CONVENTIONALISM IN GEOMETRY : AN INSTANCE OF THE IMPACT OF GEOMETRICAL SYSTEMS ON THE PHILOSOPHY OF SCIENCE

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ABSTRACT

CONVENTIONALISM IN GEOMETRY : AN INSTANCE OF THE IMPACT OF GEOMETRICAL SYSTEMS ON THE PHILOSOPHY OF SCIENCE

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Euclidean geometry,as an example of body of knowledge containing necessary a priori truths,dominated and influenced philosophical and scientific communities for more than two thousands years.The axioms of Euclidean geometry as being self-evident truths provide a strong evidence for the rationalism. And it played an important role in the philosophy of Kant;the truth of its axioms was preserved not as a representation of ultimate reality but as a unique possibility in experiencing the world. Briefly Euclidean geometry which was believed that it provides indubitable knowledge in its own field, became at the same time,an undoubted examplar model, a paradigm in attaining the real knowledge of the external world.

The emergence of non-Euclidean geometries as purely

mathematical works has changed the most important presupposition in the background of philosophical doctrines. The emergence of non-Euclidean geometries discarded the rationalist solution about the physical geometry of space and put forward instead an empiricist solution. And unsuccessful attempts to found geometry on empirical grounds necessitate the adoption of the view that there is at least some ingredients in physical geometry so that they can be accepted as true only by convention.

This essay considered mainly the views of three philosophers,-that of Poincaré,Reichenbach and Grünbaum-, who maintained that there is a certain role (more or less) played by conventions in physical geometry.

By examining these three philosophers,I presumed that the conventionalist thesis can be divided into two subtheses in the problem of physical space.The first one , the first-order conventionalism is the conventionalist approach of Reichenbach and Grünbaum maintaining an empirical determination of physical geometry after the conventional choice of congruence.The second subthesis,the second-order conventionalism,is the quasi-Poincarean conventionalism maintaining that even after the physical stipulation of congruence has been fixed conventionally,the metric geometry of physical space is still a matter of convention.

I examined Grünbaum's argument, a counter-example to the Duhemian hypothesis,as a strong support for the first-order conventionalism.And I hope that the failure of Grünbaum's

argument will be shown, and consequently, I hope that I set forth that the first-order conventionalism cannot be a solution to the problem.

The present essay, then, maintains two main theses : firstly the impact of the emergence of non-Euclidean geometries on the formation of conventionalist views; secondly and relatedly the second-order conventionalist approach is relatively the only acceptable claim among the various approaches in the philosophy of space and geometry.

Key words: Convention, Conventionalism, Congruence, Physical geometry of space.

Science Code: 209.01.00.

ÖZ

GEOMETRİDE UZLAŞIMSALCILIK :

GEOMETRİK DİZGELERİN BİLİM FELSEFESİNE ETKİSİNE

BİR ÖRNEK

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Euclides geometrisi zorunlu a priori doğruluklar içeren bir bilgiler bütünü örneği olarak felsefi ve bilimsel çevrelerde 2000 yıl boyunca egemen oldu. Euclides geometrisinin aksiyomları kendiliğinden açık doğrular olarak ussalcılık için sağlam bir dayanak oluşturdu. Kant'ın felsefesinde de Euclides geometrisinin önemli bir yeri oldu; aksiyomlarının doğruluğu bu kez sonul gerçekliğin bir betimi olarak değil, ancak dış dünyayı bilebilmemizin tek koşulu olarak korundu. Kısaca, Euclides geometrisi kendi alanında kuşku duyulmayacak bilgilere ulaştığına inanılan bir dizge olarak, dış dünyanın gerçek bilgisine ulaşmakta da kuşku duyulmayacak bir örnek modeli, bir paradigmayı oluşturdu.

Arı matematiksel çalışmalar olarak Euclides-dışı geometrilerin ortaya çıkışı,felsefi öğretilerin temelinde bulunan en önemli öndayanakların değişmesine neden oldu.Euclides-

dışı geometrilerin ortaya çıkışı uzayın fiziksel geometrisi hakkındaki ussalcı çözümü gözden düşürdü ve yerine deneyci çözümü öne çıkardı. Ve geometriyi deneysel temeller üzerinde kurmak çabalarının başarısız olması, fiziksel geometride en azından bazı ögelerin ancak uzlaşım olarak doğru kabul edilebilecekleri görüşünün kabul edilmesini gerektirdi.

Bu çalışmada, fiziksel geometride uzlaşımların belirlirli (az ya da çok) bir rol oynadıklarını kabul eden üç felsefecinin -Poincaré H., Reichenbach H. ve Grünbaum A. -görüşleri ele alınıyor.

Bu üç felsefecinin görüşlerini de inceleyerek sanıyorum ki, fiziksel uzayın geometrisi sorunundaki uzlaşımsalcı sav,iki altsava ayrılabilir.Birincisi,birinci dereceden uzlaşımsalcılık, yani çakışmanın (congruence) uzlaşımsal olarak seçiminden sonra fiziksel geometrinin deneysel olarak belirlenebileceğini düşünen Reichenbach ve Grünbaum'un uzlaşımsalcı yaklaşımı; ikincisi,ikinci dereceden uzlaşımsalcılık, yani çakışmanın fiziksel belirleniminin uzlaşımsal olarak yapılmasından sonra da, fiziksel uzayın metrik geometrisinin hâlâ bir uzlaşım olduğunu savunan kısmen Poincareci uzlaşımsalcılık.

Birinci dereceden uzlaşımsalcılığa sağlam bir destek olarak görünen Grünbaum'un uslamlamasını -Duhem hipotezine bir karşı-örnek oluşturan uslamlamasını- inceledim. Grünbaum'un uslamlamasının yanlış olduğunu gösterdiğimi,dolayısıyla birinci dereceden uzlaşımsalcılığın soruna bir çözüm olamayacağını ortaya koyduğumu umuyorum.

Böylelikle, bu çalışma iki ana savı savunmuş oluyor: İlkin, uzlaşımsalcı görüşlerin oluşumunda Euclides-dışı geometrilerin ortaya çıkışının etkisi; ve buna bağlı olarak ikincisi de ikinci dereceden uzlaşımsalcı yaklaşımın, uzay ve geometri felsefesindeki diğer yaklaşımlar arasında, görece en kabul edilebilir görüş olduğu.

Anahtar sözcükler: Uzlaşım, Uzlaşımsalcılık, çakışma, Uzayın fiziksel geometrisi.

Bilim Dalı Sayısal Kodu :209.01.00.

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"I am becoming more and more convinced that the necessity of our [Euclidean] geometry cannot be proved, at least not by human reason nor for human reason. Perhaps in another life we will be able to obtain insight into the nature of space, which is now unattainable."

Carl F. Gauss, Werke, Vol.8, p.177.

CHAPTER I

INTRODUCTION

Is the cosmos just what appears to be, i.e. is there any difference between what exists and the knowledge of it? In other words, is any expressed human knowledge a "description"or an "explication" of the world(1). What is the status of scientific theories from the epistemological standpoint?Do they describe the world removing the cover of ultimate reality,i.e. do they completely describe the ultimate things and their relations as really they are,or do they explicate only the world of appearance as we conceive it without touching the secret of ultimate reality and without arguing that this explication is unchangeable.

The present essay is a study on the philosophy of geometry and space, so the general epistemological problems cited hereinbefore will be focused on the problem of knowledge about space and geometry.Does space have a geometry in itself? Does space impose the acceptance of a certain geometry? Can men know the structure of space? Is there true geometry? Are the principles or axioms of geometry a priori truths, or empirical hypothesis, or,arbitrary or guided conventions.

For two thousands years Euclidean geometry was considered as a real description of space , and "Elements" as a book having indubitable truths about space. Then,can any geometry describe the world as it is presupposed in the Euclidean case?

What is the importance of the emergence of non-Euclidean geometries in undermining the sanctity of Euclidean geometry as well as the belief that mind can acquire a priori and indubitable or infallible knowledge about the world?Considering non-Euclidean geometries can we say that Euclid's axioms are true? If they are not true can we show that they are false? Since the emergence of non-Euclidean geometries shows the non-uniqueness of Euclidean axioms,are geometrical axioms empirical hypotheses? If so, are there final objective grounds to test empirically any physical geometry?If not,what is the status of axioms of geometry,i.e.if they are neither a priori nor empirical,are they conventions?

Considering these questions, and trying some possible replies, the present essay maintains two main theses: firstly the impact of the emergence of non-Euclidean geometries on the formation of conventionalist views,and,relatedly the conventionalist approach is relatively the only acceptable claim among the various approaches in the philosophy of space and geometry.

However, I presumed that the conventionalist thesis can be divided to two subtheses in the problem of physical space.The first one , the first-order conventionalism is the conventionalist approach of Reichenbach and Grünbaum maintaining an empirical determination of physical geometry after the conventional choice of congruence. The second subthesis, the second-order conventionalism, is the quasi-Poincarean conventionalism maintaining that even after the physical stipulation of congruence has been fixed conventionally, the metric geometry of physical space is still a matter of convention. I hope that I showed that the first-order conventionalism cannot be a solution to the problem and the only approach to support among the others is the second- order conventionalist approach.

For these purposes, I examined briefly the Euclidean geometry, its history, its paradigmatic importance as a cosmological world view. I examined, then, briefly the emergence of non-Euclidean geometries which were pure mathematical works at the beginning. Later, I tried to indicate why and how these pure mathematical works are important in the formation of new philosophical tendencies such as conventionalism. And, finally, I tried to summarize uptodate discussions on the conventionalist version of space and geometry concluding that why the second-order conventionalist approach is relatively the only acceptable claim among the others.

CHAPTER II

EUCLIDEAN GEOMETRY : ITS HISTORY and IMPACT

2.1. Introduction :

Euclidean geometry or Euclidean cosmology has dominated the conception of the world of man for more than two thousand years.Its influence is not only essential in the formulation of physical theories, but also in the construction of philosophical systems. The domination and influence of this one of the most effective paradigm in the history of thought is continued until the time when the non-Euclidean geometries are constructed and approved by scientific and philosophical communities.

The first thing to be considered, then,is the characteristics of Euclidean geometry or Euclidean cosmology which make it so powerful and influential.

2.2. Euclidean geometry and its history :

There are certain hypothesis about the foundation of Euclidean geometry and its axiomatic method. According to Proclus, Euclid's Elements is a systematic collection of the works of ancient mathematicians. "Euclid,who put together the Elements,collected many of the theorems of Eudoxus.He perfected many of the theorem of Theatetus.."(2).However,Euclid did not collect previous theorems in a random way, instead he organized them so that all the theorems of his `Elements'can be deduced logically from a few number of definitions and axioms Euclidean geometry was the first axiomatic system.

For the hypothesis about the origin of axiomatic method one can be mentioned the thesis of Szabo who argued that Euclidean axiomatic has taken its origin from the Eleatic dialectic.Szabo says :"My problem is to explain the change in the criterion of truth in mathematics from justification by practice to justifications by theoretical reasons.My solution is that this change was due to the impact of philosophy, and more precisely of Eleatic dialectic upon mathematics."(3). Szabo reminding that the Greek word `axioma' originally means `request',explains that in a philosophical debate one partner requested the other to accept his assertion as the starting point.

A detailed history of Euclidean geometry is far beyond the scope of this essay and also not necessary,however ,since my concern is on the cosmological background of Euclidean geometry,I will tell some opinion about its origin.

Euclid's Elements contains thirteen books. Book I starts with the list of definitions, axioms and postulates.In that period, there was a distinction between postulates and axioms. By axioms they understand common notions which are basic to all science (for example :"The whole is greater than its parts) and by postulates, some self-evident statements of a specific field, like geometry(4). Euclid's postulates (I will call hereafter Euclid's axioms.) are as following :

1- A straight line can be drawn from any point to any any other point.

2- Any straight line can be extended continuously in a straight line.

3- Given any point and any distance, a circle can be drawn with that point as its center and that distance as its radius.

4- All right angles are equal to one another.

5- If a straight line crosses two other straight lines so that the sum of the two interior angles on one side of it, is less than two right angles, then the two straight lines, if extended far enough, cross on that same side.(5)

The basic characteristic common to all axiomatic systems is to choose some primitive or undefined terms, so that all the other terms of the system could be defined by means of these primitive terms, and, to choose some basic statements or axioms taken without proof so that all the other statements or theorems of the system could be proved by means of these definitions, axioms and derivation rules. The fixation of undefined terms and axioms serves to avoid an infinite regress in the process of demonstration.

Since Aristotle, it was known that in a syllogistic argument the truth of the conclusion is logically related to the truth of premisses and there is no need to test empirically any conclusion if it is accepted that such a conclusion is followed logically by true premisses. It was believed that Euclid's axioms were true, self-evident premisses and consequently Euclid's system was considered as a true system concerning truth about the structure of space and the universe. The geometry of space became then,a rational science without requiring any appeal to empirical justification.

Euclid's axioms provided the requirements to deduce all the theorems of his system although these requirement were not logically sufficient (6). For the criteria about the selection of Euclid's axioms, two points can be mentioned : firstly, for a better integration of previous theorems before Euclid into a single set of statements; this is the mathematical requirement, and, secondly, historical and philosophical situation and problems of that period.

The philosophy of Pythagoras who had the aim to built up a cosmology based on numbers, had been impacted by the emergence of irrational numbers.This purely mathematical work (discovery of irrational numbers)undermined the philosophy of Pythagoras(in that time this work might be seen as cosmological).Plato, being the witness of Pythagorean disaster, tried to found his cosmology on geometry (7). The properties of space believed to be depicted by Euclidean geometry, were the properties of space in itself (of absolute space) and were not related to matter and motion. It was unchangeable and therefore timeless structure. I suppose that there is a connection between this timeless structure of Euclid and the timeless realm of ideas of Plato(8).The sprit of Plato's philosophy and cosmology can be felt in the selection and formation of Euclid's axioms. Contrary to the Aristotelian scheme of universe formed by spheres of finite size, Euclid's second and fifth axioms imply an infinite universe as philosophically founded by Archytas, Plato's contemporary(9). Furthermore, the mentioned common notion of Euclid (The whole is greater than its part),was a reply to one of the Zeno paradox (A time interval is equal to its half). In that period this paradox could not have been refuted. May be for that reason Euclid included this common notion to his system as a statement which its proof is not required(10).

Also, necessary methodological idea for constructing Euclidean geometry came from Aristotle who held that at the foundation of all knowledge some self-evident truths must be founded so that the truths of theoretical sciences might be deduced from them.According to Aristotle self-evident truths had to be defined so that no properly educated man could deny them.

What this brief outlook shows is that the Euclidean system is not only a limited geometrical system, but rather a cosmological system as well trying to give certain answers to previous philosophical problems, and, also it proposes a certain cosmological view.

The power and consequently the source of the influence of Euclidean geometry comes from its provability. Although there is no single proposition which could be refuted by experience,i.e. all the propositions seems to be empirically true,there is no need to justify them empirically;all the theorems of Euclid can be proved logically (some logical defects of the original Euclidean system must be considered).

In the next sections of that chapter, the impact of Euclidean geometry will be considered. Henceforth, I will use Euclidean geometry and Euclidean cosmology almost synonymously but with the following distinction : Euclidean geometry as the mathematical and physical space implied by Euclid's system,and, Euclidean as the impact of Euclidean geometry on philosophy,physics and on general conception of the world of man.

2.3 The Impact of Euclidean Geometry :

There are several historical examples to illustrate the impact of Euclidean cosmology on the conception of the world of man.As it is stated earlier,the power of its influence comes the idea that although all the assertions of the system may be empirically true,there is no need to experience or to any observation for their justification. Their truths could be obtained by deductive proof basing on self-evident axioms and logical laws.Until the time of non-Euclidean geometries,there was no doubt about the self-evident truth of Euclid's axioms including the fifth one. Among all the theorems which are derived from self-evident truths, there was no a single one which can be refuted by experience and observation. It was believed that the body of these statements contains the truths of mind as well as the truths of nature and reality. The powerful support of Euclidean cosmology as a body of knowledge concerning indubitable truths, was a fruitful example for all other works of humanity, and, one of the consequence of Euclidean cosmology is the idea to use mathematical method in the search for truth.

Usually,modern philosophy up to Kant has been divided into two camps,namely Continental Rationalism culminately represented by Descartes, Spinoza and Leibniz;and,English Empiricism represented by Locke, Berkeley and Hume. For both camp Euclidean geometry played an important role in the formation of philosophical ideas.As Davis and Hersh say:"For the rationalists, mathematics was the best example to confirm their view of the world.For empiricists it was an embrassing counterexample which had to be ignored or somehow explained away."(11). By and large, the central philosophical doctrines which preceded the 19^{th} century could not be constructed without accounting for Euclidean cosmology.

Descartes was looking for indubitable, infallible knowledge to build up a system of thought on it.For Descartes the source of this indubitable knowledge can be attained rationally and applying a mathematical method (only the method, not the mathematics itself)(12).Spinoza's most important work had the title "Ethica Ordine Geometrica Demonstrata". It was an attempt to express his philosophical ideas by using the method of Euclidean geometry. In fact, 'more geometrico' was being used synonymously with 'more logico'.According to Leibniz,geometric truths are "innate,and in us virtually, so that we can find them there if we consider attentively and set in order what we already have in mind, without making use of any truth learned through experience or through the tradition of another."(13). Leibniz considered metaphysics as 'Mathesis Universalis' where the laws of logic are the laws governing the realm of being. He also envisaged, using the heritage of Aristotle as well as that of Euclid,to construct a 'Characteristica Universalis' so that it would be possible to discover a number of simple concepts by means of which all other concepts could be expressed and assertions could be proved.It is also interesting to state Leibniz's successor Wolff's own words to understand the spirit of the century:

> "The principles of philosophy must be derived from experience.The principles are demonstrated by experiments and observations.Also philosophy must use mathematical knowledge. For in philosophy we wish to have complete certitude...The rules of philosophical method are the same as the rules of mathematical method.. Finally,in many cases,complete certitude depends on mathematical knowledge and demonstrations.And who would deny that those things in philosophy,by which truth is made known,ought to be such that no one could doubt them?"(14)

After this quotation of Wolff which reflects,by and large, the mind of the rationalist philosophers of 17^{th} and 18^{th} century,it is not possible not to acknowledge Kant to be right in writing about Wolff "the greatest of all dogmatic philosophers."(15)

The culmination point of the English empiricism,Hume, exempted only book of mathematics and natural sciences from "committing to the flames"(16).

Briefly,these historical notes shows us how Euclidean cosmology and methodology was influential in philosophical systems up to Kant. I will try to examine the place of Euclidean geometry in Kant's philosophy in the next section. I will consider now the situation in scientific theories of 17^{th} and 18^{th} century.

Although the mathematical method of Euclidean geometry was so influential in philosophical system and in natural philosophy, mathematics itself was not being used in science until Galileo and Newton (Of course, not only geometry but various branches of mathematics were being used in scientific theories of those centuries. I emphasized Euclidean geometry because I am concerned with the cosmological ideas behind the use of mathematics and its method. The paradigmatic importance of Euclidean geometry comes from its cosmological background) Prior to Galileo and then, prior to Newton, mathematics has been studied as a fine art without any consideration to its physical application (excepting some rare and trivial cases such as Archimede's works on hydrostatics). Beginning with Galileo (Galileo's famous dictum: "The great book of nature can be read only by those who know the language in which it is written, and this language is mathematics."), especially with Newton all the background of mathematical knowledge were turned to advantages in the solution to physical problems. Physical problems of the external world, then, are transformed into mathematical problems.

It is accustomed to state that the scientific revolution in the 17^{th} century beginning with Copernicus, is characterized by two complementary feature: first, the unification of physica coelistis and physica terrestris, i.e. scientific laws govern the whole universe,and,secondly the geometrization of space (17), "that is,the substitution of the homogeneous and abstract -however now considered as real- dimension space of the Euclidean geometry for the concrete and differentiated place-continuum of pre-Galilean physics and astronomy."(18). Beginning with Galileo and especially with Newton we can talk about mathematical reconstruction of nature.

The geometry which is considered in the mathematical reconstruction of nature and geometrization of space could not be other than the one which is considered as the body of knowledge concerning indubitable truth for two thousands years, namely the Euclidean geometry. As Nagel pointed out:

> "The Newtonian conception of geometry as the simplest branch of mechanics was based on the tacit assumption that Euclidean geometry is the only theory of spatial relations that can be provide a theory of mensuration."(19)

Newton's first laws ("Every body continues in its state of rest , or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it." (20)),is based on straight line notion which is apprehendable from Euclid's first and second axioms. Nagel says :

> "Neither Newton nor his contemporaries had any reason for supposing that a doubt could arise as two what is to be understood by `straight line' in his formulations of the axioms of motion, for the only theory of geometry known at that time was the system of Euclid. It was therefore taken for granted that a line is straight if it conforms to the conditions specified in Eulidean geometry."(21).

According to Newton, the axioms of geometry were true statements about physical bodies and the geometry was a part of universal mechanics.

> "To describe right lines and circles are problems, but not geometrical problems.The solution of these problems is required from mechanics... Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of mechanics."(22).

Newton's first law, the law of inertia, presupposes the existence of fixed inertial frames providing that the particles continue their uniform motion on a straight line indefinitely. This uniform motion is real motion in Newton's system and must be distinguished from relative motion. This conception necessitates another important concept of Newtonian mechanics,that of `absolute space' (23). "To Newton, absolute space is a logical and ontological necessity. For one thing, is a necessary prerequisite for the validity of the first law of motion..."(24) .

As is stated earlier,the second and fifth axiom of Euclid necessitates the concept of infinity. Koyré states : "Geometrization of space implies necessarily its infinitization:we can not assign to limits of Euclidean space."(25).

It is perhaps not an exaggeration to say, then, that important cosmological considerations such as infinity, absolute space, idealized motion, the shape and the structure of space take their roots from the Euclidean cosmology.As Eddington says:"The only thing,that can be urged against spherical space is that more than twenty centuries ago a certain Greek published a set of axioms which (inferentially) stated that spherical space is impossible."(26)

Naturally, although in the background of Euclidean cosmology, there are numerous thought that of Pre-Socratic philosophers,that of Eleatics, that of Plato, we can announce with Cornford that : "There was a pre-Euclidean common sense, whose conception of the world in space had to be transformed into the Euclidean conception..",Cornford continues his statement with an important remark which will be interesting for the next chapters,"...just as our Euclidean common sense has now transformed into the post-Euclidean scheme of relativity" (27).

2.4. Euclidean Geometry and Kant :

In 1763 The Berlin Academy of Science offered a prize for an essay on the question of "Whether metaphysical truths in general, and especially the first principles of natural theology and of morals, are capable of just the same certain demonstration as are the truths of geometry...?"(28).This question is an interesting one to show the spirit of the century. Euclidean geometry was providing one of the support for the ground in arguing rationalistic views.

Pre-Kantian period was under such effects of the Euclidean cosmology and en plus,Newtonian mechanics was becoming dominant in the domain of natural sciences by its success to `discover'natural phenomena using mathematics and its method.

Kant had certain dissatisfactions both with rationalist conceptions which state that the principles of metaphysics are the same that of logic, and that is possible to obtain genuine knowledge of the world regarding certain laws of logic;and with empiricist views for the explanation of mathematical knowledge,causality principle when mind is considered as a passive receptive on acquiring knowledge of the external world.

In his essay for the Berlin Academy prize (29), Kant made a distinction between argument in mathematics and argument in philosophy. Kant thought that mathematicians starts from definitions and philosophers must work towards definitions.Also,according to Kant, "metaphysics had in the past made the mistake of thinking that it ought to be like mathematics" (30).After such methodological designations, Kant made a distinction between analytic and synthetic judgements, and noted that from analytic premisses can only analytic consequences be obtained,not synthetical ones. Therefore,the principles of logic can not provide by themselves synthetical knowledge, knowledge of the natural world.However,if one starts with non -logical principles, what will be the ground for accepting such principles as true. How, then, can be given account for problematic cases such as geometry which is considered as a body of knowledge concerning a priori and necessary truths and still providing knowledge about space and external world.

Kant was impressed by the success of Newtonian mechanics which use mathematics and mathematical method in the investigation of natural phenomena.He had written that :"..Newton's method in natural science turned the unstable nature of physical hypotheses into a certain method according to experience and geometry."(31)

According to Kant,geometrical truths can not be analytical since they represent the nature of real external entities. Either,they can not be purely synthetical since they provide necessary knowledge of the external world.The concept of 'synthetic a priori' provide Kant to develop a solution to that problem.

In the axioms of geometry, the predicate does not belong to subject, then they are not analytical, therefore they are synthetical. "All mathematical judgements, without exception are synthetic...That 'the straight line between two points is the shortest', is a synthetic proposition."(32). But although axioms are synthetical,they provide necessary statements about the external world, there is no need to empirical justification for the truth of any geometrical theorem, i.e. they are a priori.

> "For geometrical propositions are one and all apodeictic,that is,are bound up with the consciousness of their necessity; for instance, that space has only three dimensions. Such propositions can not be empirical or,in other words, judgements of experience, nor can they be derived from any such judgements."(33)

That leads to the consequence that the axioms of geometry are synthetic a priori truths.

> "How,then,can there exist in the mind an outer intuition which precedes the objects themselves,and in which the concept of these objects can be determined a priori ? Manifest-

> ly, not otherwise than in so far as the intuition has its seat in the subject only, as the formal character of the subject, in virtue of which, in being affected by objects, it obtains immediate representation, that is, intuition of them; and only in so far, therefore, as it is merely the form of outer sense in general. Our explanation is thus the only explanation that makes intelligible the possibility of geometry, as a body of a priori synthetic knowledge."(34)

Kant's theory of space is different from the two rival theories of space, that of Newton's and Leibniz's theory of space. According to Newton, there is a real absolute space. The objects are inside the receptable (or container) space, they do not constitute this receptable space but they acquire their existence in this container space. Container space is absolute and different than relative space (35). On the other hand, Leibniz had denied such a real and absolute space. He was holding a relational theory of space, whereby space is merely a system of relation in which indivisible substances or 'monads' stand one another. Leibniz also held that we arrive to the concept of space by experience.

> "I will here show, how men come to from to themselves the Notion of Space. They consider that many things exist at once, and they observe in them a certain Order of Co-existence according to which the relation of one thing to another is more or less simple. This Order is their Situation or Distance."(36)

Kant is distinguished from both of them. He denies that we can arrive to the conception of space by experience as Leibniz hold. Instead he says:

"Space is not an empirical concept which has been derived from outer experience... The representation of space cannot,therefore,be empirically obtained from the relations of outer appearance."(37)

Although he is closer to Newton's conception of absolute space,he denied to give real existence to absolute space. He showed in the first Critique how to consider space and time as objectively real,led to antinomies.

According to Kant, the concept of space is not an empirical one.One cannot arrive to the idea of space by experiencing facts and objects. On the contrary, no experience is possible without the concept of space. Space is a form of a priori external intuition, for this reason, "[g]eometry is a science which determines the properties of space synthetically,and yet a priori."(38). We can know the external world as we conceive it by a priori forms of our mind.What we observe, apart the real properties of external world,is our properties of mind in experiencing the world. Since the form of conceiving the world is originated from us, the principles of geometry being the formal expression in conceiving space,must be true and necessary.

We have seen how Euclidean geometry and cosmology,and epistemological roots of the axioms of geometry constitute an important side of Kant's philosophy.

Most of the recent philosophers have agreed on the view that the emergence of non-Euclidean geometries has under mined Kant's conception of synthetic a priori. It can be thought, however that Kant's consideration of synthetical parts of geometry leaves room for alternative non-Euclidean geometries.Kant's 'synthetic a priori' version of geometry does not preclude the mathematical possibility of non-Euclidean geometries.The mathematical impossibility of non-Euclidean geometries means that they can be self-contradictory and then,to be self-contradictory will imply that are analytical; and consequently that Euclidean geometry is analytical. Kant had not such an intention.But,since all our possible experience of space is represented by Euclidean geometry, for Kant we have no way to experience the world in a non-Euclidean way

One side of great importance of Kant,I think,is lying in his way of explanation the role of mathematics in natural sciences,i.e. the relation between a priori truths of geometry and mathematics, and empirical knowledge of the external world.In this way,he put some fundamental stones of the bridge which leads later to conventionalistic approach in the modern philosophy of science. Moreover, Kant can be seen as an important avant-garde of conventionalistic approach, although he was not a conventionalist. In the next chapter, that point will be reconsidered.

CHAPTER III

NON-EUCLIDEAN GEOMETRY : ITS HISTORY and IMPACT

3.1. Introduction :

The emergence of non-Euclidean geometries is the result of purely mathematical works. As it will be seen in detail in the next section, the doubt about the deducibility of the `parallel axiom' from the other axioms, leads to the construction of internally consistent (regarding to Euclidean geometry) geometrical systems which are independent of Euclidean geometry.

The impact of the emergence of non-Euclidean geometries and their approval by mathematical and scientific communities were so profound that we can name this impact regarding to its consequences as a revolution in the domain of mathematics,science, philosophy and in general world view of man.The reason of why those purely mathematical works had had revolutionary consequences in general, comes from the cosmological influence of Euclidean geometry both in the domain of science and philosophy. That is the corollary for showing that Euclidean geometry is not only a geometrical system,but a cosmological world view as well.

After this revolution,the concept of truth, the range of necessary statements,the foundation of mathematics and geometry,the relation between mathematics and physics had to be reexamined and revised.Since the legitimacy of consistency of non-Euclidean geometries was approved,it was not possible to take Euclidean geometry as the only true one. The conception of unconditional truth of Euclidean geometry is being changed by the conception of alternative geometrization of space. The truth of Euclidean geometry ,i.e. the truth of its axioms was no longer self-evident or a priori nor synthetic a priori.It remains no reason to consider its statements as necessary statements. This situation led to the idea that the decision for the truth of alternative geometrical systems could be established by experimental methods. However,as it will be seen in this chapter, empiricist approach could not provide permanent solution to the problem. The epistemological problems of geometry and space raised after the emergence of non-Euclidean geometries, has forced to pose certain alternative views to rationalism and empiricism,namely the conventionalist approach.

3.2. Non-Euclidean Geometry and its History :

Euclid's common notions and first four axioms were simple and easy to admit as self-evident truths.They were coherent with immediate experience and with common sense. However, the fifth axiom was not seem to be self-evident as the others. One reason for that may be the concept of infinity which is implicitly mentioned at the fifth axiom and which goes beyond the limit if immediate experience. After the time of Euclid,many mathematician tried to deduce the fifth axiom from the other axioms instead of accepting it as self-evident truth, and, until about 1820 great efforts had been made to prove the fifth axiom as a theorem resulting from the other four axioms.

Works to eliminate doubts about the fifth axiom,started with Ptolemy (second century A.D.) and Proclus (fifth century A.D.) and continued with Saccheri and Lambert in the 18^{th} century. It is interesting to note that in all these works there was not a single sentence about the doubt for the truth of the fifth axiom. We understand that during all these works of proving the fifth axiom, the belief to the truth of it,is preserved. Mathematician had tried only to deduce it from the other axioms without interrogating its truth or falsity.

Saccheri's works is interesting here to mention.Saccheri tried to prove the fifth axiom by considering its negation and seeking for a contradictory statement in the system, and,by that way he became closer to non-Euclidean geometries. But,since he never got a contradiction,he concluded that the fifth axiom is a necessary truth because it follows even from its own negation.That was a remarkable result showing the paradigmatic importance of Euclidean cosmology.

It was clearly and certainly understood that the fifth axiom is not a theorem of Euclidean system but an independent axiom as the others, only after the independent works of Lobatchevsky, Bolyai and Gauss.If the fifth axiom were a theorem.contradictions should have found when the negation of the fifth axiom is added to the other four. Since no contradiction has been found the fifth axiom cannot be a theorem. All the three founders of non-Euclidean geometry have noticed that when the fifth axiom (Through a given point P not on a given line L, there is only one line in the plane of P which does not meet L (39)) is replaced by its negation (Through a point P on a line L,there are more than one line in the plane of P which does not meet L (That is the case of hyperbolic geometry)),new geometrical systems can be constructed. A different parallel axiom has enabled them to prove several different theorems which are incompatible those of Euclid's.

Although non-Euclidean geometry was ignored at the beginning, a thirty years after the first publication of Lobatchevsky's paper,the idea of non-Euclidean geometry became to be acceptable for mathematical and scientific communities. Riemann,later,developed a new kind of non-Euclidean geometry ,elliptical geometry,by denying both the second and the fifth axioms.

Although proofs of some theorems of non-Euclidean geometry had been established without leading to any contradiction, that situation was not sufficient to conclude that no contradiction would be ever founded. Were non-Euclidean geometries internally consistent?This problem of consistency had been solved by Felix Klein.He showed that hyperbolic geometry is consistent if Euclidean geometry is. He established the consistency of present non-Euclidean geometries by devising a certain type of geometrical dictionary in which the terms of Euclidean geometry were corresponding to the terms of non-Euclidean geometry. By that way, a relative proof of consistency was being established, i.e. accepting Euclidean geometry as consistent. Later Hilbert made a relative proof of consistency of Euclidean geometry regarding to arithmetic, considering that arithmetic is consistent.

The consistency proof of non-Euclidean geometry raised it to the level of Euclidean geometry in mathematical sense; there are more than one system which are internally consistent. But, if Euclidean geometry and non-Euclidean geometry are both consistent, is it possible, then, to consider both systems are true, i.e. can one argue that Euclid's parallel axiom and its negation are both true? If not, which system was the true geometry of space? It has been understood that within the domain of mathematics alone these questions could not be answered. Mathematics can only decide to the consistency of systems, not to their truth. The idea of distinction between the formal and physical character of mathematics, and, between consistency and truth are the other important consequences of non-Euclidean geometries.

3.3. <u>The Impact of Non-Euclidean Geometry :</u>

In general, it was usual until Kant to think both for philosophers and scientists that the properties which are attributed to objects and their relations, are real properties of objects and their relations. It had been thought that what was explicated, was described too, and, hence mind can remove the cover of reality. (Certain empricist objections can be considered against that unification before Kant.The principle of causality,for example,was held as an ontological necessity for for rationalists and realists. For Hume, the principle of causality was neither a real property governing the realm of objects nor a property of human mind; Hume considered it as a psychological assumption being a result of our habit.)

Kant, emphasizing the role of universal properties of human mind in conceiving the world,proposed and prepared the way of thought that the order which we seek in the world is not independent of human mind.Also,Kant, emphasizing the role of human mind in experiencing the world,distinguished a realm of phenomenon which is subject to experience, and a realm of noumenon, `things in themselves', which are outside of space and time, and consequently which can not be subject to any possible experience.The knowledge about the object of possible experience,phenomena,can be interpreted then,as an explication as mind perceives and conceives phenomena.

However,in Kant's philosophy the roots and possibility of those explications remain invariant because there is no any other way to experience the world.Although in Kant's philosophy,Euclid's axioms were no more the description of the external world, alternative explications were not possible either,because Euclidean geometry was the only formal expression of our outer intuition,viz. space.

The emergence of non-Euclidean geometry have changed these considerations.Since it was known that there are alternative geometries as consistent as Euclidean geometry, it became necessary to give up from the conception that the truth of any geometrical system is attainable by a priori cognitions of mind.

However,although the mathematical possibility of non-Euclidean geometries were approved, there was no evidence yet that those geometries could be used in physical theories, or in the explication of physical phenomena.

In the first decades after the approval of non-Euclidean geometries some attempts have been made to decide whether space is Euclidean or non-.Euclidean.But during all these attempts geometry was still considered as an independent science without considering the assumption of physical theories which are inherent in geometrical systems. And, to such questions as:"How we decide which geometry is the real geometry of physical space?",the founders of non-.Euclidean geometry, Lobatchevsky,Gauss and later Riemann, Helmholtz and Clifford answered clearly;by experience.

Gauss made for example such a `crucial' experiment by observing three tips of distant mountains to see whether the sum of internal angles of this triangle is equal to two right angles or not.The result is approximately 180° within the limits of error.Of course,such naive observations were far from to be conclusive, for these experiments and observations had made by presupposing Euclidean and physical(or philosophical) assumptions such that light rays travel along straight lines, or measuring instruments do not alter their original dimensions when their locations are changed (the axiom of free mobility was presupposed).

Also, observations of stellar parallax are based on similar presuppositions.On the assumption that space is Euclidean, i.e. flat, the parallax will be a small observable angle.If space is Lobatchevskian,i.e.hyperbolic, the parallax will be a little greater.If space is Riemannian,i.e. elliptic or spheric, this parallax will be less or negative.Hence, any interstallar observation for finding out the sum of internal angles of a triangle formed by a distant star and by two locations of the world which is symmetric according to the sun,would not be tested independently of those presuppositions so that each set of assumptions will result his own findings.These examples show that it will be an error to assume that the test for deciding the truth of any geometrical system can be done apart from its physical content. Historically,carrying the traces of spatial intuition and immediate experience,geometry was the first physical theory, and, consequently,since the notion of space is essential for any physical theory, geometry formed the base of other successive physical theories. Any interpreted geometrical system is not independent form physical theories and their assumptions,and, any physical theory is not independent from interpreted geometrical systems and their assumptions.

Riemann, on the other hand, although he adopted under the influence of his teacher, Gauss, the empiricist account about the decision of true geometry of physical space, he had another point of view which will constitute the `point de de-part' for conventionalistic approaches, especially that of Grünbaum's.

Riemann, in his famous inaugural dissertation "On the Hypotheses Which Lie at the Foundation of Geometry" (40), in 1854, introduced certain new and fruitful concepts to discussion about geometry and space. For example, the problem of discrete and/or continuous structure of space and the problem of measurement related to them; intrinsic and extrinsic features of space (intrinsic features of any space (or manifold) are those that could be determined only in that space without considering that space is embedded in a higher dimensional space; extrinsic features are the properties of this embedding); the problem of metric, intrinsic and/or extrinsic metric. The approach to these concepts signifies a new understanding.

According to Riemann, the approach to the question of intrinsic or extrinsic metric will relate the problem of physial space to experimental and observational results. Riemann must have had implicitly the idea that the approach for the existence of `intrinsic metric, or its non-existence has to be fixed before experiments and observations, for such experiments and observations have meaning.

The concept of `intrinsic metric' and the problem of alternative metrizability as being one of the central concept of that essay, and also the problem of discrete space and continuous space will be discussed later in detail. Now, I will consider briefly the crisis in the epistemology of geometry and its influence to the other domain of philosophy and science after the emergence of non-Euclidean geometries.

The scientific paradigm of 19^{th} century maintained that Newtonian mechanics having geometrical base of Euclid,s system in the mathematical formulation of nature, is the real description of ultimate entity.The belief that Euclidean geometry represents the ultimate reality of things,i.e. there is a correspondence between the ultimate reality and the principle of Euclidean geometry,was undermined by the approval of consistency of non-Euclidean geometries though they are still considered as only a mathematical possibility. But the doubts about the uniqueness of Euclidean geometry after the approval of non-Euclidean geometries prepared the way for the doubts about the uniqueness of Newtonian mechanics and its geometrization.There could be alternative possibilities for representing the realm of reality.By this way,the distinction between form and content became explicit. First prerequisite for any representative system of the world is to be internally consistent.One more dictum,then,by non-Euclidean geometries -may be historically taking its roots from Parmenides- that `logical coherence is essential to any explanation of the world' (41).Accordingly, which internally consistent and physically interpreted theoretical system is the true representation of the world? The scientific paradigm of late 19^{th} century maintained that this endeavour should be carried out by scientists.

Maxwell's (1831-1879) electromagnetic theory is constructed in this spirit. Maxwell's theory is a collection of consistent set of differential equations. A similar attempt had been done by Heinrich Hertz (1857-1894) in his`Principles of Mechanics'. Toulmin and Janik describe that situation as follows:

> "It occurred to Hertz that,in actual fact,Maxwell was saying nothing at all about the physical nature of these phenomena.His equations were logical formulas which enabled him to deal with the phenomena and to understand how they operated.In short,Hertz realized that `Maxwell's theory is Maxwell's system of equation'.He thus became aware that mathematical formulas could provide a framework for dealing with all the problems of physics,and,so, confer a logical structure on physical reality."(42)

Hertz's book consists of two parts.In the first part, mathematical framework of the system is given and it is physically uninterpreted,and definitions, axioms, proofs of the propositions are also given. The second part is the physical interpretation of the first part as a system of mechanics.The comments of Hertz for his own book are so interesting and also influential both for convetionalist philosophers, especially for Poincaré, and for logical positivists that I quote some passages from it.

In his prefatory note to `Principles of Mechanics', Hertz writes:

> "The subject-matter of the first book is completely independent of experience. All the assertions made are a priori judgements in Kant's sense.They are based upon the laws of internal intuition of, and upon the logical forms followed by, the person who makes the assertions;with his external experience they have no other connection than these intuition and forms may have."(43)

In concluding note of the first part he writes:

> "It is true that the formation of the ideas and the development of their relations has only been performed with a view to possible experiences; it is true none the less true that experience alone must decide on the value or worthlesness of our investigations. But the correctness or incorrectness of these investigations can be neither confirmed nor contradicted by any possible future experiences."(44)

Although there is a structural similarity between Newton's Principia and Hertz's book, especially with its second part,there are important differences from the epistemic point. Newtonian mechanics was considered as the representation of the structure of ultimate reality(Indeed,Newton,himself did not consider his system of mechanics as a discovery of laws of nature; but after him the essentialist interpretation became dominant until to critics made by philosophers and scientists at the second half of 19^{th} century.). On the other hand,as we understood from Hertz's prefatory note,Hertz had not such a contention, his claim is that his system of mechanics is constructed independently of experience; it was only a net,one of the possible nets. As Wittgenstein says who is profoundly influenced by Hertz's work:

> "The network,...,is purely geometrical;all its properties can be given a priori.Laws like the principle of sufficient reason, etc.are about the net and not about what the net describes."(45)

Kantian influence in Hertz's work is clear, but Hertz also is the witness of non-Euclidean geometries. Hertz's comments on his book leaves room for alternative formulation of physical phenomena.Hertz was thinking for his system as being one of the possible formulation and so for Maxwell's system. It will not be wrong,then,to argue that the idea of alternative formulation of physical theories is the heritage of already realized and approved alternative non-Euclidean geometries for which the philosophical roots of their applicability were established by Riemann and suggested by W.K. Clifford(46).

Besides the doubts about the uniqueness of Newtonian mechanics as a whole, some important concepts of Newtonian mechanics, like the concept of absolute space were also criticized by Ernst Mach (1838-1916).According to Mach, physical theories with their metaphysical concepts as in the case of Newtonian mechanics (absolute space, absolute time, `absolute motion'are all metaphysical concept for Mach and they must be eliminated from physical theories) can not represent the ultimate reality of things. For Mach metaphysical conceptual elements are mere `embellishments' which have no bearing on ascertainable facts(47).

Mach seems to be committed to the reality of things and to their sensory data but not committed to the reality of physical theories in which he was finding certain metaphysical concept, i.e. unobservable, which are not the outcome of immediate experience, and therefore they had to be eliminated from physical theories. According to Mach, physical theories are mere instruments serving to make predictions about phenomena. By this considerations the physical terms correspond to the things of phenomena,but theories do not correspond to ulmate reality, i.e. we can change our theory for any other one for better predictions of phenomena, and for better `economy of thought'.(48)

I think that the influence of the emergence of non-Euclidean geometries can be added to Toulmin's remark:

> "Neither Planck nor Mach, at any rate, was in any mood to claim that the science of physics can arrive directly at any real knowledge of `external reality' or `things-in-themselves'; this much caution Kant had taught them both."(49)

Kantian views together with the possibility of non-Euclidean geometries have the impact on both to the philosophy of science and to the formulation of physical theories since 1850.

If we summarize the historical perspective so far, we see the following scheme: Euclidean geometry was considered until the first half of the 19^{th} century as a body of knowledge containing necessary truths which are a priori and independent of experience. The axioms of Euclidean geometry as being self-evident truths provide a strong evidence for the rationalism that mind can acquire the knowledge of ultimate reality. And, with Kant, the principles of Euclidean geometry became the formal expression of a priori external intuition in experiencing the world, like the unremovable glasses so that without them no experience is possible. The sanctity of axioms are preserved this time not as a representation of ultimate reality but as a unique possibility in experiencing the world.

Mathematical works which lead to non-Euclidean geometries have changed most important presuppositions in the background of philosophical doctrines, in the conception of `natural philosophy'. By the negation of at least one of the Euclid's axioms, it became possible to construct logically consistent geometrical systems as Euclid's system. Neither Euclidean geometry was the unique system which describes the real structure of space nor its statements were universal truths.A belief of two thousands of years has been destroyed.

The loss of certainty in geometry has had mathematical, scientific and philosophical impact. The understanding of that the foundation of mathematics which was believed so firm and sound,are not so accurate indeed,made turned certain mathematician from geometry to arithmetic in the foundation of mathematics.The works of Weierstrass, Dedekind and Peano had that aim.And set theory constructed by Cantor requires no recourse to geometrical intuition. Needless to say, these details are far beyond the scope of this essay.

Furthermore, Newtonian mechanics which is considered as unique and true description of the universe has had in its base the unique Euclidean geometry. The loss of uniqueness of Euclidean geometry has implied the probable non-uniqueness of Newtonian mechanics.

Briefly, using a Hegelian terminology, Euclidean geometry provide the thesis:the rationalist account of geometry; non- Euclidean geometry provide the empiricist anti-thesis, and,the synthesis came from the conventionalist approach.

In the next chapter,I will examine how these developments in geometry and consequently in scientific theories lead to conventionalist approaches.

CHAPTER IV

CONVENTIONALISM in GEOMETRY

I. Introduction :

Are Euclid's axioms true ?If they are true from where comes their truth ? Can we justify their truth ? If they are wrong,can we show that they are wrong ? If reality is described by a certain geometry,can the Euclidean and the non-Euclidean geometries be both true ? Which geometry is the true and real geometry of space ? If Euclidean geometry is not the true one,what will be the status of Newtonian mechanics ?

These are general questions raised after the emergence of non-Euclidean geometries and their approval by scientific communities. Even the possibility of asking such questions proclaimed the discard of the rationalistic solution. It was no more possible to consider the epistemology of geometry in a rationalistic way.On the other hand,the empiricist approach before the time of non-Euclidean geometries was not seeming to be satisfactory for their philosophical account of Euclidean geometry and mathematics in general, because they were thinking within the scope of the rationalist, i.e. accepting the universal truth of geometry and mathematics.

The second round of the empiricist approach, however,was not also fully satisfactory after the emergence of non-Euclidean geometries, for the final empirical justification of geometrical systems had certain inherent limitations (see chapter III, section 3.3; this point will also be reconsidered in the next sections).

As I have briefly mentioned before, Kant has prepared the conventionalist way of thinking by emphasizing that the knowledge of the external world and the order which we are seeking for, are not totally independent of human mind, i.e. there is a difference between the ultimate reality of things and our knowledge of them(50).With the influence of Kant and with the emergence of non-Euclidean geometries, some scientists and philosophers as Mach, Hertz, Duhem,... are inclined towards the view that as far as we concern with the empirical data obtained by experience,experiment and observation , and without considering a priori or self-evident truths, one can not be sure whether our knowledge of the external world is a description of ultimate reality.It would be better, then, to conceive theorized knowledge of the external world as explications about the world and to conceive scientific theories as instrument for making related predictions.

This idea of dichotomy between reality as it is and the knowledge of it which is suggested by the emergence of non-Euclidean geometries, has opened the way to the conventionalist thesis which had been opened part way by Kant.

Briefly and vaguely put, conventionalism replaces the criterion of truth with convenience in choosing among alternative ways of 'describing' the natural world.

In the conventionalist approach, both some empirical ingredients and a priori or self-evident ingredients of old approaches or theories are replaced by conventional ingredients. As Amsterdamsky says:

> "Conventionalism,...,is a doctrine according to which some empirical problems can be solved only if we accept the experimental data together with some empirical statements asserted as true by convention."(51)

Which ingredients are considered as conventional and which ones are empirical and the way to give account for such considerations make consist the difference among the philosophers who uphold at least certain conventional elements in their analysis of the problem of physical space.

The upholders of the view that there is certain role (more or less) played by conventions in geometry,the views of Poincaré, Reichenbach and Grünbaum will be presented in the next sections,and,in this way, I suppose, it will be possible for the problem of physical geometry to subdivide the conventionalist thesis into two subtheses: the first-order conventionalism and the second-order conventionalism. The first one is the conventionalist approach of Reichenbach and Grünbaum maintaining an empirical determination of physical geometry after the conventional choice of congruence. The second subthesis,the second-order conventionalism, is the quasi-Poincarean conventionalism maintaining that even after the physical stipulation of congruence has been fixed conventionally, the metric geometry of physical space is still a matter of convention. But, before, I will examine one of the fundamental concept of geometry, viz. congruence which is the central point for the discussions on conventionality in geometry.

4.2. Problem of Congruence and Metric :

Measurement is a fundamental deal for all scientific, engineering and technical works, and for everyday life. The idea of measurement is based on finding an equality between the object to be measured and a measuring rod selected as unity having a certain scale. Each scale has certain unit depending on arbitrary selection (or, convention) like metric unit system or British unit system for example. The main idea for every measurement whatever is the unit, is that the initial and the end point of any object to be measured must be touched with certain part of the measuring rod.These initial and end point give the length of the object. It is said that part of the measuring rod is \`congruent' with the object. \`Congruence' means the equality or/and superposability of two or more objects.

Another kind of measurement is the one which is made by using optical instruments for distant intervals.

In all these measuring operations certain assumptions are made. The homogeneity and isotropy of space are presupposed; the size of measuring rod and measured object are supposed to remain unaltered before,during and after the measuring operation, and also light rays are supposed to be straight everywhere(52). Both measuring rods and objects can be effected by direct and unobserved forces, or by differential and universal forces respectively as defined by Reichenbach(53). Direct forces can be detected and calculated within the system (like heat effecting different materials in different ways), unobserved forces can not be detected if they effect all the materials and objects in similar ways (in that case there will be no measuring difference tough there is factual difference).

Rigid body assumption is essential for measuring operation. Reichenbach defines the concept of rigid body as follows:

> "Rigid bodies are solid bodies which are not effected by differential forces,or concerning which the influence of differential forces has been eliminated by corrections;universal forces are disregarded."(54).

Rigid body assumption is used in defining `congruence'. The assumption of homogeneity and isotropy of space make `rigid body' the essential instrument of measuring. This homogeneity and isotropy is also preserved in visual field. It is accepted that our perception of rigid bodies is not effected by any force affecting the isotropy of visual field. (This does not mean to accept that visual field is Euclidean; for example,Luneburg argues that `the binocular sensory space is Lobatchevskian'(55). Rigid body assumption is consistent with a non-Euclidean space of constant curvature). For the measurement both with measuring rods and with optical devices the straightness of light rays is presupposed. Then,it is accepted and assumed that we live in a homogeneous and isotropic universe;space is flat (of zero curvature), the size of rigid bodies is not afected by their motion in space.

The general name of this assumption is `the axiom of free mobility'.It was first time explicitly defined by Helmholtz as : "If two figures can be brought into coincidence in one position,this is also possible in any other position." (56)

The idea of this assumption had been expressed in the fourth axiom of Euclid and also implicitly in the second and the fifth axiom,probably not as an assumption but as a self-evident principle about the real nature of things. Einstein says:

> "If two tracts are found to be equal once and anywhere,they are also equal always and everywhere.Not only the practical geometry of Euclid,but also its nearest generalization,the practical geometry of Riemann,and therewith the general theory of relativity,rest upon this assumption."(57)

This assumption, firstly, is a practical assumption which makes life simpler,but beyond that it is a philosophical adventure which can be summarized as the search of invariants under continuous change or transformation. This philosophical attitude,of course, is broader than the problem of invariance in dimensions which is apart of the aim. As far as we know,since Pre-Socratic philosophers, the aim was to reach the general order in cosmos which is considered to exist under the rough world of appearance,under the world of becoming and change.

But,what is the source of this assumption philosophically?Is it an a priori principle or is it an empirical generalisation,or a convention?

For a possible rationalist account of geometry after the emergence of non-Euclidean geometry, some philosophers have tried to reconcile non-Euclidean geometries with Kantian synthetic a priori approach. Although it is understood that not all the axioms of any geometrical system are totally a priori,they seeked some common principles which are a priori for both Euclidean geometry and for non-Euclidean ones. They claimed that such a principle common to all geometries is the axiom of free mobility. Bertrand Russell, for example, argued that "the denial of this axiom would involve logical and philosophical absurdities" and concluded that this axiom "is an a priori condition of metrical geometry"(58). Russell's claim could be supported by the mathematical works of Sophus Lie who proved that there are only four metric geometry which are consistent with the axiom of free mobility :Euclidean, hyperbolic,spherical and elliptical; all with constant curvature (59),i.e. preserving the homogeneity and isotropy of space.

But Russell was unlucky in arguing that "the axiom of free mobility is a priori and its denial would involve logical and philosophical absurdities", because he was not the witness of the general theory of relativity which will appear twenty years after Russell's claim.Einstein used a non-Euclidean geometry with non-constant positive curvature in his theory. As a predecessor of Einstein,Riemann had already talked about the homogeneity of space as an abstraction without taking into account the existence of matter. He had argued that the homogeneity will disappear when matter in space is taken into consideration (Clifford developed the same point of Riemann as early as in 1876 in his essay `On the Space Theory of Matter', defining motion as an intrinsic change in the curvature of space)(60).This is the case in the general theory of relativity. The non-costancy of curvature of space implies at least some local unhomogeneity and unisotropy of space. That means the denial of the axiom of free mobility at these locations.

I think that it is no more possible to support that the axiom of free mobility is a priori,and also it is no more possible to support the rationalist account for the axioms of geometry and for geometrical system in general.

Almost the same things can be said for the empiricist approach too.It is no more possible to justify or falsify the axiom of free mobility since every attempt will necessitate the acceptance or the denial of that axiom in a vicious way.

I will discuss the conventional account of that assumption and the congruence problem in the next sections.Now, I will discuss the congruence problem in the light of discrete space and continuous space approaches.

4.3. Discrete Space or Continuous Space :

In his famous dissertation, 'On the Hypothesis Which at the foundation of geometry',Riemann says :

> "Notions of quantity are possible only where there exists already a general concept which allows various modes of determination...they [modes of determination] form a continuous or a continuous or a discrete manifold...Determinate parts of a manifold, distinguished by a mark or by a boundary,are called quanta. Their comparison as to quantity comes in discrete magnitudes by counting, in continuous magnitude by measurement...while in a discrete manifold the principle of metric relations is implicit in the notion of this manifold, it must come from somewhere else in the case of a continuous manifold."(61)

What is understood from Riemann's remark is that in the case of a discrete manifold (space), the distance of two things already exists in this manifold or the measure of something is determinated by the number of elements belonging to it. The size of the elements may be consisted of some particular length,either the fundamental length of Heisenberg (10^{-13} cm) or Planck length (1.6×10^{-33} cm) or some another length which makes space discrete. Then it becomes obvious that in such a case congruence relation is an existing pro-property among things,i.e. two things having the same cardinal number of elements will be congruent.

The conception of discrete space invokes a certain metric; an intrinsic metric of space.This existing intrinsic metric makes congruence relation a factual one,i.e.the length of two things exists independently of any metric geometry and before the construction of them end of their congruence classes.

In the case of discrete space conventionality of congruence becomes trivial depending on unit system used.One can give the measuring of something either by metric system or by British system or by some another system. Also in that case the name used for congruence becomes only a semantical convention and not a physical stipulation. That is what Grünbaum calls 'Trivial Semantical Conventionality' (TSC)(62). We can label any word whatever we like to a symbol in a syntactical formal system which denotes distance equality.

In the case of discrete space,then,there must be metric,an 'intrinsic metric' which is independent of alternative metrizability with different congruence classes and also independent of unit systems.

One point must be mentioned; mathematically speaking there are sets neither discrete nor continuous.Set of discrete spaces are denumerable, but the inverse is not necessary : for example the set of natural numbers ($\mathbb{N}$) is discrete and denumerable,but the set of rational numbers ($\mathbb{Q}$) is also denumerable but not discrete (therefore the lack of intrinsic metric), $\mathbb{Q}$ is a dense denumerable set.(As it will be considered later, Grünbaum explains the reasons why not to invoke a dense denumerable sets though such sets has no intrinsic metric. Also, I will discuss the difficulties with such sets in the case of Zeno paradoxes.) Denumerability and non-denumerability (in the Cantorian sense) becomes a criterion for non-continuous and continuous sets respectively.

In the case of continuous space on the other hand, there is no specific concept of distance, and as Riemann says, `it must come from somewhere else'. Mathematically speaking, we can represent continuous space by the properties of real numbers (R). The set of R is a dense and non-denumerable continuum. A line can be put to one-to-one correspondence with a plane, and even with three or more dimensional space. (But this transformation, itself, is not continuous; dimension is a topological invariant.)

It is obvious then, no specific distance 'by itself' can be attributed to continuous space. There is no metric which is intrinsic to continuum. Grünbaum calls that 'intrinsic metrical amorphousness'(63). If there is no metric intrinsic to space, then, congruence relation and metric 'must come from somewhere else' as Riemann had pointed out. This 'from somewhere else' means that congruence and therefore metric must be conventional. This latter remark can be elucidated by examining the correlation between 'congruence' and 'metric'.

A metric must satisfy certain properties. Let us take three points x, y, z all non-negative real numbers satisfying the following conditions:

a) $d(x,y) = d(y,x)$, where $d(x,y)$ denotes the distance from x to y ;

b) $d(x,y) = 0$ if and only if $x = y$;

c) $d(x,y) + d(y,z) \geq d(x,z)$.

A general distance function can be written in two dimensional space as $ds = \sqrt{g_{ik}dx^i dx^k}$,where g_{ik} is a surface function according to the properties of surface. Different g_{ik} functions give different metrics.For example:g_{ik} = 1 when i=k and g_{ik} = 0 when $i \neq k$ gives the Euclidean metric, viz. $ds=\sqrt{dx^2+dy^2}$ or $g_{ik} = 1/y^2$ when i=k and g_{ik} = 0 when $i \neq k$ gives a hyperbolic metric, viz. $ds = \sqrt{dx^2+dy^2} / y$. Different metrics have different congruence classes. Euclidean geometry and non-Euclidean geometries have different metrics,hence different congruence relations.

Can we ask now which metric is true or which metric is the real metric of physical space, and which congruence classes are true representative classes?

As it is mentioned before, in the case of discrete space an intrinsic metric exist and consequently `congruence' is factual. But in the case of continuous space the lack of intrinsic metric is factual.Hence there is no congruence relation,in other words congruence relation is not a real relation as 'betweenness' is.And as Grünbaum says:"...the congruence of two segments is a matter of convention,stipulation,or definition and not a factual matter..."(64), congruence standard can be stipulated conventionally in the case of continuous space. Alternative metrizability of physical space is the implication of conventional stipulation of congruence.And the result:the choice of any metric is conventional.

But do we know that space is either discrete,or dense denumerable (not continuous),or continuous?A plausible answer that I agree is :no. However, I think that we can have some general opinion in favour of continuous space approach by considering Euclidean and non-Euclidean geometries,Zeno paradox of extension and some attempts to construct models based on discrete space approach.

As it is said before if space were discrete, then, there must be a true 'unique' metric, an 'intrinsic metric'. Let us consider that any geometrical system either Euclidean or one of the non-Euclidean geometry describe fully or partly this 'intrinsic metric'.Since distinct geometrical systems are logically incompatible with each other, we have to expect that one and only one of them can be the true geometry of space. In other words,if Euclidean metric is the true account in the case of discrete space, then non-Euclidean geometries can not,or vice versa.

But we have a famous example by the general theory of relativity that a non-Euclidean geometry(Riemannian geometry) with non-constant curvature is used,giving accessible results with empirical findings. However, in general, it has been thought that Newtonian mechanics is a limiting case of the general theory of relativity;for example Nagel says:"It is .. evident that the comprehensive geometry of the general theory of relativity containing the geometry of Newtonian mechanics as a limiting case,is a branch of physics."(65). But,although differential geometry comprises both the Euclidean and Riemannian geometry,it seems to me it is overlooked that synthetically the geometry of Newtonian mechanics and the geometry of the general theory of relativity are logically incompatible with each other.

Although the mathematical possibility of self-consistency of non-Euclidean geometries does not tell us whether space is a continuum or discrete, a successful use of a non-Euclidean metric apart from the successful use of Euclidean metric in Newtonian mechanics,can suggest us that alternative metrizability of space with incompatible geometries, is a result of its continuous structure.

Dense denumerable spaces,being a third alternative to discrete and continuous spaces, have no intrinsic metric. But such spaces have certain difficulties as Grünbaum says:

> "...ordinary analytic geometry allows the deduction that the length of a denumerably infinite point set is intrinsically zero.More generally,the measure of a denumerable point set is always zero...(66).For the length(measure) of an denumerably infinite point-set (like the set of rational poits between and including 0 and 1) is zero (upon denumeration of the set)..Thus,if any set of rational points were regarded as constituting an extended line segment,then the customary mathematical theory under consideration could assert the length of that segment to be greater than zero only at the cost of permitting itself to become self-contradictory(67)...These considerations show incidentally that space-intervals cannot be held to be merely denumerable aggregates within the context of the usual mathematical theory(68)."

In Zeno paradox of extension and 'Achilles and Tortoise' space was considered as dense (therefore not discrete, but also not continuous like a space composed of rational numbers Q). In that not continuous space,for each step of Achilles at an instant of time, Tortoise has exactly one, and, then,if Tortoise starts competition ahead, Achilles can never catch Tortoise. In that not continuous space one cannot take take the derivative of trajectory function with respect to time at every point of that function.And since the individual denumerable point has zero length,they are not additive. But, if we consider a continuous space of real numbers R, then, there is no Zeno paradox;one can take the derivative of trajectory function at every point,and,in that continuum Achilles can catch Tortoise.

The attempts for modelling theories based on discrete space concept have several difficulties (69), and also a complete modelling has not been found yet as some scientists say who are working on that modeling:"...it is unlikely that a completely general and intrinsic quantization construction will ever be found..."(70). Considering of these attempts Schild says :

> "There ls simple model of discrete space-time which although not invariant under all Lorentz transformations , does admit a surprisinngly large number of Lorentz transformations."(71)

While in a discrete space modelling, there are "surprisingly large Lorentz transformations",a continuous space-time modelling are invariant under all Lorentz transformations. And as Einstein has pointed out :

> "For all these theories it is essential to operate with a space-time continuum of four dimensions. ..a renunciation of the continuum would imply a break with all the fundamental concepts of the theories considered hitherto."(72)

Although these remarks do not `prove' that space is a continuum rather than discrete or not continuous, there are sufficient reason to admit (and commit to) the opinion that it is a continuum.

In the following pages I will accept the opinion that space is a topological continuum as Grünbaum does.

4.4. Some Approaches to Conventionalism in Geometry :

Consider the following two questions :

i) Are the axioms of Euclidean geometry or axioms of any geometry in general, a priori or self-evident truth?

ii) Are the axioms of geometry propositions about experimental facts which can be justified or refuted by experience, i.e. are all the statements of geometry empirical statements?

In general, philosophers who reply these questions negatively can be characterized as philosophers appropriating the view that there is certain role (more or less) played by conventions in physical geometry. Such views of the three philosophers, the views of Poincaré, Reichenbach and Grünbaum, their similarity, differences, and, their solution to the problem of physical geometry of space will be examined in the next sub-sections.

4.4.1. Henri Poincaré :

Poincaré's answers to these questions are very clear: "The axioms of geometry ... are neither synthetic a priori judgements nor experimental facts.They are conventions.."(73)

According to Poincaré,the axioms of geometry are 'definition en disguis'(disguised definitions)(74).

The axioms of geometry, though they are conventions, they are not totally arbitrary according to Poincaré, experience plays an important role :"..our choice among all possible conventions is guided by experimental facts.."(75).But the role of experience has not to be overwhelmed in geometry :

> "...it would be an error ...to conclude that geometry is, even in part, an experimental science..Experience guides us in this choice without forcing it upon us [(76)];it tells us not which is the truest geometry, but which is the most convenient".(77)

Poincaré held that experience cannot be conclusive about the truth of geometrical propositions:

> "..two courses would be open to us;either renounce Euclidean geometry,or else modify the laws of optics and suppose that light does not travel rigorously in a straight line.
>
> It is needless to add that all the world regard the latter solution as the more advantageous.
>
> The Euclidean geometry has,therefore,nothing to fear from fresh experiments."(78)

His attitude against empiricist account and his preference for the conventionalist approach become very clear with the following words :"No experience will ever be in contradiction to Euclid's postulate; nor,on the other hand, will any experience ever contradict the postulate of Lobatchevsky. "(79)

If we summarize Poincaré's account for geometry and physical sciences;any geometrical structure can be imposed on the natural world with some revisions in physical laws when it is needed. These revisions will make possible to hold any geometrical system as in the case of Duhem's thesis. The reason why to hold certain geometrical systems rather than the others, is not because the formers are true and the latters are false;but for the formers are more convenient and simpler than the latters "just as a polynomial of the first degree is simpler than one of the second"(80).According to Poincaré,the simplest geometry is Euclidean geometry, therefore it should be convenient to hold Euclidean geometry revising some laws of the related physical theory.Poincaré express his philosophical position as follows:

> "It is impossible to imagine a concrete experiment which can be interpreted in the Euclidean system and not in the Lobatchevskian system,so that I may conclude: No experience will ever be in contradiction to Euclid's postulate;nor,on the other hand,will ever contradict the postulate of Lobatchevski.(81) ...experiment cannot decide between Euclid and Lobatchevski...To sum up,..it is impossible to discover in geometric empiricism a rational meaning."(82)

The name given to this philosophical position is conventionalism.

Poincaré is usually called as a founder of conventionalism.However, his conventionalism is not the one beyond all limits,and,he criticized such extreme conventionalist approaches calling them as nominalism and he says in his `La Science et L'Hypothèse' :

> "They have wished to generalize beyond measure,and, at the same time,they have forgotten that liberty is not license. Thus they have reached what is called `nominalism'..."(83).

Poincaré did not intend to consider the whole of science as conventional;for example he gave a special importance to inductive reasoning in mathematics (84),and also,he said in his `La Science et L'Hypothèse':"The method of the physical science rests on the induction which makes us expect the repetition of a phenomenon..."(85), though he distinguished carefully conventional,experimental and hypothetical elements in physical sciences.

Where,then,does the source of conventionalism lie in Poincaré? The answer is not the other than that; in his analysis of geometry.

In 1887, Poincaré has published his first article on the philosophy of geometry (86) asking the question :"Sont-ce des faits experimentaux,des jugements analytiques ou synthetiques a priori ?"(87).The words 'convention' and 'disguised definitions appear first time in his article "Les Géométries non-Euclidiennes" in 1891 (88). His negative answers to the questions of his first article on that subject have forced him to adopt conventionalist approach. And subsequent conventional ideas came from the acceptance of conventionality of geometrical axioms as he says :

> "In mechanics we should led to analogous conclusions,and should see that the principles of this science,though more directly based on experiment,still partake of the conventional character of the geometric postulates."(89)

The emergence of non-Euclidean geometries would have played an important role in the formation of conventionalist thesis of Poincaré. As it is said before philosophical roots of conventionalism goes back to Kant. In general, it can be said that Poincaré's conventionalist approach has many roots beginning from Kant and non-Euclidean geometry to Mach, Riemann and Hertz. He owes to Kant about his distinction of phenomenon and noumenon,and about the active role played by mind in conceiving natural phenomena; to non-Euclidean geometry about the non-uniqueness of such conceptions; to Mach for his consideration that physics is an instrument to make prediction about natural phenomena and not a body of knowledge to conceive the ultimate reality; to Riemann about his theoretical consideration of a non-Euclidaen world and for his assumptions leading to the idea of alternative metrizability of space (90);and to Hertz for his "Principles of Mechanics"(91) as an example to a physical system that the logical structure of a physical theory can be constructed without any appeal to experience (92).

Poincaré's conventionalist approach has been criticized in certain respects.For example,his consideration about the simplicity of geometry alone as a criteria for convenient choice of a physical theory has been criticized.Secondly,some thinkers considered Poincaré's conventionalist account of geometry meaningful only in trivial sense, i.e. as a convenient semantical interpretation of uninterpreted signs of a formal geometry.One of the advocate of that view,Eddington says : "I admit that space is conventional -for that matter [about a passage from Poincaré's "La Science et L'Hypothèse"(93)], the meaning of every word in the language is conventional."(94). For Nagel disputable part of Poincaré's conventionalism comes from "by his not distinguishing clearly between pure and applied geometry"(95). Nagel accepted that Poincaré's conventionalism of geometry "is simply the thesis that choice of notation in formulating a system of pure geometry is a convention"(96). But the contention of Poincaré about the conventionality of applied or physical geometry,is "far from clear" (97) according to Nagel. I will consider Reichenbach's critiism of Poincaré which is on the line somewhat different that of Nagel's,in the next section.

While Poincaré emphasized the interdependence of geometry to physics, he held that the simplicity of geometry alone will provide the ground for decision about the convenient physical theory plus geometry.These points have been criticized by many philosophers and scientist including Eddington, Reichenbach, Hempel, Nagel,..., each of them maintaining that the simplicity of total theory, i.e. physics + geometry, must be considered.They had naturally superiority to Poincaré to be the witness of the general theory of relativity which makes the total theory simpler by using a non-Euclidean (Riemannian) geometry.And they were right in general.

But,for the second part of the criticism, can we say that Poincaré's conventionalism is only in trivial sense in such a way that the meaning of every word in the language is conventional? Or,can we say that the conventionality of physical geometry is far from to be clear in Poincaré?

The criticism were right in general when they argue that Poincaré did not make an adequate distinction between formal geometry and physical geometry. But when Poincaré saying :

> "In space we know rectilinear triangles the sum of whose angles is equal to two right angles;but equally we know curvilinear triangles the sum of whose angles is less than two right angles. The existence of the one sort is not more doubtful than that of the other.To give the name of straights to the sides of the first is to adopt Euclidean geometry; to give the of straights to the sides of the latter is to adopt the non-Euclidean geometry.So that to ask what geometry it is proper to adopt is to ask,to what line is it proper to give the name straight?
>
> It is evident that experiment can not settle such a question.."(98)

,he is talking about physical space and conventionality of physical geometry,not about a `proper' semantical interpreta- of formal geometry (99).Also Poincaré,using the ideas of Riemann about the amorphous continuum structure of space, imagined a network of lines and surfaces in that amorphous continuum which he considered as a common basis to all the geometries and said that:".we may then convene to regard the meshes of this net as equal to one another,and it is only after this convention that this continuum become measurable."(100).Thus, he intended very clearly to argue the conventionality of geometry after the physical interpretation of formal geometry and after the physical stipulation of a measurability criterion,i.e. of congruence. Modifying this approach of Poincaré regarding to the general theory of relativity maintaining that only the whole system both physical theory and geometrical system can specify the metric geometry, I will call this quasi-Poincarean approach as the second-order conventionalism and I will later reconsider that point.

4.4.2. Hans Reichenbach :

The philosophical approach of Reichenbach certainly cannot be mentioned as conventionalism.Reichenbach was one of the important philosophers of Logical Positivism, and he was considering himself as Logical Empiricist.However,although he emphasized an empiricist conception of geometry of space, he also distinguished conventional ingredients in his analysis of space and geometry. That kind of analysis was due to the idea of non-Euclidean geometries. He says:

> "After the discoveries of non-Euclidean geometries the duality of physical and possible space was recognized.Mathematics reveals the possible spaces;physics decides which among them corresponds to physical space."(101)

To the first question of section 4.4. he replies,then negatively;but for a possible reply to the second question,he seems to be closer to empiricism (not in the sense of 19th century empiricism about space and geometry) although he emphasizes some conventional ingredients in his analysis.But,in any way, these conventional ingredients does not lead to a conventionalist conception of geometry in Poincaré's sense since Reichenbach accepts that a true geometry of physical space is attainable. Rather his qualified empiricist conception can be mentioned as 'relativism in geometry'.

Although Poincaré argued that we can adopt conventionally any geometrical system by considering its simplicity and convenience, this selection was not totally arbitrary for

Poincaré: it was guided by experience. But Poincaré did not give any further technical detail to explain how experience guides us. Secondly he assumed that final falsification of physical geometry is not possible;the criterion is convenience rather than truth.

Reichenbach tried to develop these technical details introducing the concept of 'coordinative definition'. In that way, he tried to draw limits to empiricist approach. For Reichenbach, there is no an empirical warrant for the axiom of free mobility, i.e. we cannot be sure whether our measuring rod or optical devices do not alter their original shape during a transportation , or light rays remain straight. This problem is not an empirical problem for Reichenbach, but a problem of definition. He says :"Is really equal a meaningful concept?..it is impossible to settle this question if we admit universal forces."(102). For Reichenbach the elimination of universal forces (which can cause measuring rod to expand, expand,contract or shrink,or can cause light rays to bend,and that we cannot know if such distortions or bending occurred or not)is essential.Only after a 'coordinative definition' of congruence,the problem of the geometry of physical space becomes an empirical problem.

According to Reichenbach, physical knowledge is cha-characterized by coordinating concept to real objects,and,"in general this coordination is not arbitrary"(103).But Reichenbach maintains the determination of certain preliminary coordinations to carry the method of coordination through any further. He calls this first coordinations `coordinative definitions.He says:"They are arbitrary like all definitions;on their choice depends the conceptual system which develops with the progress of science"(104).But the coordinative definition of congruence has an clear aim: to eliminate universal forces. According to Reichenbach :"Properties of reality are discovered only by a combination of the results of measurement with the underlying coordinative definition"(104). Reichenbach gives the name 'the relativity of geometry' to the possibility of arbitrary selection of coordinative definition of congruence. He says:"If we change the coordinative definition of congruence,a different geometry will result.This fact is called 'the relativity of geometry'"(105). The preference of this coordinative definition of congruence rather than that is guided by experience,i.e. by the convenience of coordinative definition to eliminate universal forces.For example ,Riemann geometry is preferable in Einstein's theory according to Reichenbach since it eliminates universal forces which cause light rays to bend.But once the coordinative definition of congruence has been fixed there is no more conventionality of geometry,"it is determined through objective reality alone which is the actual geometry."(106)

Reichenbach criticized Poincaré's conventionalism with the following words:

> "From conventionalism the consequence was derived that it is impossible to make an objective statement about the geometry of physical space,and that we are dealing with subjective arbitrariness only;the concept of geometry was called meaningless.This is a misunderstanding."(107)

Reichenbach is against to consider the physical geometry of space a matter of convention even after the physical definition of congruence has been fixed.(108)

One can talk about a distinction between real geometry and true geometry in Reichenbach's conception of geometry. In Poincaré there is no any problem of truth but convenience in the case of geometry of space. For Reichenbach also, real geometry of physical space is not a question to be able to answer since we start with arbitrary definitions, but once coordinative definitions are fixed,the true geometry of space is attainable.

Reichenbach express, in his 'The Rise of Scientific Philosophy',the difference between his conception and that of Poincaré's in the following way:Let us consider two groups of explication having each two subgroups:

I. Group:

a) Our space is Euclidean, but there are universal forces which affect our measuring rods and light rays.

b) Our space is non-Euclidean, and there are no universal forces.

II. Group:

a) Our space is Euclidean, and there are no universal forces.

b) Our space is non-Euclidean, but there are universal forces which affect our measuring rods and light rays.

The a) or b) of the first group leads to a non-Euclidean space while the a) or b) of the second group leads to Euclidean space.

Reichenbach says that if Poincaré held that we can conventionally decide between the subgroups of any group, he would be right; but we cannot conventionally decide between two main groups since they are factually different,i.e. these are two possible world which cannot be reduced to each other. The decision between two main groups can be determined with empirical observations(109).

In his 'Experience and Prediction', Reichenbach distinguishes two kind of simplicity: one is the 'descriptive simplicity' for which the selection of this or that system does not make any difference for their truth-character as in the case of choice between metrical unit system or British unit system; the other kind of simplicity is the 'inductive simplicity' for which the selection of this or that system "has a truth-character and demands a justification within the theory of probability and induction."(110). For Reichenbach, the choice between Euclidean geometry and non-Euclidean geometry is a matter of 'descriptive simplicity'.(111)

The adoption of non-Euclidean geometry will eliminate universal forces which cause light rays to bend in the neighborhood of heavenly bodies while Euclidean geometry will necessitate the existence of universal forces for such bending of light rays gives appropriate results with empirical findings. According to Reichenbach, the choice between two subgroups a) or b) of either group I or II is a matter of 'descriptive simplicity'; but the choice between two groups I or II is not a matter of convenience but of truth, and hence, matter of 'inductive simplicity' having a truth-character which can be justified by empirical observations.(112)

Reichenbach's approach of 'relativism in geometry' is a qualified empiricist approach. I call that approach as the first-order conventionalism.His qualified empiricism can also be construed as a qualified realism, or saying it otherwise, from a qualified empiricist position he passes gradually to a qualified realist position by eliminating gradually conventional ingredients in his analysis of geometry.That can be seen in the last pages of his 'The Philosophy of Space and Time' :

> "The essence of space-time order,its topology,remains an ultimate fact of nature,.. The three-dimensionality represents one of the topological properties of space and time, and any explanation would have to start with the assumption that some continuous order of space and time exists."(113)

The conventionality of metric also has only a definitional character for Reichenbach:

> "The coordinate system assigns to the system of coincidence,of point-events,a mutual order that is independent of any metric.This order of coincidences must therefore be understood as an ultimate fact."(114)

These last pages make clear that the idea of conventionality of congruence is only a methodological idea to establish the empirical truth of geometry of space :

> "The reality of space and time turns out to be the irrefutable consequence of our epis-

temological analyses, which have led us through many important individual problems. This result is somewhat obscured by the appearence of an element of arbitrariness in the choice of description.But in showing that the arbitrariness pertains to coordinative definitions we could make a precise statement about the empirical component of all space-time descriptions."(115)

His qualified realism manifests itself with the following words:

"Mathematical space is a conceptual structure,and as such ideal.Physics has the task of coordinating one of these mathematical structure to reality.In fulfilling this task, physics makes statement about reality.."(116)

So,the question of whether the problem of geometry of space is a description or explication can be answered for Poincaré and Reichenbach as follows: Non-Euclidean geometry has enabled Poincaré to deduce the consequence that the answers concerning the geometry of space can only be conventional explications which are guided by experience. But for Reichenbach, non-Euclidean geometries have forced to maintain conventional ingredients in any analysis of geometry of space, the problem must be a description of facts once the definitional part of the theory has been fixed. However this conception of 'description' is not in the sense of realism but in the sense of 'qualified realism' since universal forces are eliminated by arbitrary decisions. But after these definitions have been fixed, our knowledge of physical space becomes gradually a 'description'.

4.4.3. Adolf Grünbaum:

In the first pages of his "Geometry, Chronometry and Empiricism" Grünbaum asks the following question:

> "In what sense and to what extent can the ascription of a particular metric geometry to physical space and the chronometry ingredient in physical theory be held to have an empirical warrant?"(117).

In the last page of the same essay he gives a general answer to his question:

> "Our analysis of the logical status of the concept of a rigid body ... leads to the conclusion that once the physical meaning of congruence has been stipulated by reference to a solid body ... for whose distortions allowance has been made computationally as outlined,then,the geometry...is determined uniquely by the totality of the relevant empirical facts."(118).

So Grünbaum's position seems very close to Reichenbach's approach though there are differences in certain respect.And also,although Grünbaum's approach to the conventionality of metric geometry has the same Riemannian root with Poincaré,Grünbaum's conception of conventionality of geometry is different and a more limited one than the conventionalist conception of Poincaré.

Both Poincaré and Reichenbach admitted a conventional standard of congruence arbitrarily in the guide of experience.But for Grünbaum the reason to adopt a conventional choice of congruence is because of the topological structure ture of continuous space. He "ha[s] grounded the conventionality of spatial ... congruence on the continuity of the manifolds of space.."(119)

Grünbaum's approach to the philosophical problems of space can be considered in two stages. In the first approach what Grünbaum calls `geo-chronometric conventionalism' (GC), Grünbaum explains the reasons for adopting conventionality ofcongruence and hence metric,and,later he explains how the problem of physical space becomes an empirical problem after the conventional stipulation of congruence. In the second stage, Grünbaum tries to refute the Duhemian argument about the impossibility of separate testing of scientific hypothesis with a counter-exemple from physical geometry to show how the problem of physical space can be established empirically.

The GC approach is based on Riemannian view about the distinction between intrinsic and extrinsic metric (120). He says: "..the continuity we postulate for physical space and time furnishes a sufficient condition for their intrinsic metrical amorphousness."(121). As it is said before, the non-existence of intrinsic metric due to the continuity of space eliminate the existence of a real congruence standard.

Grünbaum's approach to the conventionality of congruence in determining a particular unique metric geometry is different that of Reichenbach who maintained that "the desired metric geometry would uniquely specify a metric tensor under given factual circumstances"(122). Reichenbach arguing: "If we change the coordinative definition of congruence, a different geometry will result."(123),he was holding that any kind of metric geometry determines a unique congruence class and vice versa.Grünbaum showed that only congruence definition can uniquely determine a particular metric geometry, but the converse is not valid. Grünbaum showed that there are distance functions yielding different congruence classes but each having Euclidean metric with zero Gaussian curvature;the same thing holds also for non-Euclidean geometries (124). He concluded then:

> "Only the choice of a particular congruence standard which is extrinsic to the continuum itself can determine a unique congruence class,the rigidity or self-congruence of that standard under transport being decreed by convention." (125)

For Grünbaum the conventionality of congruence is not the result of our free decision,we have to stipulate congruence standard by convention as a consequence of the continuity of space (126). He says:

> "..even disregarding inductive imprecision, sion,the empirical facts themselves do not uniquely dictate the truth of either Euclidean or of one of its non-Euclidean rivals in virtue of the lack of an intrinsic metric."(127)

Once the ineludictability of conventionality of congruence is granted,and consequently a particular metric geometry is fixed by choosing a particular congruence standard, the problem becomes an empirical problem :"..there can be no question at all of an empirically or factually determinate metric geometry .. until after a physical stipulation of congruence."(128).

Grünbaum maintains that by choosing a particular distance function $ds = \sqrt{g_{ik}dx^i dx^k}$, it has been specified not only what segments are congruent but the entire geometry(129). And although the choice of this or that metric 'within a class of equivalent description is a matter of convention' (130), of 'descriptive simplicity', he also gives a special attention to the choice of a particular metric function -for which the decision among non-equivalent descriptions is true and not at all conventional- , as a matter of empirical fact, a matter of 'inductive simplicity' in Reichenbach sense. Grünbaum says:

> "..the criterion of inductive simplicity... governs the free creation of the geometer's imagination in his choice of a particular metric tensor..And choices made on the basis of such inductive simplicity are in principle true or false, unlike those springing from considerations of descriptive simplicity, which merely reflect conventions."(131)

Also, he distinguishes carefully the criteria of 'descriptive simplicity' and 'inductive simplicity' for the topological properties of space. He considers the alternative choice among different metrics and consequently among their associated metric geometries as a matter of 'descriptive simplicity' within the confines of the same topology. Similarly, he considers the alternative between a particular topological property as against another, again as a matter of 'descriptive simplicity'. However, he admits that the collection of topological properties comprised in the topology of space as against an alternative set of them is not a matter of 'descriptive simplicity'.(132)

Grünbaum's analysis of the problem of space and time has been criticized by many philosophers and writers.GC views of Grünbaum,for example,has been criticized by Hilary Putnam. Putnam stated that GC is a subthesis of 'trivial semantical conventionalism', (abbreviated as (TSC)). TSC is the general view that any meaning assignment to any linguistic sign or symbol is conventional.In the case of syntactical formal systems,more specifically in the case of pure geometry,any meaning can be assigned to symbols x_1 or to the relation R_1 depending on the context.The advocates of the view that GC is a subthesis of TSC argued that as we can interpret $x_1$as 'point' or R_1 as 'incidence' relation for example, we can interpret a R_2 relation as congruence;then 'congruence' relation does not occupy epistemologically distinguished position among the other terms and relations of geometry.(133)

According to Grünbaum,because of 'intrinsic metrical amorphousness' of space due to its topologically continuous character,congruence occupies a different position.Conventionality of congruence is not as that of the conventionality of the word "famous" is. Grünbaum says:

> "..the conventionality of congruence is a claim not about the noise 'congruent'... For alternative metrizability ..is a matter of the non-uniqueness of a relation term already pre-empted as the physico-spatial equality predicate. And this non-uniqueness arises from the lack of an intrinsic metric in the continuous manifolds of physical space and time."(134)

Putnam,in an argument during a meeting in 1958 (135), argued that by using new color words of a language -say Spenglish - a space-dependent use of phenomenalist color predicates can be introduced to denote the color of a given object in various place under different light conditions in such a way that when a white piece of chalk is moved in a room, its appearance can change under different conditions of light; for example,it can be seen as green,blue or yellow and can be named as such.(136)

Later,replying Grünbaum's criticism, Putnam says:"... 'phenomenalist colors' are intrinsically color amorphous..." (137).Then, although piece of chalk has different appearance of color we call it 'white' conventionally. We stipulate conventionally that it is color-congruent under different illumination conditions. The result of Putnam's argument is then: 'color-congruence' of a piece of chalk is an example of TSC, and, since 'phenomenalist colors' are intrinsically color amorphous, this is exactly the same case with what Grünbaum calls GC. Therefore, according to Putnam GC is nothing but a subthesis of TSC.

Grünbaum denying the thesis that GC is a subthesis of TSC tries to show its falsity by making a distinction between (A)-conventionality and (B)-conventionality.He gives the following sentence as example to the case (A) : "Person X does not have a gall bladder",and,for the case (B) :"The platinum-iridium bar in the custody of the Bureau of Weights and measures in Paris (Sèvres) is one meter long everywhere rather than some other number of meters (after allowance of 'differential forces')". Grünbaum maintained that (A) is a case of TSC since only the use of the given sentence is conventional, not the factual proposition expressed by the sentence'. But he argued that is not the case for (B) since it is a conventional and not a factual proposition "even after we have specified what sentence or string of noises will express this proposition".(138)

He argued, then, that statements about phenomenalist colors in Putnam's example are empirical statements, and are conventional only in the (A)-conventional sense and hence in TSC sense. In Putnam's argument, there is no conventionality about the structural properties of things but only about the names of them.

> "Only the color-words are conventional,not the obtaining of specified color-properties and of color-congruence... the alternative color descriptions do not render any structural facts of the color domain and are therefore purely trivial."(139)

I agree with Grünbaum in his claim against Putnam's argument. The analogy of color space and discrete space is founded I suppose. As in the case of discrete space, since 'congruence' is factual and definite we can describe it with any unit system we like and use any word we like to denote 'congruence';in the case of color problem, since 'to have the same color' is a factual property, i.e.'to have the same wave length', the conventionality can be only (A)-conventionality, not (B)-conventionality. For 'to have the same wave length' is a factual property in the extensional sense, but this property can be factual only after the conventional stipulation of congruence standard has been fixed.

In the case of continuum, the problem is not to name 'congruence' only,because there is no congruence by itself.We apply 'congruence' to amorphous space,conventionality starts with which congruence class will be stipulated. Alternative congruence classes and consequently,alternative metrizability is the result of the continuity of space.

Grünbaum also considered that the conventionalist conception of Poincaré can not be construed as a version of TSC. He interpreted Poincaré's approach as congruence is conventional even after the semantical interpretation of congruence relation.(140)

Apart from his particular answer to Hilary Putnam (141),Grünbaum tried to answer all the critics on his work in one of his last writings on the philosophy of space and time, in his "Space,Time and Falsifiability"(142).

Grünbaum made there clear that he adopted the conventionalselection of congruence standard because of the structural properties of space but in the intensional sense. So, that conventionality of self-congruence under transport does not mean that there is a conventionality in the extensional sense, for when we perform any measurement with a rigid rod which became rigid with a particular congruence standard,"... .the concordance of two or more kinds of standards of congruence,..is a matter of fact(empirical law),as opposed to being stipulational."(143). Thus,although an extrinsic metric standard is self-congruent under transport as a matter of convention, any concordance between its congruence findings is a matter of fact.(144)

Furthermore, he is clearly against to a Poincaréan understanding of conventionality,accepting the following contention as incorrect:

> "When a physical theory provides two or more concordant test conditions for the applicability of a particular term like `spatially congruent' by means of a conjunction of sentences, then each and every one of these sentences is true by stipulation such that no new facts can falsify their conjunction."(145)

Now that the time seems ripe,Grünbaum considers Poincaré's conventionalism as a qualified empiricism, in the contrary to customed understanding, regarding to his relatively less known writings(146).Yet,I am considering Grünbaum's very personal approach as doubtful,since when Poincaré's writings are considered as a whole, it is very difficult to find any room for estimating Poincaré's position as a qualified empiricist position.And when we consider this particular passage on the Special Theory of Relativity what Poincaré calls "Le principe de relativité de Lorentz" from his "Dernières Pensées" (one of his last works):

> "Quelle va être notre position en face de ces nouvelles conceptions?Allons-nous être forcés de modifier nos conclusions? Non certes: nous avions adopté une convention parce qu'elle nous semblait commode,et nous disions que rien ne pourrait nous contraindre

> à l'abandonner.Aujourd'hui certains physiciens veulent adopter une convention nouvelle. Ce n'est pas qu'ils y soient contraints;ils jugent cette conception nouvelle plus commode,voilà tout;..." (147)

,it is evident that Grünbaum's claim can be very hardly supported.

If we summarize Grünbaum's GC views so far, one can say that he attempted to show the objective reasons for adopting conventionality in space-time theories;however he draws limits to conventionality and he declares in his "Space,Time and Falsifiability "the full extent of [his] rejection both the substance and the spirit of unbridled conventionalism." (148)

So ,conventionality in space-time theories depends on alternative metrizability. Alternative metrizability depends on the non-uniqueness of congruence classes. And the non-uniqueness of congruence classes depends on the lack of intrinsic metric in continuous space. Congruence classes are non-unique because self-congruency under transport is stipulated by convention. Grünbaum,like Reichenbach accepted that alternative metrizability is a matter of `descriptive simplicity' among equivalent descriptions.But, once the conventionality of any particular congruence class has been fixed, further concordance between its congruence findings is a matter of empirical fact, and hence the problem of physical space becomes an empirical problem.

Grünbaum's approach,then ,can be called as a first-order conventionalism similarly to Reichenbach's approach.

The second stage of Grünbaum's approach starts at that point. Grünbaum tries to elaborate that how we can decide empirically to the truth or falsity of any geometry about physical space after the stipulation of congruence standard. To do that Grünbaum tries to refute the well-known Duhemian hypothesis with a counter-example from physical geometry.Duhem, who maintained that ".the physicist can never subject an isolated hypothesis to experimental test, but only a whole group of hypothesis;.."(149),would also point out that no hypothesis of physical geometry is falsifiable separately from the related physical theory. Grünbaum, contrary to the Duhemian hypothesis,argued that geometry is separable from physics,and it is possible to isolate at least one geometrical hypothesis from the collateral physical theory.

The Duhemian hypothesis can be written in the following logical form: Let 'H' be a certain hypothesis, 'A' an auxilliary assumption and 'O' the observational statement implied by both 'H' and 'A' ;so,if [H.A -> O] and [∿O] then we obtain by Modus Tollens [∿H v∿A].But this consequence does not allow us to deduce the falsity of an isolated hypothesis 'H'.

Grünbaum argued that if we interpret 'H' as a particular metric geometry of a certain surface 'S' and 'A' as the the assumption that the surface 'S' is free from perturbing influences,then we can provide a counter-example to the Duhemian hypothesis. But to do that "...freedom from deforming influences [could] be asserted and ascertained independently of collateral theory."(150). Grünbaum,accepting that this can be done, founded his counter-example on the separate verification of 'A'.

I will examine below Grünbaum's counter-example (151) in detail. And I suppose that its failure will be shown.

Grünbaum supposes that a certain Duhemian wants to uphold the following hypothesis 'H' about any arbitrary surface 'S': if lengths $ds = \sqrt{g_{ik}dx^i dx^k}$ are assigned to space intervals by means of rigid unit rods, then Euclidean geometry is the metric geometry which prevails physically on the given surface 'S'. In the antecedent of 'H' there is an assumption such that "there is concordance among rigid rods such that all rigid rods alike yield the same metric ds and thereby the same geometry." (152). This assumption is the axiom of free mobility. Grünbaum referred to Einstein's formulation of this assumption calling it as 'Riemann concordance assumption' or briefly 'R'. So, mentioned by Grünbaum as empirical assumption 'R' becomes the following assumption : "If two tracts are found to be equal once and anywhere, they are equal always and everywhere." (153). And Grünbaum maintains that if 'S' is indeed free from perturbing influences then it follows from 'R' that "any two rods of different chemical constitution which initially coincide in S will coincide everywhere else in S, independently of their respective paths of transport." (154). He called 'A' the auxiliary assumption that 'S' is free from perturbing influences. Provided these conditions, the conjunction H.A will entail that 'measurements carried out on S should yield the findings required by Euclidean geometry' (155). Later Grünbaum supposed that if the surface 'S' is actually a sphere then the measurements will yield incompatible results with Euclidean geometry such as certain π values which are less then the Euclidean π value. In that case, Grünbaum says that if the Duhemian wants to uphold the hypothesis 'H' then he will argue that 'A' is false. Thus, if one could be possible to show the truth of 'A', then, Grünbaum will be able to establish the falsity of 'H', i.e. the falsity of an isolated hypothesis. In that case, the actual logical schema will be :

$$[\{(H.A) \rightarrow O \}. \sim O.\ A] \rightarrow \sim H.$$

Then, Grünbaum "seek[s] to establish A on the strength of the following further experimental findings: However different in chemical appearance, all rods which are found to coincide initially in S preserve their coincidences everywhere in S to within experimental accuracy [underline is mine] independently of their paths of transport."(156)

To do that, Grünbaum proposed the statement 'C' : "whatever their chemical constitution, any and all rods invariably preserve their initial coincidences under transport in S" so that the conjunction R.A will entail 'C', i.e. 'C' is the prediction (156). Then, "the observed preservation of coincidence or concordance in S"(157) ,i.e. 'C', will establish the conjunction R.A . Grünbaum assumed that `since the Duhemian wants to uphold 'H', he could not contest 'R' but only 'A', and this is because 'R' is assumed both by the Duhemian in the case of plane 'S' and by the anti-Duhemian who maintains that 'S' is a sphere.

Grünbaum's conclusion,then,is that since'R'is granted both by the Duhemian and by the anti-Duhemian, the observed concordance stated by 'C' will establish the truth of 'A'. So the separate falsibiality of 'H' will be established.

Grünbaum's argument has been criticized by certain philosophers especially on the ground of inductive uncertainty to establish the truth of 'A' (158). Grünbaum replied to objections by a detailed analysis from the probability theory ,and led to the conclusion that "in the context of R,the inductive confirmation of A by C is so enormously high that A can be regarded as well-nigh established by R.C ." (159). However, Grünbaum accepted a possible inconclusiveness of the separate falsification of 'H' because of the inductive uncertainty of the verification of 'A'. But, although he accepts that "its [A] VERIFICATION suffers from inductive uncertainty" (160),he continues to argue that:

> "Our verification of A did proceed in the the context of the assumption of R. And while we saw that R is ingredient in the Euclidean H of our example, as specified, R is similarly ingredient in the rival hypothesis that our surface S is not a Euclidean plane but a spherical surface. Thus, our verification of A was separate from the assumption of the distinctive physical content of the particular Duhemian H."(161).

My argument against Grünbaum's counter-example is not related with inductive uncertainties,that is an already discussed question. I agree provisionally that 'C' provides an inductive basis for the truth of 'A'.My question is: How and by which means the preservation of concordance stated by 'C' will be observed? By defining 'A' (A: the surface S is free from perturbing influences),we admit the Euclidean surface S (by 'H')and the observation of concordance stated by 'C' will be observed on S .But the concordance measurement can not be done without assuming and using a certain geometry. Then, the inductive confirmation of 'A' requires a certain geometry.But if we use,in our case,the geometry stated by 'H',then 'A' can not be verified since the actual surface is a sphere ,and so, since we want to establish 'A',we have to use a non-Euclidean sphere geometry, i.e. '∼H', to verify 'A'.

Then,in establishing the separate falsification of'H' ,i.e. to establish '∼H',we have used the verified 'A',but its verification is already established by '∼H', so the verification of 'A' cannot be made isolately from 'H' ,and this circularity shows that 'A' is inseparable from 'H'.

The actual logical schema is not , then ,

$$[\{ (H.A) \rightarrow O \}. \sim O.\ A] \rightarrow \sim H$$

as Grünbaum argued (162), but it can only be written as :

$$[\{ (H.A) \rightarrow O \}. \sim O.\ (\sim H \rightarrow A)] \rightarrow \sim H$$

But,it is evident that this last formula is not valid, i.e. this formula does not entail '∼H' but '∼H v∼A ' ; then the correct logical schema must be :

$$[\{ (H.A) \rightarrow O \}. \sim O.\ (\sim H \rightarrow A)] \rightarrow \sim H \vee \sim A$$

Consequently,the Duhemian(the one who is mentioned at the counter-example of Grünbaum) will say that `your intention of separate falsification of 'H' has failed since the separate verification of 'A' is based on the assumption of '∾H', and this result will not allow to deduce that 'H' is separately falsifiable.

Furthermore, when Grünbaum maintaining the separate verication of 'A', does he share the same position with Reichenbach who assumed that the elimination of the effects of perturbational forces could be made without resorting to a metric and that the metric is implicitly defined by the condition that universal forces vanish? Putnam argued that this assumption is based on an error of Reichenbach and he showed that the elimination of universal forces "does not single out a unique geometry plus physics"(163)

Grünbaum's counter-example was not only against to Duhem's conception but also Einstein's philosophy of geometry as well. For Einstein have endorsed the inseparability of geometry from the collateral physical theory and he objected to the idea of separate falsifiability or testability of geometry. He exposed his views in the form of a fictional dialogue between Poincaré and Reichenbach. Einstein-Poincaré replying Reichenbach who maintained (or, as Einstein interpreted him) that `the empirically given body realizes the concept of "distance" ', and the elimination of perturbating forces will enable the "real definition" of rigid body, say that :

> "In gaining the real definition improved by yourself you have made use of physical laws, the formulation of which presuppose (in this case) Euclidean geometry. The verification, of which you have spoken, refers, therefore, not merely to geometry but to the entire

> system of physical laws which constitute its foundation. An examination of geometry by itself is consequently not thinkable."(164)

And since Grünbaum's counter-example to that conception of geometry described above is shown to be false, I quote one of Grünbaum's sentence who write it to argue just the opposite : "..the rigid body is not even defined without first decreeing the validity of Euclidean geometry(or of some other particular geometry)."(165)

The only moral to be taken from the failure of Grünbaum's counter-example is not that the Duhemian argument is not false but even more : any geometry can not be falsified by experience and then, only the conventional choice of congruence class does not provide an empirical basis to justify or falsify by experience. The conclusion is that the problem of physical geometry is an empirical matter, a matter of fact to the extent that all the physical theory is empirical, and, then, the first-order conventionalism cannot be a solution to the problem of the geometry of physical space.

CHAPTER IV

CONCLUSION

The examination of the problem of physical geometry and physical space showed that which solutions cannot be supported;namely,the rationalist, empiricist and even ˜qualified empiricist'solutions cannot be supported.Therefore,the axioms of geometry as historically taken and the axiom of free mobility or choice of congruence standard can be considered neither as a priori, self-evident truth nor as empirical or experimental facts; even a 'qualified empiricist' approach is not a solution to the problem.

Historically,the geometry case, by itself, did one of the most influential impact on philosophical doctrine and on scientific theories with the idea behind the use of mathematics and geometry in the search for truth and indubitable knowledge.

Euclidean geometry,as an example of body of knowledge containing necessary truths which are a priori and independent of experience,dominated and influenced philosophical and scientific communities for more than two thousands years. The axioms of Euclidean geometry as being self-evident truths provide a strong evidence for the rationalism.And with Kant,the truth of its axioms was preserved not as a representation of ultimate reality but as a unique possibility in experiencing the world, or in other words, Euclidean geometry was the only possible one among the other alternatives.

The emergence of non-Euclidean geometries as purely mathematical works has changed the most important presupposiition in the background of philosophical doctrines. The emergence of non-Euclidean geometries discarded the rationalist solution and put forward instead an empiricist solution. And unsuccessful attempts to found geometry on empirical grounds necessitate the adoption of the view that there is at least some ingredients in physical geometry so that they can be accepted as true only by convention.Hence,citing it once more schematically by using a Hegelian terminology: Euclidean geometry provided the thesis,the rationalist account of geometry; non-Euclidean geometry provided the empiricist antithesis; and, the synthesis came from the conventionalist approach.

This essay considered mainly the views of three philosophers,-that of Poincaré,Reichenbach and Grünbaum-, who maintained that there is a certain role (more or less) played by conventions in physical geometry.

As it is seen before,as far as we accept a continuous topology for physical space, congruence standard can only be stipulated by convention as in the Grünbaum's approach.I called the conventionalist view which limits the role of convention with the conventional definition of congruence -which is necessary for the elimination of universal forces as in

Reichenbach's approach, and is necessary as a result of intrinsical metrical amorphousness as in Grünbaum's approach –, as the first-order conventionalism which is clearly different than the TSC (or zero-order conventionalism) which holds for any and all linguistic signs or symbols.

Then Reichenbach's and Grünbaum's conventionalism rest on the first-order conventionalism. The first-order conventionalism corresponds to the 'qualified empiricism'in the problem of physical space. The 'qualified empiricism' is the view that although we have to define congruence standard conventionally,once this physical stipulation has been given the question of which metric geometry prevails in physical space is an empirical question, a matter of fact.

But as it has been seen with the failure of Grünbaum's attempt (his counter-example for the Duhemian argument) to found geometry on empirical ground and hence with the failure of Reichenbach's position, the first-order conventionalism cannot provide a solution to the question of the geometry of physical space, i.e. the question of the geometry of physical space cannot be established by the first-order conventionalism. Thus, Grünbaum's inquietude became realized at least for the time being when he is saying :

> "If my proposed method of escaping from the web of the Duhemian ambiguity were shown to be unsuccessful,and if there should happen to be no other scientifically viable means of escape, then, it seems to me, we would unflinchingly have to resign ourselves to this relatively unmitigable type of uncertainty."(166)

Then, a Poincarean conventionalism modified after the General Theory of Relativity maintaining that only the whole system both physical theory and geometrical system can specify the metric geometry, will be the approach prevailing in the question of physical space. I called this modified Poincarean conventionalism - quasi-Poincarean conventionalism- as the second-order conventionalism , and I will argue for the second-order conventionalism which maintains that even after the physical meaning of 'congruence' has been stipulated, the geometry of physical space is still a matter of convention. Also it becomes clear that "an examination of geometry by itself is not thinkable"(167).

And considering congruence, and consequently metric and geometry of physical space, can we argue any geometry as a real description of physical space? We have seen that considering the underdetermination of both congruence and metric, a realist account of geometry is not convincing. Therefore any possible geometry of physical space cannot be established as a description of real space, and only a conventional choice of metric geometry of physical space is possible. But to what extent can be this conventionality of the choice of metric geometry?

The second-order conventionalism, then, emphasizing a rational choice of the entire system, of both physical theory and geometry, will provide the conclusion that the problem of physical geometry is an empirical matter, a matter of fact to the extent that all the physical theory is empirical.

NOTES

(1) I define the use of the words 'description' and 'explications' as follows: by 'description of the world' I label the view that human knowledge can attain the true picture of reality as it is,and properties attributed to objects and their relations are real attributes and relations as they are.On the other hand,by 'explication of the world' I label the view that human knowledge cannot attain the ultimate reality, instead he explicates phenomena as we conceive them via some appropriate theories without arguing that explications are real and unchangeable. The use of these two words will have this defined meaning for the whole text excepting the quotations.

(2) Proclus,1970."A Commentary on the First Book of Euclid", (transl. by G.R. Morrow),Princeton Un. Press,New Jersey.

(3) Szabo A.,1967.pp.1-2

(4) Aristotle had made a distinction between an axiom and a postulate so that an axiom was common to all sciences, whereas a postulate was related to a particular science.

(5) Among the equivalent propositions for the fifth axiom, the most well-known one is the Playfair's axiom: "From a point not on a given line, one and only one line can be drawn parallel to the given line".

(6) Originally Euclidean geometry was not logically perfect.

There are some defects in the original Euclid's system such that in the proof of some theorems spatial intuition are used implicitly,i.e. there are some theorems in the original system which cannot be proved within the formal system without adding some other axioms to the system. This defect of Euclidean geometry has been corrected and complemented by David Hilbert in his 'Gründladen der Geometrie' so that the theorems of Euclidean geometry could be proved without any appeal to spatial intuition,i.e. in a purely logical way.

(7) Plato,1953. Timaeus 53a-54a.

(8) "[T]he knowledge at which geometry aims is the knowledge of eternal being,and not of aught which at a particular time comes into being and perish. Then, geometry will draw the soul toward truth,...". Plato,1953. The Republic. 527-b.

(9) Cornford F.M.,1976.,p. 14.

(10) Szabo A.,1967.,p. 7.

(11) Davis P.J. & Hersh R.,1981."The Mathematical Experience" ,Birkhäuser,Boston,p. 328.

(12) "Je me plaisais surtout aux mathématiques, à cause de la certitude et de l'évidence de leurs raisons;...je m'étonnais de ce que leurs fondements étant si fermes et si solides,on n'avait rien bâti dessus de plus relevé." (p.17)..."Ces longues chaînes de raisons,..dont les géomètres ont coutume de se servir pour parvenir à leurs

plus difficiles démonstrations,m'avaient donné occasion de m'imaginer que toutes les choses qui peuvent tomber sous la connaissance des hommes s'entresuivent en même façon..."(p. 26). Descartes, "Discours de la Méthode", Librairie Joseph Gibert,1940.

(13) Leibniz G.W.,"New Essays Concerning Human Understanding" (transl. by Langley A.G.),Chicago,1916,p. 78.

(14) Wolff C., "Preliminary Discourse on Philosophy in General" (transl. by Blackwell R.J.), The Bobbs-Merrill Company,1963,New York,pp.18-19 and 76.

(15) Kant I.,1976,p. 33.

(16) Hume D.,1965."An Inquiry Concerning Human Understanding" The Bobbs-Merrill Company,New york,p. 173.

(17) Koyré A.,1965,pp. 6-7.

(18) Ibid.,pp. 6-7.

(19) Nagel E.,1961,p. 234

(20) Newton I.,1947,p. 13

(21) Nagel E.,1961,p. 203-204

(22) Newton I.,1947,p.XVIII

(23) Ibid.,p. 6.

(24) Jammer M.,1960,p. 101.

(25) Koyré A.,1965,pp. 6-7.

(26) Eddington A.,1933."The Expanding Universe",p. 40. Quoted from Cornford F.M.,1976,p. 4.

(27) Cornford F.M.,1976.,p. 5.

(28) Walsh W.H.."Kant" in Encyclopedia of Philosophy,p.307.

(29) Lindsay ,1970."Kant",Greenwood Press,London.Pp. 25-26.

(30) Ibid.

(31) Ibid.

(32) Kant I.,1976,pp. 52-53.

(33) Ibid.,p. 70.

(34) Ibid.,pp. 70-71.

(35) Newton I.,1947,p.

(36) Jammer M.,1960,p. 117.

(37) Kant I.,1976,p. 68.

(38) Ibid.,p. 70.

(39) Playfair's axiom,see note 5.

(40) Riemann B.,1959.

(41) Toulmin S. & Janik A.,1973,p. 147.

(42) Ibid.,p. 142.

(43) Hertz H.,I965,p. 45.

(44) Ibid.,p. 135.

(45) Wittgenstein L.,1978. "Tractatus Logico-Philosophicus", (transl. by Pears D.F. & McGuinness),Routledge and Kegan Paul,London,p. 69 (6.35).

(46) Clifford W.K.,"On the Space Theory of Matter (Abstract)" ,in 'The World of Mathematics' (4 Volumes), Simon and Schuster,New York,1956.Pp. 552-569

(47) Mach E.,1974.

(48) Ibid.

(49) Toulmin S.,1970."Physical Reality",Harper Torchbook, New York,p. XV.

(50) "Space does not represent any property of things in themselves,nor does it represent them in their relations to one another"(p. 71),"...space is not a form inhering in things in themselves.."(p. 73).Kant I.,1976.

(51) Amsterdamsky S.,1975,p. 107.

(52) Misner C., Thorne K. & Wheeler J.A.,1973. "Gravitation", Freeman,San Francisco,pp. 703, 713-714.

(53) Reichenbach H.,1951,p. 24.

(54) Ibid.,p. 22.

(55) Blank A.A.,1953,"The Luneburg Theory of Binocular Visual Space," Journal of the Optical Society of America, Vol. XLIII,p. 717.; 1954, "The non-Euclidean Geometry of Binocular Visual Space," Bulletin of the American Mathematical Society,Vol. LX,p. 376.

(56) Van Frassen B.,1970,p. 118.

(57) Einstein A., "Geometry and Experience," in Readings in 'The Philosophy of Science',Feigl H.& Brodbeck M.,Appleton-Century-Crofts, New york,1953. P. 192.

(58) Russell B.,1956,pp. 150-151.

(59) Bonola R.,1955,pp. 153-154.

(60) Clifford W.K.,"On the Space Theory of Matter (Abstract)" ,in 'The World of Mathematics' (4 Volumes), Simon and Schuster,New York,1956.Pp. 568-569.

(61) Riemann B.,1959.Pp 412-413 & 424-425.

(62) Grünbaum A.,1973,p. 27.

(63) Ibid.,p. 10.

(64) Ibid.,p. 12.

(65) Nagel E.,1961,p. 270.

(66) Grünbaum A.,1973,p.10.

(67) Ibid.,p.172.

(68) Ibid.,p.10.

(69) Jammer M.,1960,p. 211.

(70) Simms D.J. & Woodhouse N.M.J.,1976. "Lectures on Geometric Quantization",Springer-Verlag,Berlin,Heidelberg.

(71) Schild A.,1949."Discrete Space-Time and Integral Lorentz

Transformations," in Canadian Journal of Mathematics,pp. 29-47,p. 29.

(72) Einsein A.,1953,p. 163.

(73) Poincaré H.,1946,p.65.

(74) Ibid.

(75) Ibid.

(76) The difference with Kant.

(77) Poincaré H.,1946,p. 79.

(78) Ibid.,p. 81.

(79) Ibid.,p. 83.

(80) Ibid.,p. 65.

(81) Ibid.,p. 83.

(82) Ibid.,p. 86.

(83) Ibid.,p. 28.

(84) Ibid.,p. 29.

(85) Ibid.,p. 30.

(86) Poincaré H., "Sur les Hypothèse fondamentale de la Géométrie," in OEvres XI,pp. 79-91.

(87) "Are they [axioms of geometry] experimental facts, analytical judgements or synthetic a priori judgements?", [Translation is mine].

(88) Mooij J.J.A.,1966,pp. 13-14.

(89) Poincaré H.,1946,p. 29.

(90) Ibid.,pp. 235-244.

(91) Hertz H.,1965.

(92) Poincaré H.,1946,p. 102 and 272.

(93) Ibid.,p. 81.

(94) Eddington A.,1953,p. 9.

(95) Nagel E.,1961,p. 261.

(96) Ibid.

(97) Ibid.

(98) Poincaré H.,1946,p. 235.

(99) See also Grünbaum A.,1973,pp. 116-117.

(100) Poincaré H.,1946,p.235.

(101) Reichenbach H.,1958,p. 6.

(102) Ibid.,p. 14.

(103) Ibid.,p. 14.

(104) Ibid.,p. 35.

(105) Reichenbach H.,1951,p. 132.

(106) Reichenbach H.,1958,p. 37.

(107) Ibid.,pp. 36-37.

(108) See also Grünbaum A.,1959.

(109) Reichenbach H.,1951,p. 135-136.

(110) Reichenbach H.,1938,p. 376.

(111) Ibid.,375.

(112) Reichenbach H.,1951,p. 136.

(113) Reichenbach H.,1958,p. 285.

(114) Ibid.

(115) Ibid.,p. 287.

(116) Ibid.

(117) Grünbaum A.,1968,p. 4.

(118) Ibid.,p. 141.

(119) Grünbaum A.,1973,p. 41.

(120) Riemann B.,1959.

(121) Grünbaum A.,1973,p. 10.

(122) Grünbaum A.,1959,p. 214.

(123) Reichenbach H.,1951,p. 132.

(124) Grünbaum A.,1973,pp. 98-105.

(125) Ibid.,p. 11.

(126) See also chapter IV,section 4.3. of this essay.

(127) Grünbaum A.,1973,p. 33.

(128) Ibid.,p. 12.

(129) Ibid.,p. 33.

(130) Grünbaum A.,1968,p. 51.

(131) Grünbaum A.,1973,p. 151.

(132) Grünbaum A.,1968,from 'Reply to Putnam',pp. 242-243.

(133) Putnam H.,1963.

(134) Grünbaum A.,1973,pp. 27-28.

(135) Grünbaum A.,1968,from 'Reply to Putnam',p. 287.

(136) Grünbaum A.,1973,pp. 34-35.

(137) Putnam H.,1963,p. 232.

(138) Grünbaum A.,1973,pp. 36-37.

(139) Ibid.,p. 37.

(140) Ibid.,p.27.

(141) Grünbaum A.,1968,'Reply to Putnam'.

(142) Grünbaum A.,1973,pp.449-568.

(143) Ibid.,p. 452.

(144) Grünbaum A.,1968,from 'Reply to Putnam',p. 217.

(145) Grünbaum A.,1973,p.452.

(146) Grünbaum A.,1973,pp. 127-131.

(147) Poincaré H.,1913,p.54."What will be our position in the face of these new conceptions? Shall we became forced to modify our conclusions? Not at all: we have adopted a convention since it seems to us more convenient and we said that nothing will constrain us to abandon it. Today certain physicists want to adopt a new convention. This is not so because they are constrained to adopt it; they judge that this new convention is more convenient, and that's all;..."[Translation is mine].

(148) Grünbaum A.,1973,p. 451.

(149) Duhem P.,1954,p. 187.

(150) Grünbaum A.,1973,p. 136.

(151) I examined the last version of his argument from "Can We Ascertain the Falsity of a Scientific Hypothesis?", Grünbaum A.,1971.

(152) Grünbaum A.,1971,p. 111.

(153) Einstein A.,1953,p. 192.

(154) Grünbaum A.,1971,p. 113.

(155) Ibid.

(156) Ibid.,p. 114.

(157) Ibid.

(158) See the articles of Giannoni C., Laudan L., Wedeking G. in Harding S.,1976 and of Hesse M. in the British Jour-

nal for the Philosophy of Science,18, 1968.

(159) Grünbaum A.,1971,p. 118.

(160) Ibid.,p. 125.

(161) Ibid.

(162) Grünbaum A.,1973,p. 137.

(163) Putnam H.,1975,p. 174.

(164) Einstein A.,1959,pp. 676-677.

(165) Grünbaum A.,1973,p. 134.

(166) Ibid.,p. 147.

(167) Einstein A.,1959,p. 677.

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