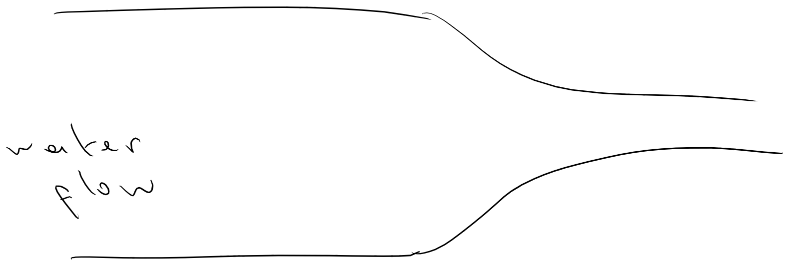
$$\int_V \left[\nabla\cdot\vec{J}(\vec{r},t)\right] dV = -\int_V \frac{\partial\rho(\vec{r},t)}{\partial t}\,dV$$

$$\int_V \left[\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J}\right] dV = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0}$$ continuity eqn.

In magneto<u>statics</u> $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} = 0}$





$$\vec{F} = q\,\vec{v}\times\vec{B}$$

$$\vec{F} = \int (\vec{v}\times\vec{B})\, dq$$

$$= \int (\vec{v}\times\vec{B})\, \lambda\, dl$$

linear charge density

$$\lambda v = I$$

$$d\vec{l} \parallel \vec{v}$$

$$\vec{F} = \int I\, d\vec{l}\times\vec{B}$$

$$\vec F = \int (\vec v \times \vec B)\underbrace{\rho\, dV}_{dq}$$

volume charge density

$$\vec F = \int (\vec J \times \vec B)\, dV$$

$$dq = \rho\, dV$$

Example

$\vec B$ is uniform

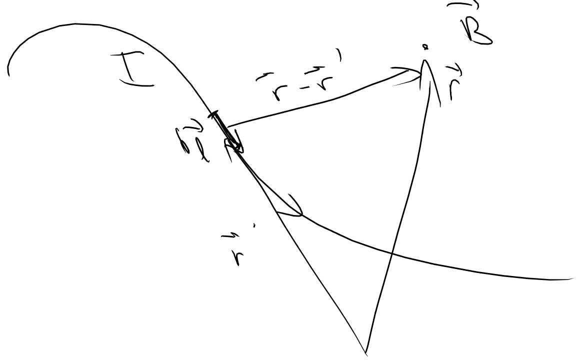
$$\vec F = \int (I\, d\vec l \times \vec B) = I\left(\int d\vec l\right) \times \vec B$$

$$\vec F = I\,(\vec l_f - \vec l_i) \times \vec B$$

$$[I] = \frac{C}{s} = A$$

$$[\vec J] = \frac{[I]}{[\text{area}]} = \frac{A}{m^2}$$

Biot-Savart Law



$$d\vec B(\vec r) = \frac{\mu_0 I\, d\vec l \times (\vec r - \vec r')}{4\pi\, |\vec r - \vec r'|^3}$$

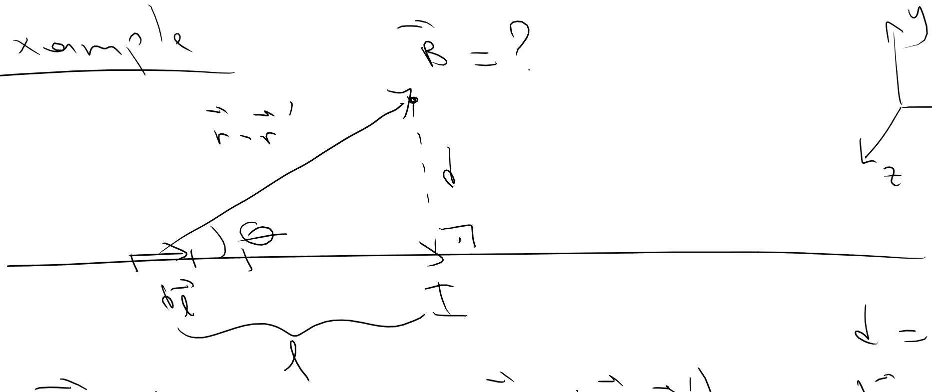
$$\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T\,m}$$

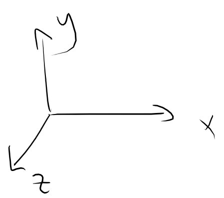
$$\vec B = \frac{\mu_0}{4\pi} \int I\, d\vec l \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} = \frac{\mu_0}{4\pi} \int \vec J \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}\, dV$$

$$I\, d\vec l = (J\, dA)\, d\vec l = \vec J\, \underbrace{(dA\, dl)}_{dV}$$

$$I\, d\vec l \Leftrightarrow \vec J\, dV$$

Example





$$d = |\vec{r} - \vec{r}'| \sin\theta$$

$$|\vec{r} - \vec{r}'| = \frac{d}{\sin\theta}$$

$$\frac{d}{l} = \tan\theta$$

$$\frac{l}{d} = \cot\theta$$

$$\boxed{\frac{dl}{d} = -\frac{1}{\sin^2\theta}\, d\theta}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$d\vec{l} \times (\vec{r} - \vec{r}') = \hat{z}\, dl\, |\vec{r} - \vec{r}'| \sin\theta$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \hat{z} \int \frac{dl}{|\vec{r} - \vec{r}'|^2} \sin\theta$$

$$= \frac{\mu_0 I}{4\pi} \hat{z} \int (dl) \sin\theta \, \frac{\sin^2\theta}{d^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi d^2} \hat{z} \int dl \sin^3\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi \cancel{d^2}} \hat{z} \int (\cancel{d}) (-1) \frac{1}{\cancel{\sin^2\theta}} d\theta \sin^3\theta$$

$$= -\frac{\mu_0 I}{4\pi d} \hat{z} \int_{\pi}^{0} \sin\theta \, d\theta$$

$$= + \frac{\mu_0 I}{4\pi d} \hat{z} \left(+\cos\theta \right|_{\theta=\pi}^{0}$$

$$= \frac{\mu_0 I}{4\pi d} \hat{z} \left(1 - (-1)\right)$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi d} \hat{z}}$$

Warning

$$\vec{B} = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}') \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \, dV'$$

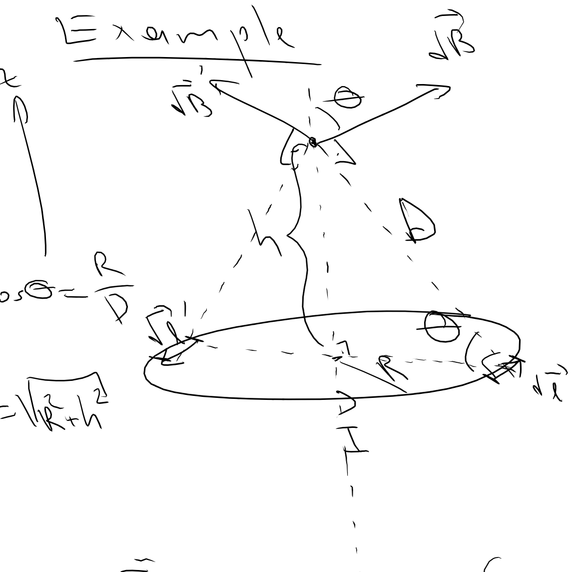
$$\vec{J} = \rho \vec{v} = q\, \delta^{(3)}(\vec{r}-\vec{r}_0)\, \vec{v}_0 \qquad \rho = q\, \delta^{(3)}(\vec{r}-\vec{r}_0) \qquad \frac{\partial \rho}{\partial t} \neq 0$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int q\, \delta^{(3)}(\vec{r}'-\vec{r}_0)\, \vec{v}_0 \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\, q\, \vec{v}_0 \times \frac{(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}}$$

not correct! only an approximation at small velocities.

Example



$\cos\theta = \frac{R}{D}$

$D = \sqrt{R^2+h^2}$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \, \frac{d\vec{l} \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl\, D}{D^3} \sin\frac{\pi}{2}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{D^2} = dB'$$

$$\vec{B} = B\hat{z} = \hat{z} \int \frac{\mu_0 I}{4\pi} \frac{dl}{D^2} \cos\theta$$

$$= \hat{z} \frac{\mu_0 I}{4\pi} \frac{\cos\theta}{D^2} \int dl$$

$$= \hat{z} \frac{\mu_0 I}{4\pi} \frac{\cos\theta}{D^2}\, 2\pi R$$

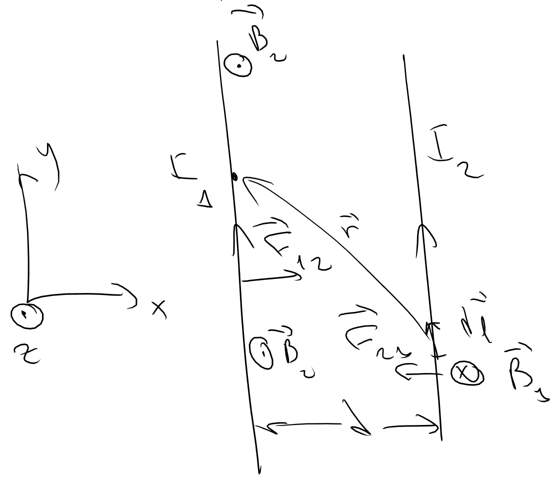
$$= \frac{\mu_0 I}{2\cancel{4\pi}} \frac{\cancel{2\pi}R}{D^2} \frac{R}{D}\, \hat{z}$$

$$\boxed{\vec{B} = \frac{\mu_0 (I \pi R^2)}{2\pi D^3}\hat{z}}$$ for $h \gg R$, $D \sim h$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{(I\pi R^2)}{h^3}\hat{z}$$ (compare with $E \sim \frac{d}{h^3}\hat{z}$ for an electric dipole $\vec{d} = d\hat{z}$)

$I\pi R^2 \equiv IA$ = magnetic dipole moment.

Example Force between two straight wires



$\vec{F}_{12} = ?$

$$|\vec{B}_2| = \frac{\mu_0 I_2}{2\pi d}$$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi d}(-\hat{z})$$

$$d\vec{F}_{12} = I_1\, d\vec{l}_1 \times \vec{B}_2 = I_1 \frac{\mu_0}{2\pi d} I_2\, dl_1 \hat{x}$$

$$\boxed{\frac{d\vec{F}_{12}}{dl_1} = \frac{\mu_0}{2\pi d} I_1 I_2 \hat{x}}$$

$$d\vec{F}_{21} = I_2\, d\vec{l}_2 \times \vec{B}_1 = (-\hat{x})\, I_2 \frac{\mu_0 I_1}{2\pi d}\, dl_2$$

$$\frac{d\vec{F}_{21}}{dl_2} = (-\hat{x})\frac{\mu_0}{2\pi d} I_1 I_2 = -\frac{d\vec{F}_{12}}{dl_2}$$

Assume $I_1 = I_2 = 1\,A$
$d = 1\,m$

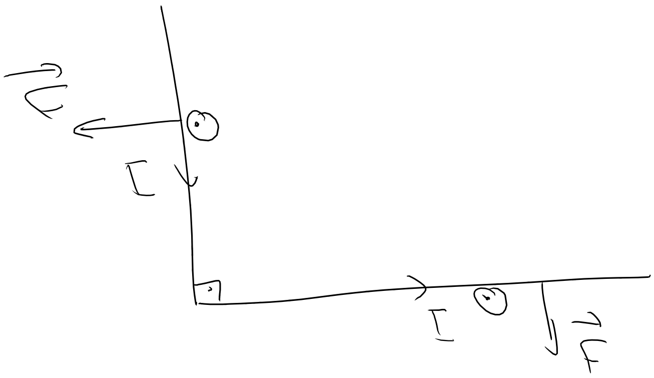
$$\frac{dF}{dl} = \frac{4\pi\, 10^{-7}\, Tm\, 1A^2}{2\pi(1m)}$$

$$\boxed{\frac{dF}{dl} = 2\times 10^{-7}\ \frac{N}{m}}$$

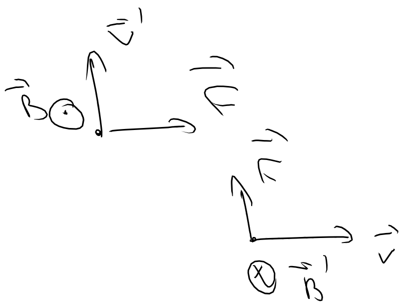
$d = 1\,cm$

$$\boxed{\frac{dF}{dl} = 2\times 10^{-5}\ \frac{N}{m}}$$

Example



Example



$$\vec B = \frac{\mu_0}{4\pi}\, q\, \frac{\vec v \times \vec r}{r^3}$$

$$\vec F = q\,\vec v \times \vec B$$