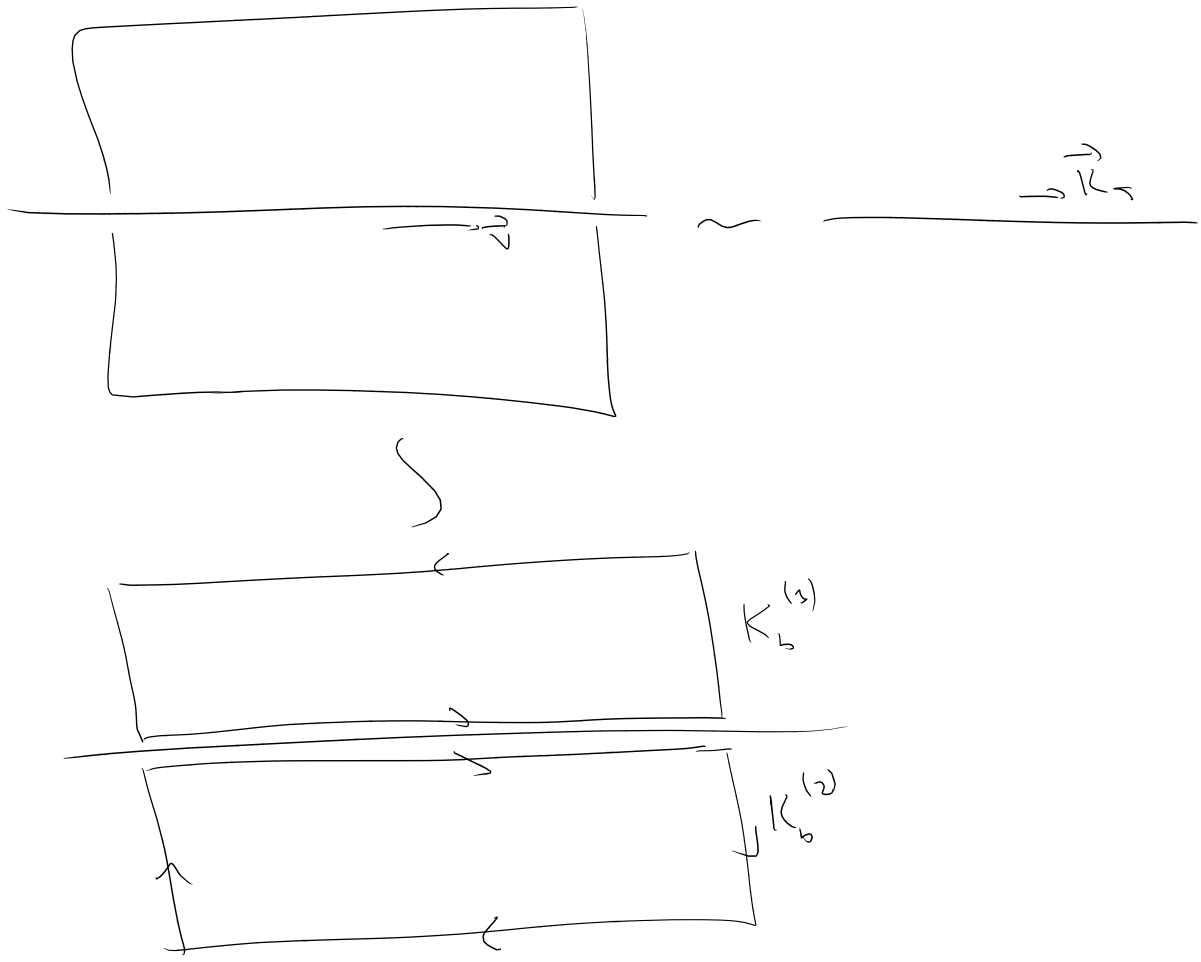


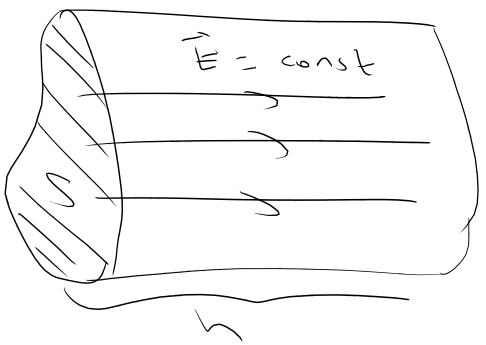
Ohm's Law $V = IR$

$$\vec{J}(\vec{r}) = \sigma \vec{E}(\vec{r})$$

σ: conductivity

$\frac{1}{\sigma} = \rho$: resistivity

Ex



$\vec{J}(\vec{r}) = \sigma \vec{E}$ is also uniform

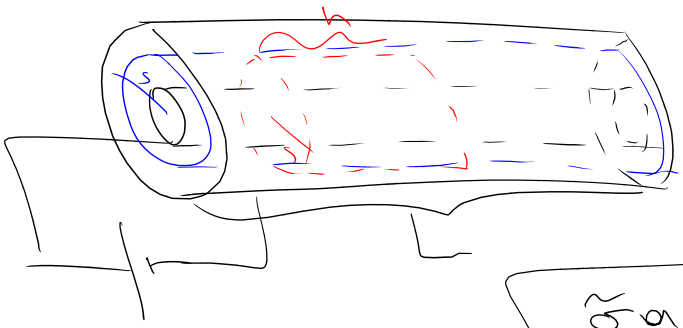
$$I = \int \vec{J} \cdot d\vec{S} = \sigma E S$$

$$V = -\int \vec{E} \cdot d\vec{x} = Eh$$

$$E = \frac{V}{h} \Rightarrow \boxed{I = \frac{\sigma S}{h} V} = \frac{V}{R}$$

$$R = \frac{1}{\sigma} \frac{h}{S} = \rho \frac{h}{S}$$

Example



$$\oint \vec{E}\cdot\vec{dS} = \frac{(2\pi a h)\sigma}{\varepsilon_0}$$

$$\boxed{\tilde{\sigma}\, a = -\tilde{\sigma}\, b}$$

$$\oint \vec{E}\cdot\vec{dS} = (2\pi s h)\, E(s)$$

$$E(s) = \frac{1}{2\pi s h}\cdot\frac{2\pi a h\,\tilde{\sigma}}{\varepsilon_0}$$

$$E(s) = \frac{a\tilde{\sigma}}{\varepsilon_0}\,\frac{1}{s}$$

$$V = -\int \vec{E}\cdot\vec{dl} = \int E\, ds = \int_a^b \frac{a\tilde{\sigma}}{\varepsilon_0}\,\frac{1}{s}\, ds = \frac{a\tilde{\sigma}}{\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$I = \int \vec{J}\cdot\vec{dS} = \int \sigma E(s)\, dS = \sigma\int\left(\frac{a\tilde{\sigma}}{\varepsilon_0}\,\frac{1}{s}\right) dS$$

$$= \sigma\,\frac{a\tilde{\sigma}}{\varepsilon_0}\,\frac{1}{s}(2\pi s)L$$

$$I = \sigma\, a\tilde{\sigma}\, 2\pi\,\frac{L}{\varepsilon_0}$$

$$V = \frac{a\tilde{\sigma}}{\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$R = \frac{V}{I} = \frac{\frac{a\tilde{\sigma}}{\varepsilon_0}\ln\left(\frac{b}{a}\right)}{\sigma\, a\tilde{\sigma}\, 2\pi\,\frac{L}{\varepsilon_0}}$$

$$\boxed{R = \frac{1}{\sigma}\,\frac{\ln\left(\frac{b}{a}\right)}{2\pi L}}$$

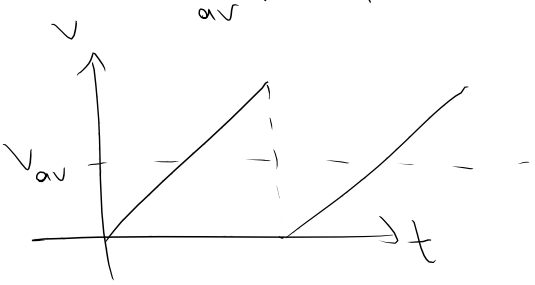
$\frac{1}{\sigma}$: resistivity

$\vec{J} = \sigma \vec{E}$

$\vec{E} \sim \vec{F} \propto m \vec{a}$

$\vec{J} \propto q \vec{v}$

$v_{av} \sim a$

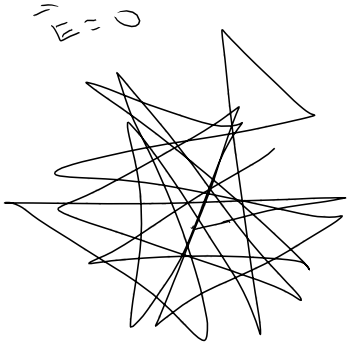


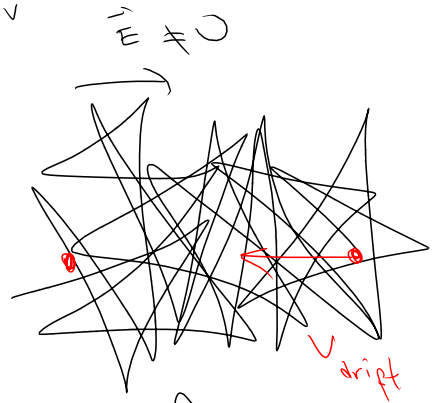
$v_{av} = \frac{1}{2}(0 + v_{max}) = \frac{1}{2} a t_{av}$

$t_{av} =$

λ: average distance between collisions

$t_{av} \neq \frac{\lambda}{v_{av}} \rightarrow v_{av} \propto \sqrt{a}$





$t_{av} \simeq \frac{\lambda}{v_{thermal}}$

$v_{drift} \simeq v_{av} \simeq \frac{1}{2} a t_{av}$

$= \frac{1}{2} a \frac{\lambda}{v_{thermal}} = \frac{1}{2} \frac{\lambda}{v_{thermal}} \left(\frac{qE}{m_q} \right)$

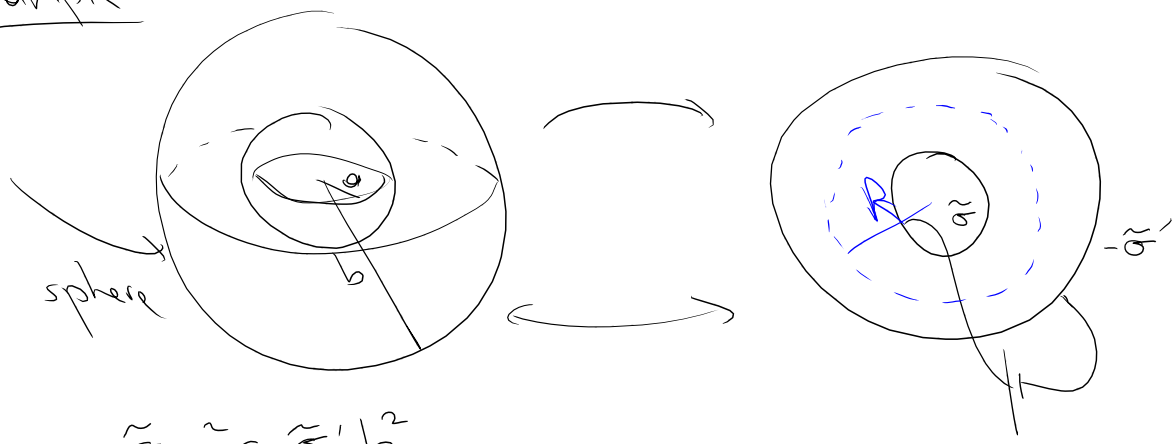
$v_{drift} \simeq \left(\frac{1}{2} \frac{\lambda q}{v_{thermal} m_q} \right) E$

$J = \rho_q v_d = \left(\frac{1}{2} \frac{n_q q^2 \lambda}{v_{th} m_q} \right) E$

σ

ρ_q : volume charge density

Example



$$\tilde{\sigma} a^2 = \tilde{\sigma}' b^2$$

$$E \cong \frac{1}{4\pi\varepsilon_0} \frac{(\tilde{\sigma} 4\pi a^2)}{r^2}$$

$$V = -\int \vec{E} \cdot d\vec{l} = \int_a^b \frac{1}{4\pi\varepsilon_0} \frac{(\tilde{\sigma} 4\pi a^2)}{r^2} dr = \frac{1}{4\pi\varepsilon_0} \tilde{\sigma} 4\pi a^2 \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$I = \iint \vec{J} \cdot d\vec{S} = \int \sigma \vec{E} \cdot d\vec{S} = \sigma \int \frac{1}{4\pi\varepsilon_0} \frac{(\tilde{\sigma} 4\pi a^2)}{R^2} dS$$

$$I = \sigma \frac{1}{4\pi\varepsilon_0} \frac{(\tilde{\sigma} 4\pi a^2)}{R^2} \cancel{4\pi R^2}$$

$$\boxed{(\tilde{\sigma} 4\pi a^2) = \frac{I \varepsilon_0}{\sigma}}$$

$$V = \frac{1}{4\pi\varepsilon_0} \tilde{\sigma} 4\pi a^2 \left(\frac{1}{a} - \frac{1}{b}\right)$$

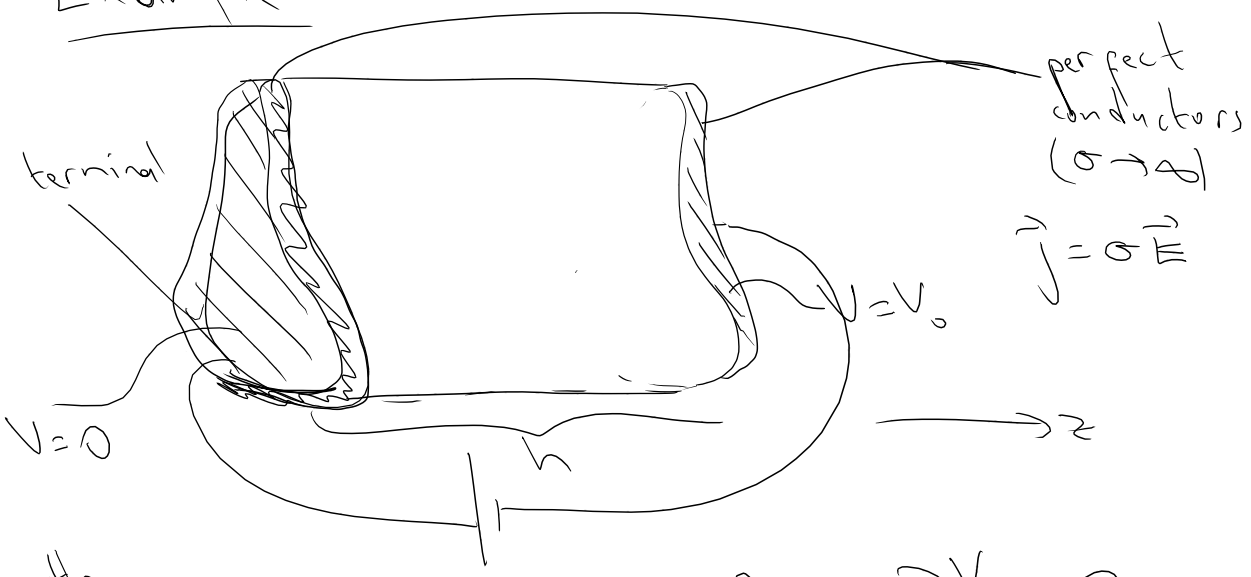
$$V = \frac{1}{4\pi\varepsilon_0} \frac{I\varepsilon_0}{\sigma} \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= IR$$

$$\boxed{R = \frac{1}{\sigma} \frac{1}{4\pi} \left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$\boxed{V = IR}$$

Example

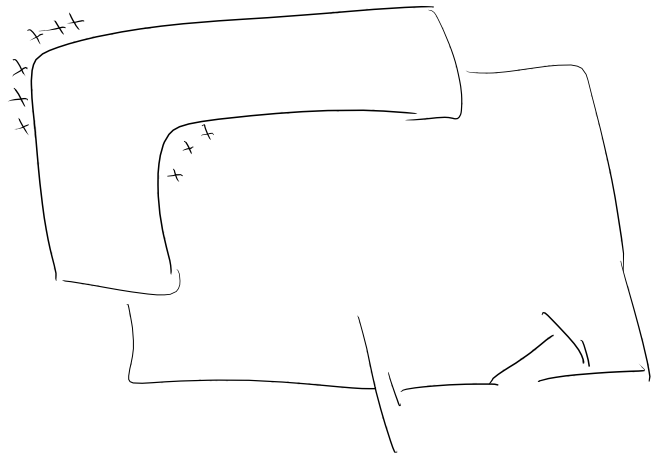


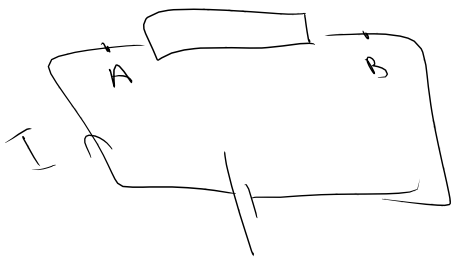
on the surface $\vec{J} \cdot \hat{n} = 0 \Rightarrow E_n = 0 \Rightarrow \frac{\partial V}{\partial n} = 0$

$$V = \frac{V_0}{h} z \Rightarrow \vec{E} = -\frac{V_0}{h} \hat{z}$$

$$\vec{J} = \sigma \vec{E}$$

$$0 = \vec{\nabla} \cdot \vec{J} = \sigma \vec{\nabla} \cdot \vec{E} \Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = 0}$$





$P = VI$

$$\oint \left(\vec{F}_s + \vec{E}\right)\cdot d\vec{l} = \oint \vec{F}_s \cdot d\vec{l} = \mathcal{E}$$

$\vec{F}_s$: force of the source

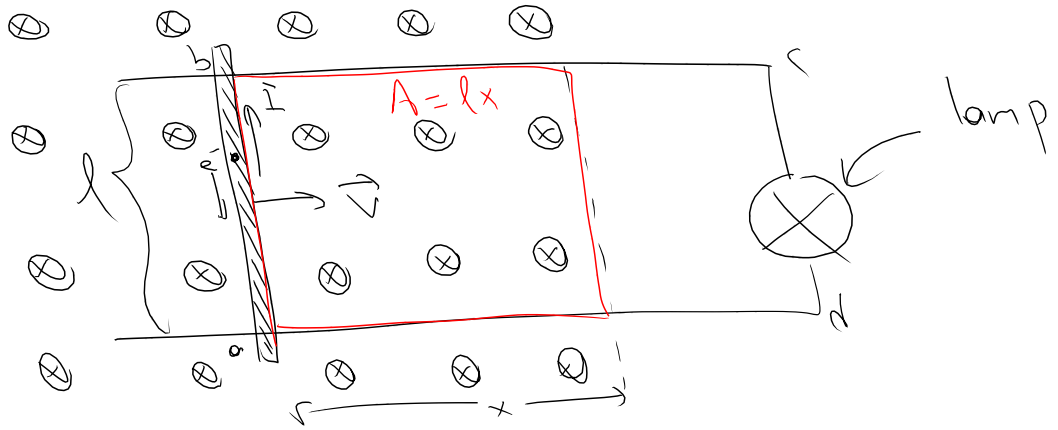
$\oint \vec{E}\cdot d\vec{l} = 0$

$\mathcal{E}$: emf

emf: electromotive force

emf is not a force.

Motional Emf



$$\mathcal{E} = \oint \vec{F}_s \cdot d\vec{l} = vBl$$

$$\oint \vec{F}_s \cdot d\vec{l} = \int_a^b \vec{F}_{mg}\cdot d\vec{l} = \int_a^b (\vec{v}\times\vec{B})\cdot d\vec{l} = vBl$$

$$v = -\frac{dx}{dt}$$

$$\mathcal{E} = -\left(\frac{dx}{dt}\right)Bl = -\frac{d}{dt}(Blx) = -\frac{d}{dt}(BA)$$

(A: area)

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}} \qquad \boxed{\Phi_B = \int \vec{B}\cdot d\vec{S}}$$

$$\mathcal{E} = \oint \vec{F}_m \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$

$$\Delta\Phi_B =$$

$$d(\Delta\vec{S}) = d\vec{l}\times\vec{v}\,dt$$

$$d(\Delta\Phi_B) = \vec{B}\cdot(d\vec{l}\times\vec{v}\,dt)$$

$$\frac{\Delta(\Phi_B)}{\Delta t} = \vec{B}\cdot\left[\left(\frac{d\vec{l}}{dt}\right)\times\vec{v}\right]$$

$$= d\vec{l}\cdot\left[\vec{B}\times\frac{\vec{v}}{dt}\right]$$

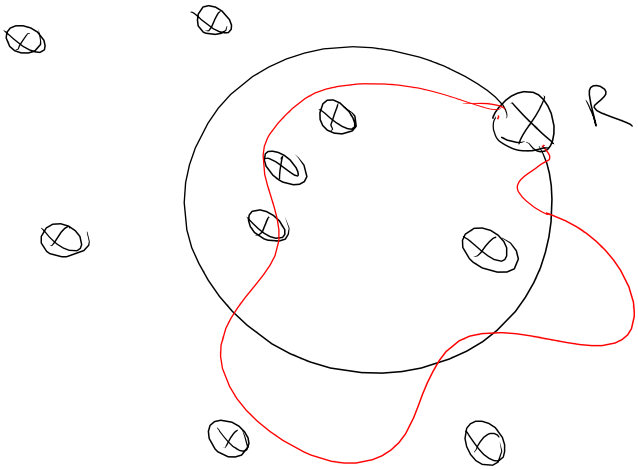
$$(\vec{a}\times\vec{b})\cdot\vec{c} = (\vec{c}\times\vec{a})\cdot\vec{b} = (\vec{b}\times\vec{c})\cdot\vec{a}$$

$$\frac{d}{dt}(\Phi_B) = -\oint \vec{F}_{mag}\cdot d\vec{l}$$

$$\Phi_B = -\int \vec{F}_{mag}\cdot d\vec{l} = -W$$

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

$$\mathcal{E} = \oint \vec{F}_m \cdot d\vec{l}$$



$\mathcal{E}I = VI$