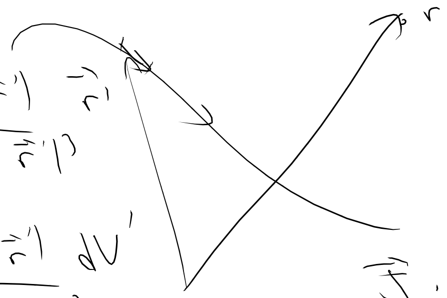
Hand in your HW!

$\vec{B}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

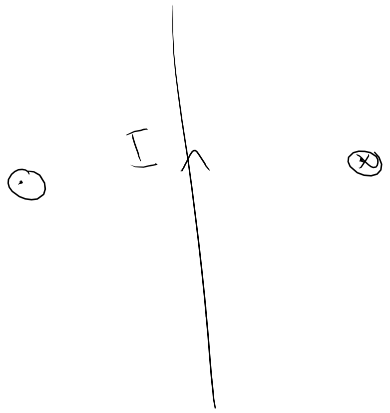


$$d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}\, dV'$$

$\vec{J}$: volume current density

$$I\, d\vec{l} \Leftrightarrow \vec{J}\, dV \Leftrightarrow \vec{K}\, dS$$

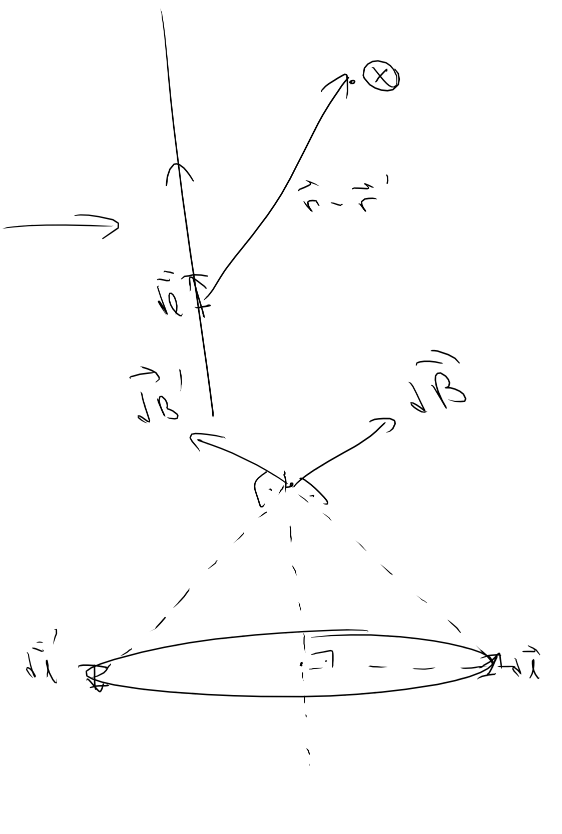
$\vec{K}$: surface current density

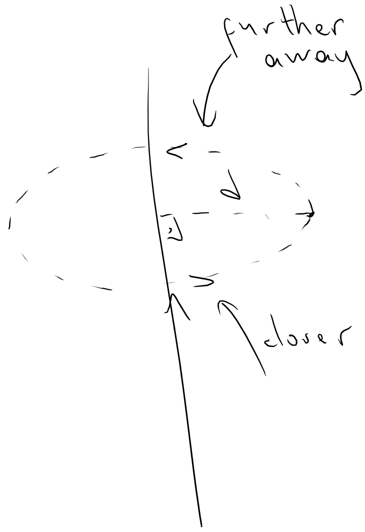


$$\boxed{B = \frac{\mu_0 I}{2\pi d}}$$

I : linear current

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$





$$B = \frac{\mu_0 I}{2\pi d}$$

$$B\, 2\pi d = \mu_0 I$$

$$\shortparallel$$

$$B \oint dl$$

$$\shortparallel$$

$$\oint B\, dl$$

$$\shortparallel$$

$$\Longrightarrow \boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}}$$ Ampere's Law

$$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 \int_S \vec{J} \cdot d\vec{A}$$

$$\shortparallel$$

$$\int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{A} = \mu_0 \int_S \vec{J} \cdot d\vec{A} \Longrightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}(\vec{r})}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}\,') \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \, dV'$$

$$\vec{\nabla} \times \vec{B}$$

$$\vec{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$$

$$\vec{\nabla}' \equiv \hat{x} \frac{\partial}{\partial x'} + \hat{y} \frac{\partial}{\partial y'} + \hat{z} \frac{\partial}{\partial z'} \qquad \vec{J}' = \vec{J}(\vec{r}\,')$$

$$\vec{\nabla} \times \vec{B} = \frac{\mu_0}{4\pi} \int dV' \, \vec{\nabla} \times \left(\vec{J}' \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right)$$

$$\vec{\nabla}\times(\vec{B}\times\vec{C})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - (\vec{A}\cdot\vec{B})\vec{C}$$

$$\vec{\nabla}\times\left(\vec{J}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\ |^3}\right) = \vec{J}(\vec{r}')\underbrace{\left(\vec{\nabla}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)}_{4\pi\,\delta^{(3)}(\vec{r}-\vec{r}')} - (\vec{J}(\vec{r}')\cdot\vec{\nabla})\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int dV'\ \vec{\nabla}\times\left(\vec{J}'\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right)$$

$$= \frac{\mu_0}{\cancel{4\pi}}\int dV'\ \vec{J}(\vec{r}')\,\cancel{4\pi}\,\delta^{(3)}(\vec{r}-\vec{r}')$$

$$- \frac{\mu_0}{4\pi}\int dV'\,(\vec{J}(\vec{r}')\cdot\vec{\nabla})\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}(\vec{r}) - \frac{\mu_0}{4\pi}\int dV'\,(\vec{J}(\vec{r}')\cdot\vec{\nabla})\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\frac{\partial}{\partial x_i}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\frac{\partial}{\partial x_i'}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\frac{\partial f(x-y)}{\partial x} = \frac{\partial f(x-y)}{\partial(x-y)}\frac{\partial(x-y)}{\partial x} = f'(x-y)\,1$$

$$\frac{\partial f(x-y)}{\partial y} = \frac{\partial f(x-y)}{\partial(x-y)}\frac{\partial(x-y)}{\partial y} = f'(x-y)\,(-1)$$

$$\int dV'\,(\vec{J}'\cdot\vec{\nabla})\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\int dV'\,(\vec{J}'\cdot\vec{\nabla}')\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$= -\int dV'\ J_i'\frac{\partial}{\partial x_i'}\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)$$

$$= -\int dV'\left[\frac{\partial}{\partial x_i'}\left(J_i'\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)\right.$$

$$-\underbrace{\left(\frac{\partial J_i'}{\partial x_i'}\right)}_{\vec{\nabla}'\cdot\vec{J}'=0}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\Bigg]$$

$$\boxed{\begin{array}{l}\dfrac{\partial \rho}{\partial t}+\vec{\nabla}\cdot\vec{J}=0 \quad \Leftarrow \text{charge conservation} \\ \dfrac{\partial \rho}{\partial t}=0 \quad \text{in magneto } \underline{\text{statics}}\end{array}} \Bigg\} \; \vec{\nabla}\cdot\vec{J}=0$$

$$=-\int dV' \frac{\partial}{\partial x_i'}\left[J_i' \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right]$$

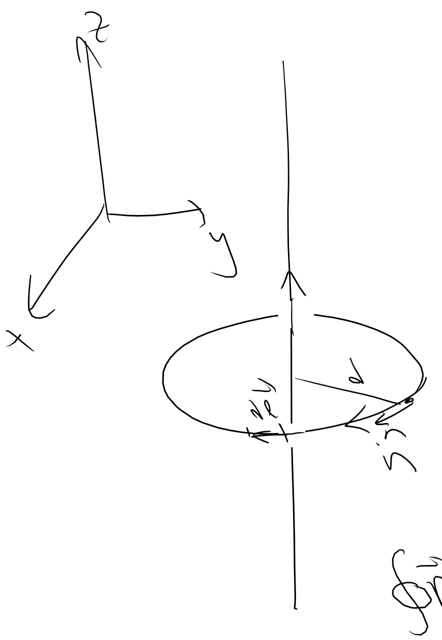
j component of last integral:

$$-\int dV' \frac{\partial}{\partial x_i'}\left[J_i' \frac{x_j-x_j'}{|\vec{r}-\vec{r}'|^3}\right]$$

$$=-\int dV' \,\vec{\nabla}'\cdot\left[\vec{J}' \frac{x_j-x_j'}{|\vec{r}-\vec{r}'|}\right]$$

$$=-\int d\vec{S}'\cdot\vec{J}' \frac{x_j-x_j'}{|\vec{r}-\vec{r}'|}=0$$

$$\boxed{\vec{\nabla}\times\vec{B}(\vec{r})=\mu_0\vec{J}}$$



$s = \sqrt{x^2 + y^2}$

$\vec{B} = ?$

$\vec{B} = \vec{B}(s, \phi, z)$

$|\vec{B}| = B(s, \phi, z) = B(s, z) = B(s)$

rotational symmetry

$\vec{B} = B(s)\,\hat{n} \qquad \vec{dl} = \hat{n}\, dl$

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$

$\oint B(s)\, dl = \mu_0 I_{enc} \Leftarrow$

$B(d) \oint dl = \mu_0 I_{enc}$

$B(d)\, 2\pi d = \mu_0 (-I)$

$$\boxed{B(d) = - \frac{\mu_0 I}{2\pi d}}$$

$$\int_S (\vec{\nabla} \times \vec{G}) \cdot d\vec{A} = \int_{\partial S} \vec{G} \cdot \vec{dl} \Leftarrow$$

$\boxed{\vec{J} = \sigma \vec{E}}$ later

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

$\vec{\nabla} \cdot \vec{B} = ?$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}\,') \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \, dV'$$

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \vec{\nabla} \cdot \left(\vec{J}' \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \right) dV'$$

$$\vec{\nabla} \cdot \left(\vec{J}' \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \right) = \frac{\partial}{\partial x_i} \left(\vec{J}' \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \right)_i$$

$$= \frac{\partial}{\partial x_i} \varepsilon_{ijk} J'_j \left(\frac{x_k - x'_k}{|\vec{r}-\vec{r}\,'|^3} \right)$$

$$= \varepsilon_{ijk} J'_j \frac{\partial}{\partial x_i} \left(\frac{x_k - x'_k}{|\vec{r}-\vec{r}\,'|^3} \right)$$

$$= -\varepsilon_{ijk} J'_j \frac{\partial}{\partial x'_i} \frac{x_k - x'_k}{|\vec{r}-\vec{r}\,'|^3}$$

$$= -\varepsilon_{ijk} \frac{\partial}{\partial x'_i} \left\{ \left[J'_j \frac{x_k - x'_k}{|\vec{r}-\vec{r}\,'|^3} \right] - \left(\frac{\partial J'_j}{\partial x'_i} \right) \frac{x_k - x'_k}{|\vec{r}-\vec{r}\,'|^3} \right\}$$

change method!

$$\vec{B} = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}\,') \times \underbrace{\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}}_{-\vec{\nabla} \frac{1}{|\vec{r}-\vec{r}\,'|}} dV'$$

$$= \frac{\mu_0}{4\pi} \vec{\nabla} \times \int \frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV'$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0 \quad \forall \vec{A}$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

vector potential

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV'}$$

$$\vec E = \frac{1}{4\pi\varepsilon_0} q \frac{\vec r - \vec r_0'}{|\vec r - \vec r_0|^3} \quad ; \quad \vec B = \frac{\mu_0 I}{2\pi d}\hat\phi$$

$$[\varepsilon_0] = \frac{C^2}{m^2 N} \Rightarrow \boxed{[\varepsilon_0] = \frac{C^2}{m^2 N}}$$

$$\vec F = \frac{q^2}{4\pi\varepsilon_0}\frac{\vec r - \vec r'}{|\vec r - \vec r'|^3}$$

$$[\mu_0]$$

$$[\vec F] = [q \vec v \times \vec B]$$

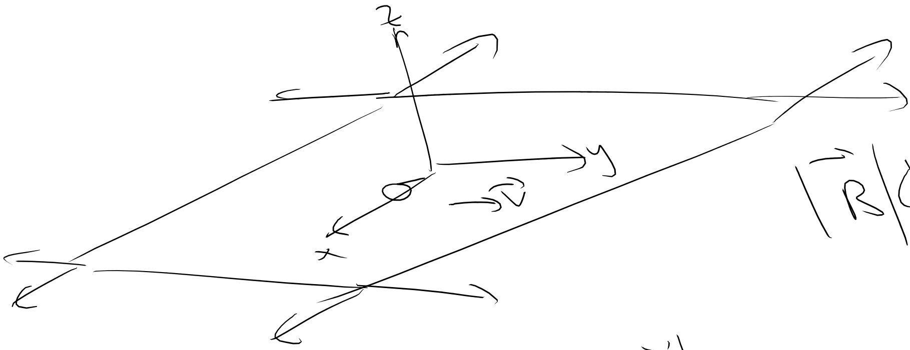
$$\vec F = q(\vec E + \vec v \times \vec B) \Rightarrow [\vec E] = [\vec v][\vec B]$$

$$[v] = \frac{[E]}{[B]} = \frac{\frac{C}{[\varepsilon_0] m^2}}{[\mu_0]\left[\frac{C}{sm}\right]} = \frac{1}{[\varepsilon_0\mu_0]}\left[\frac{s}{m}\right]$$

$$[v] = \frac{1}{[\varepsilon_0\mu_0]}\frac{1}{[v]} \Rightarrow [v] = \frac{1}{\sqrt{[\varepsilon_0\mu_0]}}$$

$\frac{1}{\sqrt{\varepsilon_0\mu_0}} \simeq 3\times10^8$ m/s = speed of light

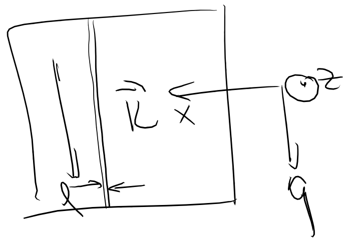
Example $\boxed{\oint \vec B\cdot d\vec l = \mu_0 I_{enc}}$



$\vec v = v\hat y$

v: constant

$|\vec B|(\cancel{x},\cancel{y},z) = |\vec B|(z)$



$$\vec B = \frac{\mu_0}{4\pi}\int \vec J(\vec r') \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}$$