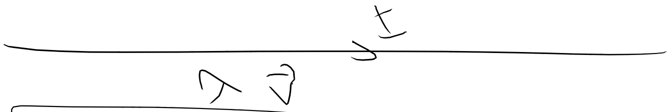
Hand in your HW!

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$



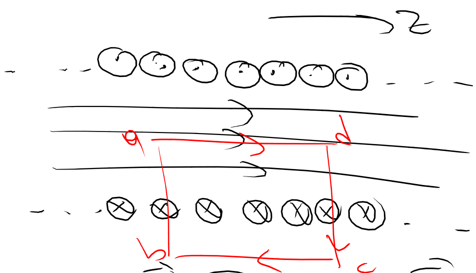
$$\boxed{I = \lambda v}$$

$$I = \lambda_+ v_+ + \lambda_- v_-$$

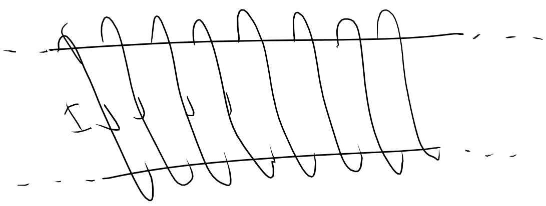
$$v_+ = 0$$

$$I = \lambda_- v_-$$

Example



$\vec{B} = 0$ L



$$\vec{B} = B(s)\hat{z}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint \vec{B}\cdot d\vec{l} = \int_a^d \vec{B}\cdot d\vec{l} + \underbrace{\cancel{\int_d^c \vec{B}\cdot d\vec{l}}}_{0} + \underbrace{\cancel{\int_c^b \vec{B}\cdot d\vec{l}}}_{0} + \underbrace{\cancel{\int_b^a \vec{B}\cdot d\vec{l}}}_{0}$$

$$d\vec{l} = dl\,\hat{z}$$

$$\vec{B}\cdot d\vec{l} = B(s)\,dl$$

$$\oint \vec{B}\cdot d\vec{l} = \int_a^b B(s)\,dl = B(s)\int_a^b dl = B(s)L = \mu_0 N I$$

$$B(s) = \mu_0 \left(\frac{N}{L}\right) I \equiv \mu_0 n I \qquad n \equiv \frac{N}{L}$$

$$\vec{B} = \hat{z} \begin{cases} \mu_0 n I & \text{if} \quad s < R \\ 0 & \text{if} \quad s > R \end{cases} = \vec{\nabla} \times \vec{A}$$

$$\vec{A} = A(s)\hat{\phi}$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA)\hat{z}$$

$$\frac{1}{s}\frac{\partial}{\partial s}(sA) = \begin{cases} \mu_0 n I & \text{if} \quad s < R \\ 0 & \text{if} \quad s > R \end{cases}$$

$s < R$

$$\frac{1}{s}\frac{d}{ds}(sA) = \mu_0 n I$$

$$\frac{d}{ds}(sA) = \mu_0 n I s$$

$$sA = \frac{1}{2}\mu_0 n I s^2 + C$$

$$\boxed{A(s<R) = \frac{1}{2}\mu_0 n I s + \frac{C}{s}} \Leftarrow$$

$s > R$

$$\frac{1}{s}\frac{d}{ds}(sA) = 0 \Rightarrow sA = D$$

$$\boxed{A(s>R) = \frac{D}{s}}$$

$A(s=R)$ is cont.

$$\frac{1}{2}\mu_0 n I R + \frac{C}{R} = \frac{D}{R} \Leftarrow \Leftarrow$$

$A(s=0)$ is finite $\Rightarrow C = 0$

$$D = \frac{1}{2}\mu_0 n I R^2$$

$$\vec{A}(\vec{r}) = \hat{\phi}\begin{cases} \frac{1}{2}\mu_0 n I s & s<R \\ \frac{1}{2}\mu_0 n I \frac{R^2}{s} & s>R \end{cases}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \Rightarrow \int(\vec{\nabla}\times\vec{A})\cdot d\vec{S} = \underbrace{\int \vec{B}\cdot d\vec{S}}_{\Phi_B}$$

$$\| \qquad\qquad$$

$$\oint \vec{A}\cdot d\vec{l}$$

magnetic flux

$$\oint \vec{A}\cdot d\vec{l} = \Phi_B$$

$$\oint \vec{A}\cdot d\vec{l} = \Phi_B \xrightarrow[d\to 0]{} 0$$

$$\Delta(A_{tan}) = 0 \Rightarrow A_{tan} \text{ is continuous.}$$

$$\oint \vec{A}\cdot d\vec{l} = (\vec{A}^{outside} - \vec{A}^{inside})\cdot\hat{l}\, l = 0$$

$$\vec{A}^{inside}\cdot\hat{l} = \vec{A}^{outside}\cdot\hat{l}$$

Coulomb gauge

$$\vec{\nabla}\cdot\vec{A} = 0 \qquad\qquad \vec{A} = \vec{A}_0 + \vec{\nabla} f$$

$$\int(\vec{\nabla}\cdot\vec{A})\,dV = 0$$

$$\|$$

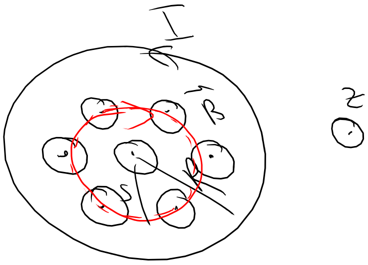
$$\oint \vec{A}\cdot d\vec{S} = 0$$

$$\oint \vec{A}\cdot d\vec{S} \xrightarrow[d\to 0]{} (\vec{A}^{outside} - \vec{A}^{insite})\cdot\hat{n}\, S = 0$$

$$\Rightarrow \boxed{\vec{A}^{outside}\cdot\hat{n} = \vec{A}^{insite}\cdot\hat{n}}$$

$$\oint \vec{A}\cdot d\vec{S} = 0 \qquad \oint \vec{A}\cdot d\vec{l} = \Phi_B$$

$$\oint \vec{A}\cdot d\vec{S} = 0 \qquad \int \vec{A}\cdot d\vec{l} = \Phi_B$$



$$\vec{B} = \mu_0 n I \hat{z} \qquad (s<R)$$

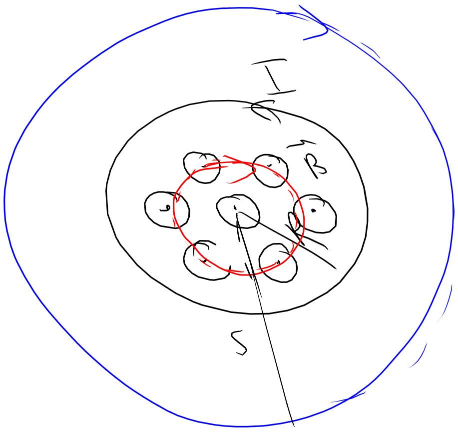
$$\vec{A} = A(s)\hat{\phi} \Leftarrow$$

$$\oint \vec{A}\cdot d\vec{l} = \oint (A(s)\hat{\phi})(dl(-\hat{\phi})) = -A(s)\,2\pi s$$

$$\Phi_B = \int \vec{B}\cdot d\vec{S} = \int (\mu_0 n I \hat{z})(dS(-\hat{z})) = -\mu_0 n I \pi s^2$$

$$A(s)\,2\pi s = \mu_0 n I \pi s^2$$

$$\boxed{A(s) = \frac{1}{2}\mu_0 n I s}$$



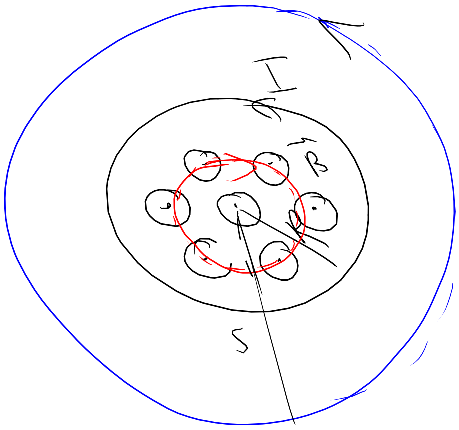
$$\int \vec{A}\cdot d\vec{l} = \int (A(s)\hat{\phi})(dl(-\hat{\phi}))$$

$$= -A(s)\,2\pi s$$

$$\Phi_B = \int \vec{B}\cdot d\vec{S} = -(\mu_0 n I)\pi R^2$$

$$A(s)\,2\pi s = (\mu_0 n I)\pi R^2$$

$$\boxed{A(s) = \frac{1}{2}\mu_0 n I \frac{R^2}{s}}$$



$$\vec A = A(s)\,\hat\phi$$

$$\oint \vec A\cdot d\vec l = \int (A(s)\hat\phi)(dl\,\hat\phi)$$

$$= A(s)\,2\pi s$$

$$\phi_B = (\mu_0 n I)\,\pi R^2$$

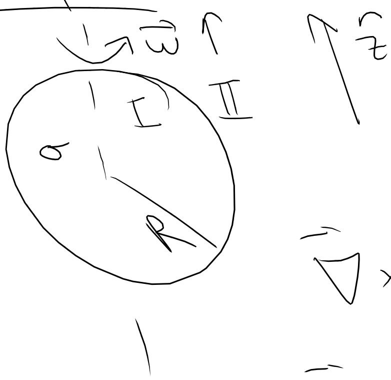
$$A(s)\,2\pi s = (\mu_0 n I)\,\pi R^2$$

$$\boxed{A(s) = \frac{1}{2}\mu_0 n I \frac{R^2}{s}}$$

$$\oint \vec A\cdot d\vec l = \int \vec B\cdot d\vec S$$

$$\boxed{\text{Aharonov-Bohm effect}}$$

Example



$$\vec\nabla\cdot\vec B = 0 \Rightarrow \vec B = \vec\nabla\times\vec A$$

$$\vec\nabla\times\vec B = \mu_0\vec J$$

$\vec\nabla\times\vec B = 0$ inside and outside the sphere

$\vec\nabla\times\vec B \neq 0$ on the sphere

$$\vec\nabla\times\vec B = 0 \Rightarrow \vec B = -\vec\nabla\phi_M$$

$$\vec\nabla\cdot\vec B = -\nabla^2\phi_M = 0 \Rightarrow \nabla^2\phi_M = 0$$

$$\phi_M = \sum \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

$$\phi_M^{I}(r=0) = \text{finite}$$

$$\Rightarrow \boxed{\phi_M^{I}(r,\theta) = \sum_l A_l r^l P_l(\cos\theta)}$$

$$\phi_M^{II}(r \to \infty) = 0$$

$$\boxed{\phi_M^{II}(r,\theta) = \sum_l \frac{B_l}{r^{l+1}} P_l(\cos\theta)}$$

$\vec{\nabla} \cdot \vec{B} = 0 \Longrightarrow \vec{B}_\perp$ is continous

B_r is continuous. $\quad B_r = -\frac{\partial \Phi_M}{\partial r}$

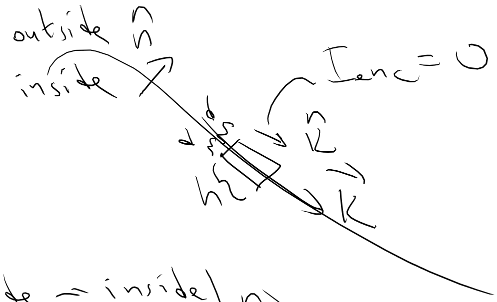
$$B_r^{I} = -\sum_l A_l \, l \, r^{l-1} P_l(\cos\theta)$$

$$B_r^{II} = \sum_l \frac{B_l}{r^{l+2}} (l+1) P_l(\cos\theta)$$

$$\sum_l \left(-A_l \, l \, R^{l-1}\right) P_l(\cos\theta) = \sum_l \left[\frac{B_l}{R^{l+2}} (l+1) \right] P_l(\cos\theta)$$

$$\boxed{-A_l \, l \, R^{l-1} = \frac{B_l}{R^{l+2}} (l+1)}$$

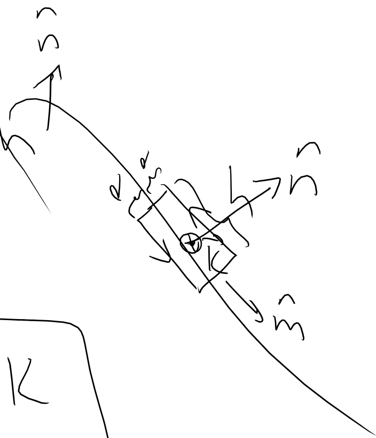
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint \vec{B} \cdot d\vec{l} \xrightarrow[d \to 0]{} \left(\vec{B}^{outside} - \vec{B}^{inside}\right) \cdot \hat{K}\, \cancel{h} = 0$$

$$\Longrightarrow \left(\vec{B}^{outside} - \vec{B}^{inside}\right) \cdot \hat{K} = 0$$



$$\oint \vec{B} \cdot d\vec{l} \xrightarrow[d \to 0]{} \left(-\vec{B}^{outside} + \vec{B}^{inside}\right) \cdot \hat{m}\, \cancel{h}$$

$$= -\mu_0 K \cancel{h}$$

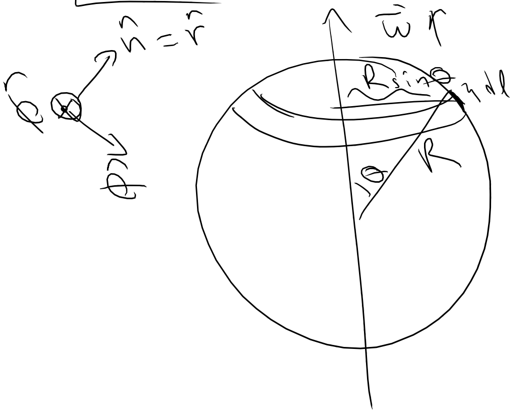
$$\boxed{\left(\vec{B}^{outside} - \vec{B}^{inside}\right) \cdot \hat{m} = \mu_0 K}$$

$$\vec{B}^{outside} - \vec{B}^{inside} = \hat{n}\, 0 + \hat{K}\, 0 + \hat{m}\, \mu_0 K$$

$$= \mu_0 K \hat{m}$$

$$= \mu_0 K \left(\hat{K} \times \hat{n}\right)$$

$$\boxed{\vec{B}^{outside} - \vec{B}^{inside} = \mu_0 \vec{K} \times \hat{n}}$$



$$dI = \frac{(\cancel{2\pi} R \sin\theta)\, dl\, \sigma}{\frac{\cancel{2\pi}}{\omega}} \equiv K\, dl$$

$$K = \omega R \sin\theta\, \sigma$$

$$\vec{K} = \vec{\omega} \times \vec{R}\, \sigma$$

$$\vec{K} = (\omega R \sin\theta\, \sigma)\, \hat{\phi}$$

$$\hat{K}\times\hat{n} = \omega R \sin\theta\, \sigma\, \hat{\theta}$$

$$\vec{B}^{outside} - \vec{B}^{inside} = \mu_0 \vec{K}\times\hat{n} = \omega\mu_0 R\sigma \sin\theta\, \hat{\theta}$$

$$B_\theta^{outside} - B_\theta^{inside} = \mu_0 \omega R\sigma \sin\theta$$

$$B_\theta = -\frac{1}{r}\frac{\partial \phi_M}{\partial\theta}$$

$$B_\theta^{I} = -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\sum_l A_l r^l P_l(\cos\theta)\right)$$

$$B_\theta^{inside} = \sum_l \left(-A_l R^{l-1}\right)\frac{\partial}{\partial\theta} P_l(\cos\theta)$$

$$B_\theta^{II} = -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\sum_l \frac{B_l}{r^{l+1}} P_l(\cos\theta)\right)$$

$$B_\theta^{outside} = \sum_l \left(-\frac{B_l}{R^{l+2}}\right)\frac{\partial}{\partial\theta} P_l(\cos\theta)$$

$$\sum_l \left(A_l R^{l-1} - \frac{B_l}{R^{l+2}}\right)\frac{\partial}{\partial\theta} P_l(\cos\theta)$$

$$= \omega\mu_0 R\sigma \sin\theta = -\mu_0 \omega R\sigma \frac{d}{d\theta}(\cos\theta)$$

$$= -\mu_0 \omega R\sigma \frac{d}{d\theta} P_1(\cos\theta)$$

$\underline{l=1}$ $$A_1 - \frac{B_1}{R^3} = -\mu_0 \omega R\sigma$$

$l\neq 1$ $$A_l R^{l-1} - \frac{B_l}{R^{l+2}} = 0$$

$$-A_l \, l \, R^{l-1} = \frac{B_l}{R^{l+2}}(l+1)$$

$$A_l = 0 \quad B_l = 0 \qquad \text{if } l \neq 1$$