$$\rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Longleftarrow$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \mu_{0m} \vec{J}_m$$

$$\rightarrow \vec{\nabla} \cdot \vec{B} = 0 = \rho_m / \varepsilon_{0m} =$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + (\ ) \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$

$$\underbrace{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{E})}_{0} = -\frac{\partial}{\partial t} \underbrace{(\vec{\nabla} \cdot \vec{B})}_{0}$$

$$\underbrace{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B})}_{0} = \vec{\nabla} \cdot (\mu_0 \vec{J}) = \mu_0 (\vec{\nabla} \cdot \vec{J}) = -\mu_0 \frac{\partial \rho}{\partial t}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + (ST)$$

$$\underset{0}{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B})} = \mu_0 \vec{\nabla} \vec{J} + \vec{\nabla} \cdot (ST) = 0$$

$$\vec{\nabla} \cdot (ST) = \mu_0 \frac{\partial \rho}{\partial t} = \mu_0 \frac{\partial}{\partial t} (\varepsilon_0 \vec{\nabla} \vec{E})$$

$$\vec{\nabla} \cdot (ST) = \vec{\nabla} \cdot \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\Rightarrow ST = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \underset{0}{\vec{\nabla} \times (STE)}$$

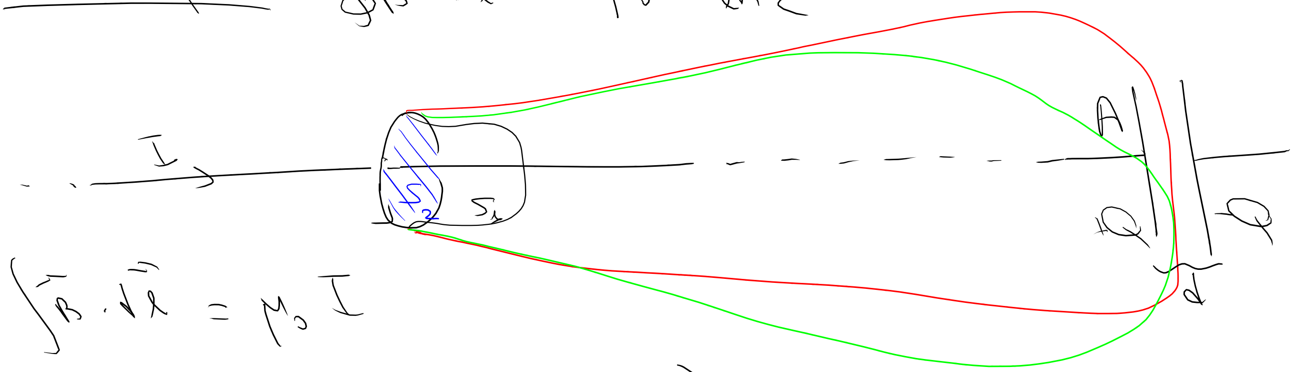
$$\vec\nabla \cdot \vec E = \rho/\varepsilon_0 \qquad \vec\nabla \times \vec E = -\frac{\partial \vec B}{\partial t}$$

$$\vec\nabla \cdot \vec B = 0 \qquad \vec\nabla \times \vec B = \mu_0 \vec J + \mu_0 \varepsilon_0 \frac{\partial \vec E}{\partial t}$$

$\vec J_D = \varepsilon_0 \frac{\partial \vec E}{\partial t}$: displacement current

$$\vec\nabla \times \vec B = \mu_0 (\vec J + \vec J_D)$$

<u>Example</u> $\oint \vec B \cdot d\vec l = \mu_0 I_{enc}$



$$\int \vec B \cdot d\vec l = \mu_0 I$$

$$\mu_0 I_{enc} = \mu_0 \int_S \vec J \cdot d\vec S = \oint_{\partial S} \vec B \cdot d\vec l$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \Rightarrow Q = (AE)\varepsilon_0 = \varepsilon_0 \int \vec E \cdot d\vec S$$

$$Q = \varepsilon_0 \oint \vec E \cdot d\vec S = \varepsilon_0 \overbrace{\int_{\text{red surface}} \vec E \cdot d\vec S}^{\Phi_E}$$

$$I = \frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \Phi_E = \int \vec E \cdot d\vec S$$

$$\oint \vec B \cdot d\vec l = \mu_0 I = \mu_0 \left[\int \vec J \cdot d\vec S + \varepsilon_0 \frac{d}{dt} \int \vec E \cdot d\vec S \right]$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

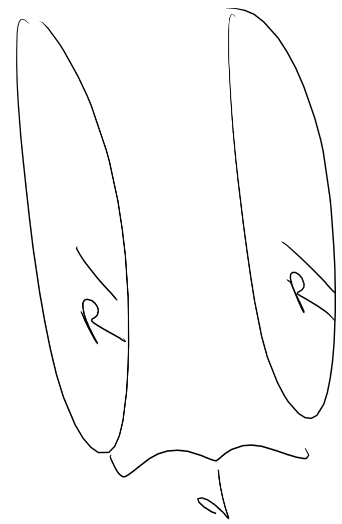
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

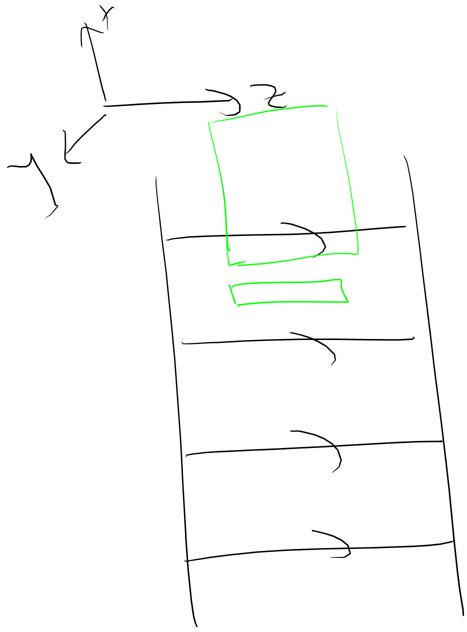
$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

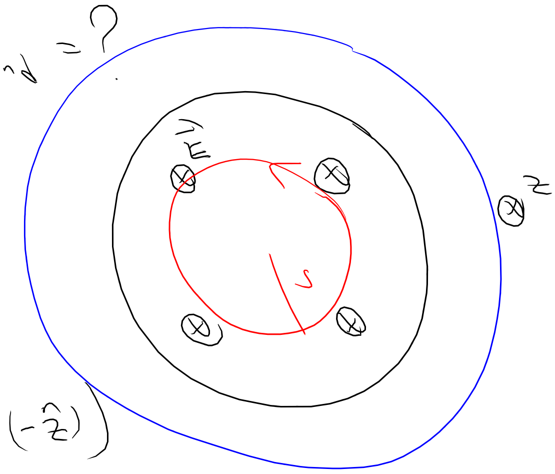
Example





$\vec{E} = E_0\cos\omega t\,\hat{z}$

$\vec{B}_{ind} = ?$



$$\Rightarrow \oint\vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d}{dt}\int\vec{E}\cdot d\vec{S}$$

$$\Rightarrow \vec{B}_{ind} = B(s,z)\,\hat{\phi}$$

$$\int\vec{E}\cdot d\vec{S} = \int(E_0\cos\omega t\,\hat{z})\cdot(dS(-\hat{z}))$$

$$\Phi_E = -E_0\cos\omega t\,\pi s^2$$

$$\oint\vec{B}\cdot d\vec{l} = \int(B(s,z)\,\hat{\phi})\cdot(dl(-\hat{\phi})) = -B(s,z)\,2\pi s$$

$$\cancel{\pi} B(s,z)\, 2 \cancel{\pi} s = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\cancel{\pi} E_0 \cos\omega t\, \cancel{\pi} s^2\right)$$

$$\boxed{B(s,z,t) = \frac{\mu_0 \varepsilon_0}{2} E_0 \frac{d}{dt}(\cos\omega t)\, s} \quad \text{if } s < R$$

$$\cancel{\pi} B(s,z)\, 2\pi s = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\cancel{\pi} E_0 \cos\omega t\, \pi R^2\right) \quad \text{if } s > R$$

$$B(s,z,t) = \frac{\mu_0 \varepsilon_0}{2} E_0 \frac{d}{dt}(\cos\omega t) \frac{R^2}{s} \left(= \frac{\mu_0 I}{2\pi s} \right)$$

$$\vec{E} = \vec{E}^{(0)} + \vec{E}^{(1)} + \vec{E}^{(2)} \ldots$$

$$\vec{B} = \vec{0} + \vec{B}^{(1)} + \vec{B}^{(2)} \ldots$$

Maxwell's Eqns in Medium

$$\vec{P} \Rightarrow \rho_b = -\vec{\nabla}\cdot\vec{P} \Rightarrow \frac{\partial \rho_b}{\partial t} = -\vec{\nabla}\cdot\left(\frac{\partial \vec{P}}{\partial t}\right)$$

$$\sigma_b = \vec{P}\cdot\hat{n}$$

$$\Downarrow$$

$$\frac{\partial \rho_b}{\partial t} + \vec{\nabla}\cdot\left(\frac{\partial \vec{P}}{\partial t}\right) = 0$$

$$\vec{M} \Longrightarrow \vec{J}_b = \vec{\nabla}\times\vec{M}$$

$$\vec{K}_b = \vec{M}\times\hat{n}$$

$$\boxed{\vec{J}_P = \frac{\partial \vec{P}}{\partial t}}$$

$$\vec{\nabla}\cdot\vec{J}_b = 0 \Longrightarrow \rho_b^M = 0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} - \frac{\vec{\nabla}\cdot\vec{P}}{\varepsilon_0} \Rightarrow \vec{\nabla}\cdot\big(\overbrace{\varepsilon_0\vec{E} + \vec{P}}^{\vec{D}}\big) = \rho$$

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \frac{\partial \vec{P}}{\partial t} + \mu_0 \vec{\nabla} \times \vec{M} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{\vec{H}} = \vec{J} + \frac{\partial}{\partial t} \underbrace{\left(\varepsilon_0 \vec{E} + \vec{P}\right)}_{\vec{D}}$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

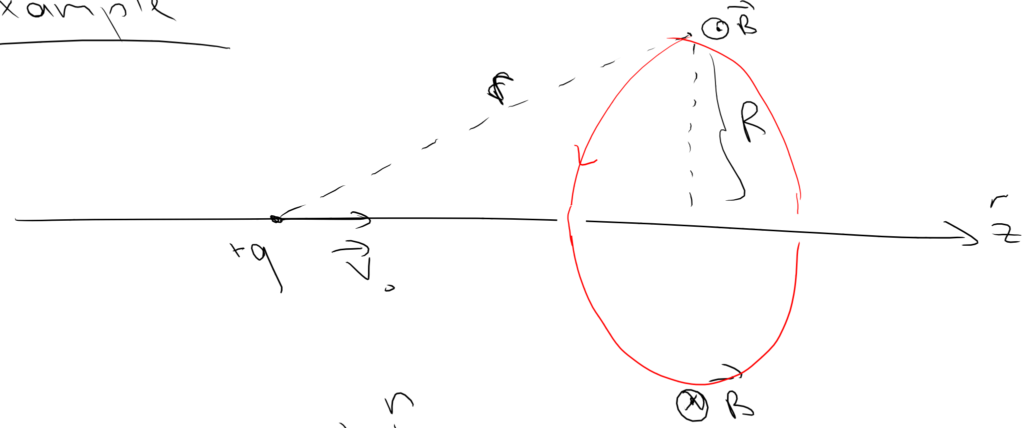
$$\boxed{\begin{array}{ll} \vec{\nabla} \cdot \vec{D} = \rho \Leftarrow & \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 \Leftarrow & \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{array}}$$

$$\vec{J}_D \equiv \frac{\partial \vec{D}}{\partial t}$$

$$[\mu_0 \varepsilon_0] = \frac{1}{[v^2]}$$

$$\boxed{\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c}$$

Example



$$\vec{B} = B(R, z, t)\,\hat{\phi}$$

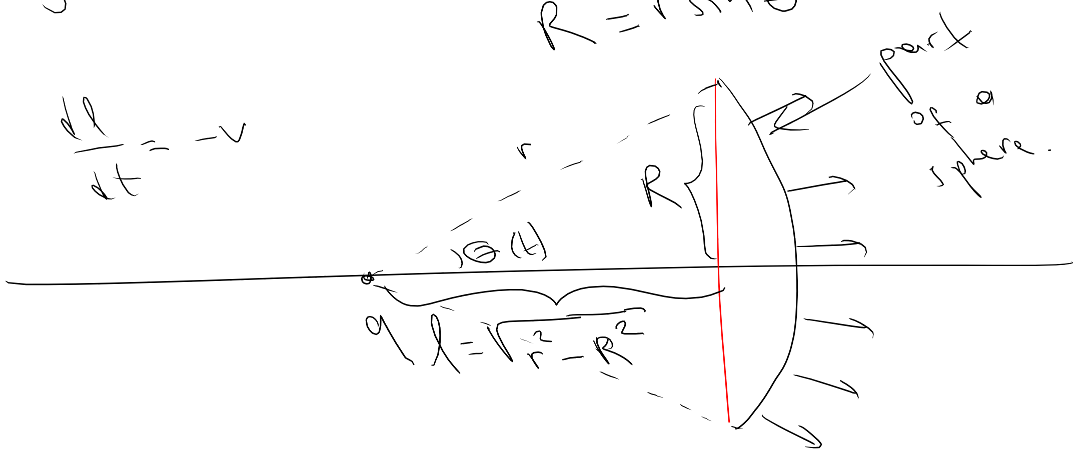
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{S}$$

$$\oint \vec{B}\cdot d\vec{l} = \oint (B\hat{\phi})\cdot(dl\,\hat{\phi}) = \boxed{2\pi R B = \oint \vec{B}\cdot d\vec{l}}$$

$$\int \vec{E}\cdot d\vec{S} = ?$$

$$R = r\sin\theta$$

$$\frac{dl}{dt} = -v$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$ quasi-static approximation

$$+ O(\varepsilon_0 \mu_0 v^2)$$

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\int dS = \frac{1}{4\pi\varepsilon_0}\frac{q}{\cancel{r^2}}\int^{\theta(t)} \cancel{r^2}\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{2\cancel{4\pi}\varepsilon_0} q \int_0 d\theta \sin\theta\ \cancel{2\pi}$$

$$\Phi_E = \frac{q}{2\varepsilon_0}\left(-\cos\theta\right)\Big|_{\theta=0}^{\theta(t)} = \frac{q}{2\varepsilon_0}\left[1-\cos\theta(t)\right]$$

$$= \frac{q}{2\varepsilon_0}\left[1-\frac{l}{\sqrt{l^2+R^2}}\right]$$

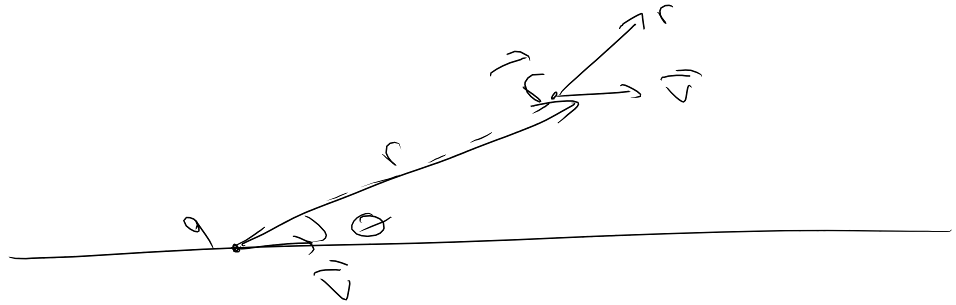
$$2\pi R B = \mu_0\varepsilon_0\frac{d}{dt}\Phi_E$$

$$B = \frac{\mu_0\varepsilon_0}{2\pi R}\left(\frac{d}{dl}\Phi_E\right)\frac{dl}{dt}$$

$$= (-v)\frac{\mu_0\cancel{\varepsilon_0}}{2\pi R}\left(\frac{-q}{2\cancel{\varepsilon_0}}\right)\left[\frac{1}{\sqrt{l^2+R^2}} + l\left(\frac{-1}{\cancel{2}}\right)\frac{\cancel{2}l}{\left(\sqrt{l^2+R^2}\right)^3}\right]$$

$$= \frac{\mu_0}{4\pi}\left(\frac{qv}{\cancel{R}}\right)\frac{1}{r^3}\underbrace{\left[r^2-l^2\right]}_{R^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{(qvR)}{r^3} = \frac{\mu_0}{4\pi}\frac{qvr\sin\theta}{r^3}$$



$$B = \frac{\mu_0}{4\pi}q\frac{|\vec v\times\vec r|}{r^3}$$

$$\vec B = B\hat\phi$$

$$\boxed{\vec B = \frac{\mu_0}{4\pi}q\frac{\vec v\times\vec r}{r^3}}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \underbrace{(\varepsilon_0\mu_0)}_{\frac{1}{c^2}}\frac{\vec{J}}{\varepsilon} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$