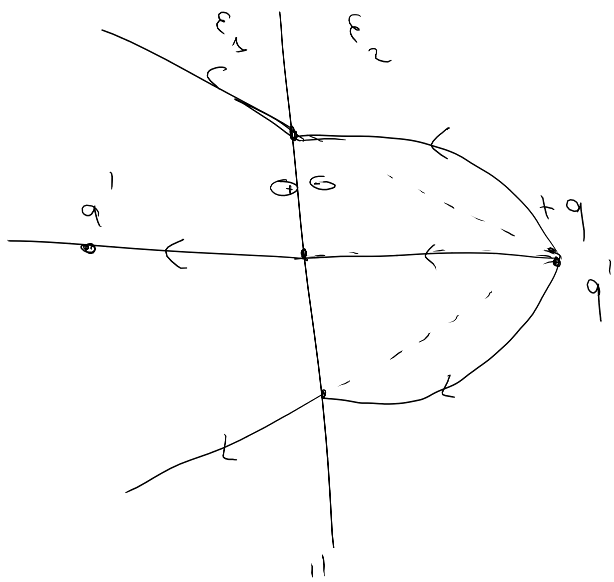
Ex

I

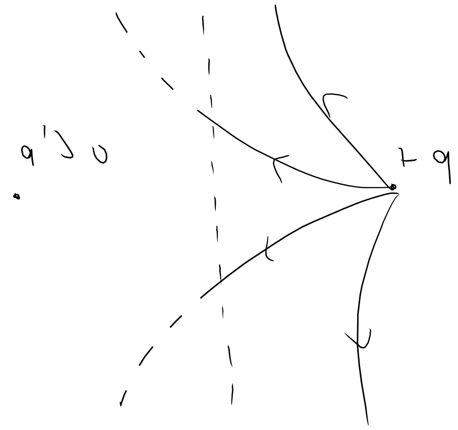
$\vec{E}$ lines $\quad \varepsilon_2 > \varepsilon_1$

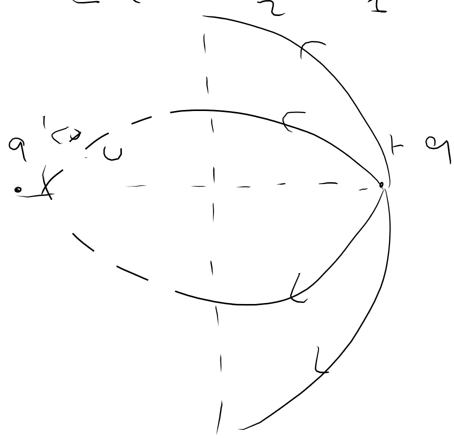



I $\Rightarrow V = \dfrac{1}{\varepsilon_1}\dfrac{q''}{|\vec{r}-\vec{r}_0|}$ $\qquad q'' = \dfrac{2\varepsilon_1}{\varepsilon_1+\varepsilon_2}$

$$q' = \frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}$$

Cf $\quad \varepsilon_2 > \varepsilon_1 \quad q' > 0$ $\qquad\qquad$ Eq $\quad \varepsilon_2 < \varepsilon_1$



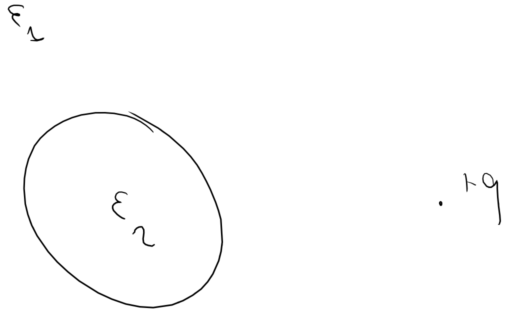




$$\vec{\nabla}\times\vec{E} = 0 \Rightarrow \Delta E_\parallel = 0$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi(\rho+\rho_b) \Rightarrow \Delta E_\perp = 4\pi(\sigma+\sigma_b)$$

$$\Delta E_\perp = 4\pi\sigma_b$$

Ex




$$\vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\vec{\nabla}\cdot\vec{P} = -\rho_b$$

$$\vec{P}\cdot\hat{n} = \sigma_b$$

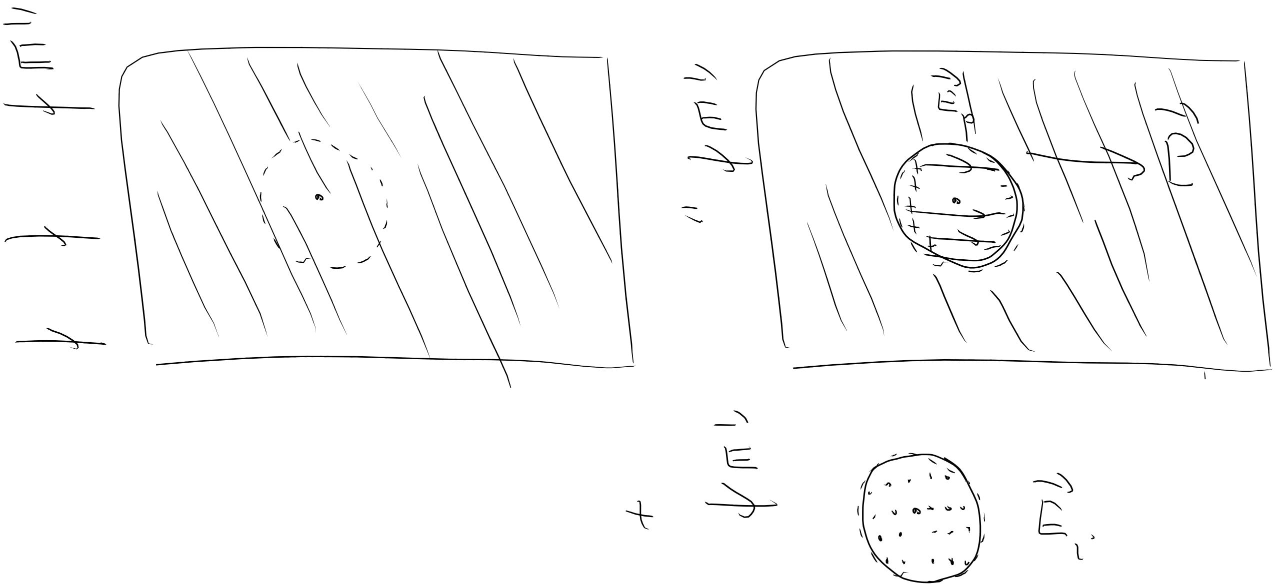
$$\vec{E}(\vec{r}) \neq \langle \vec{E}(\vec{r}) \rangle$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\vec{P} = \chi_e\vec{E}$$

$$\boxed{\varepsilon = 1 + 4\pi\chi_e}$$

$$\vec{E}_{loc} = \vec{E} + \vec{E}_p + \vec{E}_i$$



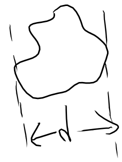
$$\boxed{V(\vec{r}) = \sum_{lm} \frac{q_{lm}}{r^{l+1}} Y_{lm}(\Omega)}$$

$$q_{lm} = \int d^3r \, \rho(\vec{r}) \, r^l \, Y^*_{lm}(\Omega)$$

$$V(\vec{r}) \simeq \frac{q_0}{r} + \sum_{m=-1}^{1} \frac{q_{1m}}{r^2} Y_{1m}(\Omega)$$

other term are negligable if

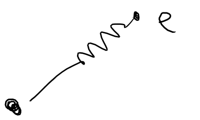
i) $\frac{d}{r} \ll 1$

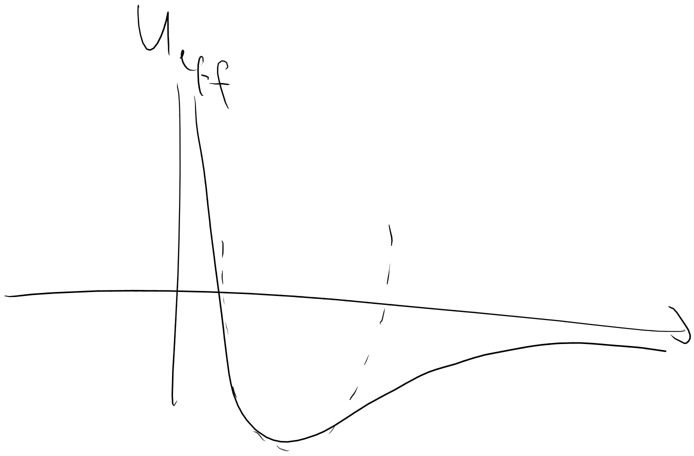



ii) $q_{lm} \simeq 0$ for $l > 1$

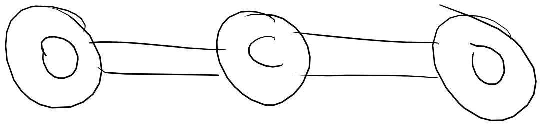
$$\vec{p}_{mol} = \gamma_{mol} \vec{E}_{loc}$$

$$\varepsilon, \chi \longrightarrow \gamma_{mol}$$



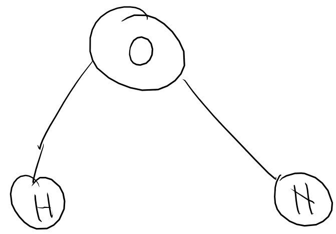



$CO_2$



$\vec{p} = 0$

$H_2O$



$\vec{p} \neq 0$

$$\boxed{P(\vec{p}) \propto e^{-\frac{V}{k_B T}}} \Rightarrow \langle \vec{p} \rangle_{mol} = \gamma_{mol} \vec{E}$$

$$\Rightarrow V = -\vec{p} \cdot \vec{E}$$

$$[\vec{p}] = [q\, d] = [q][d]$$

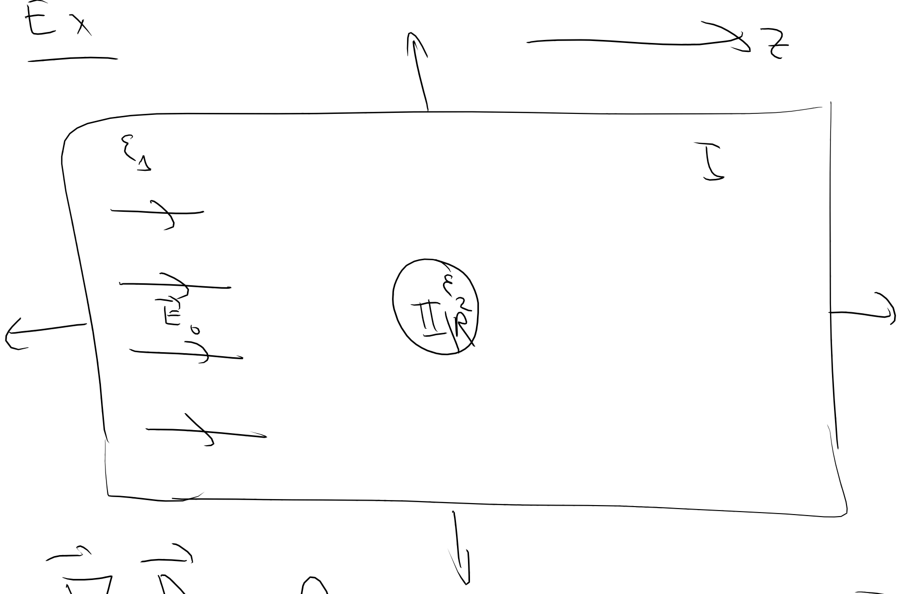
$$[\vec{E}] = \frac{[q]}{[d^2]}$$

$$[\gamma_{mol}] = \frac{[\vec{p}]}{[\vec{E}]} = \frac{\cancel{[q]}[d]}{\frac{\cancel{[q]}}{[d^2]}} = [d^3] = [\text{Volume}]$$

$$[\gamma_{mol}] \simeq v_0$$

$v_0$ : volume of the molecule

Ex




$$\vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot\vec{E} + 4\pi\underbrace{\vec{\nabla}\cdot\vec{P}}_{-\rho_b}$$

$$= 4\pi(\rho_{free} + \cancel{\rho_b}) + 4\pi(\cancel{-\rho_b})$$

$$\boxed{\vec{\nabla}\cdot\vec{D} = 4\pi\rho_{free}}$$

$$\vec{\nabla}\cdot\vec{D} = 0$$

$$\vec{\nabla}\times\vec{E} = 0 \Rightarrow \boxed{\vec{E} = -\vec{\nabla}\phi}$$

$$\vec{P} = (-\vec{E} + \vec{D})\frac{1}{4\pi}$$

$$\varepsilon = 1 + 4\pi\chi_e$$

$$\boxed{\vec{D} = \varepsilon\vec{E}}$$

$$\vec{\nabla}\cdot(\varepsilon\vec{E}) = 0$$

inside region I & II $\varepsilon$ = const

$$\varepsilon\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \boxed{\nabla^2\phi = 0}$$

The boundaries: $r = R$

$r = \infty$

$$\nabla^2\phi = 0$$

$$\phi(\vec{r}) = \sum_{l,m}\left(A_{lm}r^l + B_{lm}\frac{1}{r^{l+1}}\right)Y_{lm}(\Omega)$$

$$\phi^{II}(\vec{r}) = \sum_{lm} \left(A^{II}_{lm} r^l + B^{II}_{lm} \frac{1}{r^{l+1}}\right) Y_{lm}(\Omega)$$

$(\vec{r} = 0) \in$ Region II $\quad \phi^{II}$ should be finite

$$\Rightarrow \boxed{B^{II}_{lm} = 0}$$

$$\boxed{\phi^{II}(\vec{r}) = \sum_{lm} A^{II}_{lm} r^l Y_{lm}(\Omega)}$$

$$\phi^{I}(\vec{r}) = \sum_{lm} \left(A^{I}_{lm} r^l + \frac{B^{I}_{lm}}{r^{l+1}}\right) Y_{lm}(\Omega)$$

as $\vec{r} \to \infty$, $\vec{E}(\vec{r}) \to \vec{E}_0 = E_0 \hat{z}$

$$\phi^{I}(\vec{r}) \xrightarrow[r\to\infty]{} -E_0 z = -E_0 r\cos\theta$$

$$= -E_0 r P_1(\cos\theta)$$

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$$\phi^{II}(\vec{r}) = \sum_{lm} A^{II}_{lm} r^l Y_{lm}(\Omega)$$

$$\phi^{I}(\vec{r}) = \sum_{lm} \left( A^{I}_{lm} r^l + \frac{B^{I}_{lm}}{r^{l+1}} \right) Y_{lm}(\Omega)$$

$\phi^{I,II}(\vec{r})$ should be independent of $\varphi$ coordinate.

$$Y_{lm}(\Omega) \propto P_l^m(\cos\theta) e^{im\varphi}$$

$$A^{II}_{lm}, A^{I}_{lm}, B^{I}_{lm} = 0 \quad \text{if} \quad m \neq 0$$

$$\boxed{\begin{aligned} \phi^{I}(\vec{r}) &= \sum_{l} \left( a_l r^l + \frac{b_l}{r^{l+1}} \right) P_l(\cos\theta) \\ \phi^{II}(\vec{r}) &= \sum_{l} c_l r^l P_l(\cos\theta) \end{aligned}}$$

$$\int_{-1}^{1} d\theta \sin\theta \, P_l(\cos\theta) P_{l'}(\cos\theta) = \frac{2}{2l+1} \delta_{ll'}$$

$$\int_{-1}^{1} dx \, P_l(x) P_{l'}(x) = \delta_{ll'} \frac{2}{2l+1}$$

$$\phi^{I}(\vec{r}) \xrightarrow{r \to \infty} \sum_{l} a_l r^l P_l(\cos\theta) = -E_0 r P_1(\cos\theta)$$

$$a_1 r = -E_0 r \Rightarrow a_1 = -E_0$$

$$l \neq 1 \quad a_l r^l = 0 \Rightarrow a_l = 0 \quad \text{if} \quad l \neq 1$$

$$\boxed{\begin{aligned} \Phi^{II}(\vec{r}) &= \sum_{l=0}^{\infty} c_l r^l P_l(\cos\theta) \\ \Phi^{II}(\vec{r}) &= -E_0 r\cos\theta + \sum_{l=0}^{\infty} \frac{b_l}{r^{l+1}} P_l(\cos\theta) \end{aligned}}$$

$\vec{\nabla}\times\vec{E} = 0 \Rightarrow \Delta E_{\parallel} = 0 \Longrightarrow \Phi$ is continuous

$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \Delta D_{\perp} = 0$

$$\Phi^{I}(R) = \Phi^{II}(R) \Longleftarrow$$

$$\varepsilon_1 \left.\frac{\partial \Phi^{I}}{\partial r}\right|_{r=R} = \varepsilon_2 \left.\frac{\partial \Phi^{II}}{\partial r}\right|_{r=R} \Longleftarrow$$