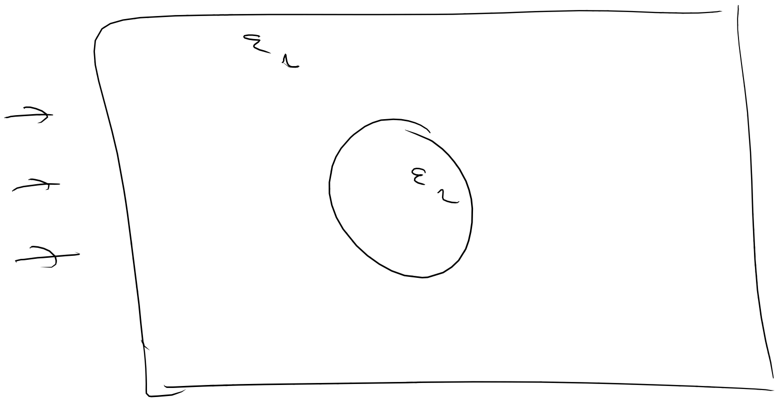
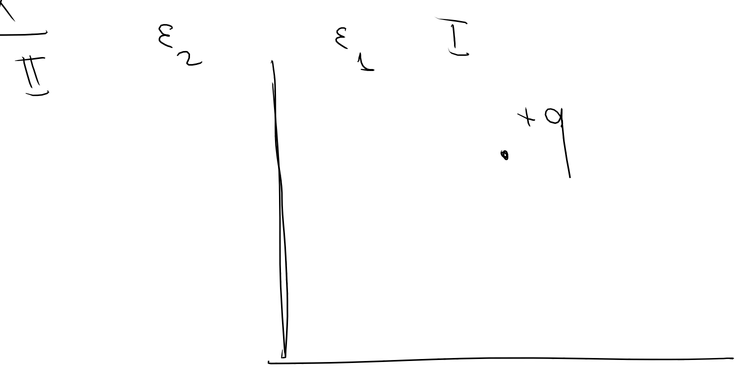
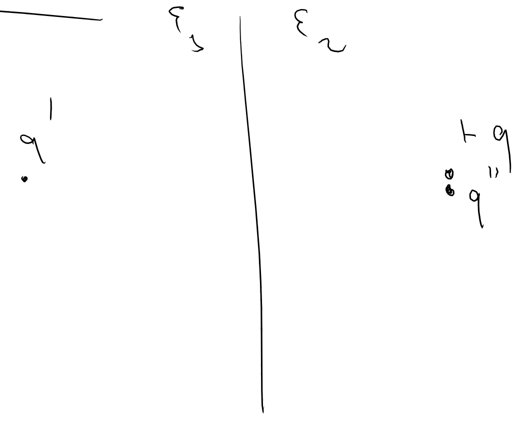
Exercise



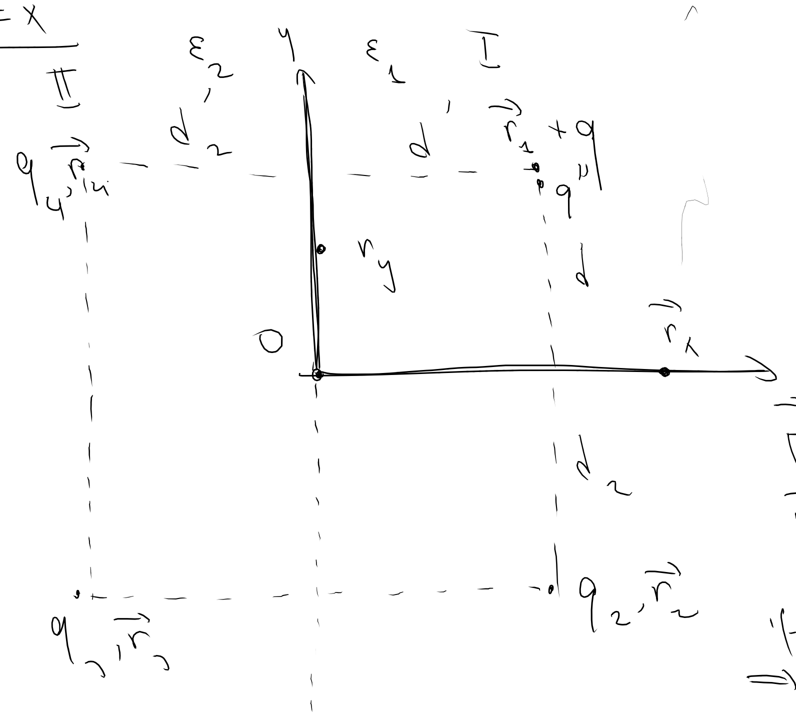
Ex



Review



Ex



q, q_1, q_2, q_3 for an observer in region 1

q'' for an observer in region 2.

$$\vec\nabla \times \vec E = 0$$

$$\vec\nabla \cdot \vec D = 4\pi \rho_{free}$$

if $\rho_{free} = 0$

$\Rightarrow D_\perp$ is continuous

- V should be continuous $\quad \vec D = \varepsilon \vec E = -\varepsilon \vec\nabla V$
- $\varepsilon \dfrac{\partial V}{\partial n}$ should be continuous

$$V^{I} = \frac{1}{\varepsilon_1} \frac{q}{|\vec r - \vec r_1|} + \frac{1}{\varepsilon_1} \frac{q_2}{|\vec r - \vec r_2|} + \frac{1}{\varepsilon_1} \frac{q_3}{|\vec r - \vec r_3|} + \frac{1}{\varepsilon_1} \frac{q_4}{|\vec r - \vec r_4|}$$

$$\vec\nabla \times \vec E = 0$$

$$\vec\nabla \cdot (\varepsilon \vec E) = 4\pi \rho_{free} \Rightarrow \vec\nabla \cdot \vec E = 4\pi \left(\frac{\rho_f}{\varepsilon}\right)$$

$$V^{II} = \frac{1}{\varepsilon_2} \frac{q''}{|\vec r - \vec r_1|}$$

$$V^{I}(\vec r_x) = V^{II}(\vec r_x)$$

$$\frac{1}{\varepsilon_1} \frac{q}{|\vec r_x - \vec r_1|} + \frac{1}{\varepsilon_1} \frac{q_2}{|\vec r_x - \vec r_2|} + \frac{1}{\varepsilon_1} \frac{q_3}{|\vec r_x - \vec r_3|} + \frac{1}{\varepsilon_1} \frac{q_4}{|\vec r_x - \vec r_4|}$$

$$= \frac{1}{\varepsilon_2} \frac{q''}{|\vec r_x - \vec r_1|}$$

$$\frac{1}{\varepsilon_1}\frac{q_1}{|\vec{r}_x-\vec{r}_1|}+\frac{1}{\varepsilon_1}\frac{q_2}{|\vec{r}_x-\vec{r}_2|}+\frac{1}{\varepsilon_1}\frac{q_3}{|\vec{r}_x-\vec{r}_3|}+\frac{1}{\varepsilon_1}\frac{q_4}{|\vec{r}_x-\vec{r}_4|}$$

$$=\frac{1}{\varepsilon_2}\frac{q''}{|\vec{r}_x-\vec{r}_1|}$$

$\vec{r}_x = r_x\hat{x}$; $r_x > 0$ $\qquad |\vec{r}_x-\vec{r}_1| = \left[(r_x-d')^2+d^2\right]^{1/2}$

$\vec{r}_1 = d'\hat{x} + d\hat{y}$ $\qquad |\vec{r}_x-\vec{r}_2| = \left[(r_x-d')^2+d_2^2\right]^{1/2}$

$\vec{r}_2 = d'\hat{x} - d_2\hat{y}$ $\qquad \left[\vec{r}_x-\vec{r}_3\right] = \left[(r_x+d_2')^2+d_2^2\right]^{1/2}$

$\vec{r}_3 = -d_2'\hat{x} - d_2\hat{y}$ $\qquad \left[\vec{r}_x-\vec{r}_4\right] = \left[(r_x+d_2')^2+d^2\right]^{1/2}$

$\vec{r}_4 = -d_2'\hat{x} + d\hat{y}$

$$\frac{1}{\varepsilon_1}\frac{q}{\left[(r_x-d')^2+d^2\right]^{1/2}}+\frac{1}{\varepsilon_1}\frac{q_2}{\left[(r_x-d')^2+d_2^2\right]^{1/2}}+\frac{1}{\varepsilon_1}\frac{q_3}{\left[(r_x+d_2')^2+d_2^2\right]^{1/2}}$$

$$+\frac{1}{\varepsilon_1}\frac{q_4}{\left[(r_x+d_2')^2+d^2\right]^{1/2}}=\frac{1}{\varepsilon_2}\frac{q''}{\left[(r_x-d')^2+d^2\right]^{1/2}}$$

$\forall\ r_x > 0$

$$\Rightarrow \boxed{\begin{array}{ll} d = d_2 & q_3 + q_4 = 0 \\ \frac{1}{\varepsilon_1}(q_1+q_2) = \frac{q''}{\varepsilon_2} & \end{array}}$$

$$\vec{r}_1' = d'\hat{x} + d\hat{y}$$
$$\vec{r}_2' = d'\hat{x} - d_2\hat{y}$$
$$\vec{r}_3' = -d_2'\hat{x} - d_2\hat{y}$$
$$\vec{r}_4' = -d_2'\hat{x} + d\hat{y}$$
$$\vec{r}_y' = r_y\hat{y}$$

$$|\vec{r}_1 - \vec{r}_y| = \left[d'^2 + (d - r_y)^2\right]^{1/2}$$
$$|\vec{r}_2 - \vec{r}_y| = \left[d'^2 + (d_2 + r_y)^2\right]^{1/2}$$
$$|\vec{r}_3 - \vec{r}_y| = \left[d_2'^2 + (d_2 + r_y)^2\right]^{1/2}$$
$$|\vec{r}_4 - \vec{r}_y| = \left[d_2'^2 + (d - r_y)^2\right]^{1/2}$$

$$\frac{1}{\varepsilon_1}\frac{q}{\left[d'^2 + (d - r_y)^2\right]^{1/2}} + \frac{1}{\varepsilon_1}\frac{q_2}{\left[d'^2 + (d_2 + r_y)^2\right]^{1/2}}$$
$$+ \frac{1}{\varepsilon_1}\frac{q_3}{\left[d_2'^2 + (d_2 + r_y)^2\right]^{1/2}} + \frac{1}{\varepsilon_1}\frac{q_4}{\left[d_2'^2 + (d - r_y)^2\right]^{1/2}}$$
$$= \frac{1}{\varepsilon_2}\frac{q''}{\left[d'^2 + (d - r_y)^2\right]^{1/2}}$$

$$\checkmark\ d' = d_2'$$

$$\boxed{\frac{1}{\varepsilon_1}(q + q_4) = \frac{1}{\varepsilon_2}q''}$$

$$\checkmark\ q_2 + q_3 = 0$$

$$\checkmark\ d = d_2 \qquad \checkmark\ q_3 + q_4 = 0 \Leftarrow$$

$$\boxed{\frac{1}{\varepsilon_1}(q + q_2) = \frac{q''}{\varepsilon_2}}$$

(1) $d' = d_2'$ (4) $q_2 = q_4$

(2) $d = d_2$ (5) $\frac{1}{\varepsilon_1}(q + q_4) = \frac{1}{\varepsilon_2}q''$ $\Leftarrow$

(3) $q_2 + q_3 = 0$

$$V^{I} = \frac{1}{\varepsilon_1}\frac{q}{|\vec{r}-\vec{r}_1|} - \frac{1}{\varepsilon_2}\frac{q_3}{\|\vec{r}-\vec{r}_2\|} + \frac{1}{\varepsilon_1}\frac{q_3}{|\vec{r}-\vec{r}_3|}$$

$$- \frac{1}{\varepsilon_1}\frac{q_3}{|\vec{r}-\vec{r}_4|}$$

$$V^{II} = \frac{1}{\varepsilon_2}\frac{q''}{|\vec{r}-\vec{r}_1|} = \frac{1}{\varepsilon_1}\frac{(q-q_3)}{|\vec{r}-\vec{r}_1|}$$

$$\vec{r}_1 = d'\hat{x} + d\hat{y} \qquad \vec{r} = x\hat{x} + y\hat{y}$$

$$\vec{r}_2 = d'\hat{x} - d\hat{y} \qquad |\vec{r}-\vec{r}_1| = \left[(x-d')^2 + (y-d)^2\right]^{1/2}$$

$$\vec{r}_3 = -d'\hat{x} - d\hat{y} \qquad |\vec{r}-\vec{r}_2| = \left[(x-d')^2 + (y+d)^2\right]^{1/2}$$

$$\vec{r}_4 = -d'\hat{x} + d\hat{y} \qquad |\vec{r}-\vec{r}_3| = \left[(x+d')^2 + (y+d)^2\right]^{1/2}$$

$$|\vec{r}-\vec{r}_4| = \left[(x+d')^2 + (y-d)^2\right]^{1/2}$$

$$V^{II} = \frac{1}{\varepsilon_1}\frac{(q-q_3)}{\left[(x-d')^2+(y-d)^2\right]^{1/2}}$$

$$V^{I} = \frac{1}{\varepsilon_1}\frac{q}{\left[(x-d')^2+(y-d)^2\right]^{1/2}} - \frac{1}{\varepsilon_1}\frac{q_3}{\left[(x-d')^2+(y+d)^2\right]^{1/2}}$$

$$+ \frac{1}{\varepsilon_1}\frac{q_3}{\left[(x+d')^2+(y+d)^2\right]^{1/2}} - \frac{1}{\varepsilon_1}\frac{q_3}{\left[(x+d')^2+(y-d)^2\right]^{1/2}}$$

$$\varepsilon_1 \frac{\partial V^{I}}{\partial y}\bigg|_{y=0} = \varepsilon_2 \frac{\partial V^{II}}{\partial y}\bigg|_{y=0}$$

$$\varepsilon_1 \frac{\partial V^{I}}{\partial x}\bigg|_{x=0} = \varepsilon_2 \frac{\partial V^{I}}{\partial x}\bigg|_{x=0}$$

$$\varepsilon_2 \frac{\partial V^{II}}{\partial y}\bigg|_{y=0} = \varepsilon_2 \frac{\partial}{\partial y} \frac{1}{\varepsilon_1} \frac{(q-q_3)}{\left[(x-d')^2+(y-d)^2\right]^{1/2}}\Bigg|_{y=0}$$

$$= \frac{\varepsilon_2}{\varepsilon_1} (q-q_3) \frac{-(y-d)}{\left[(x-d')^2+(y-d)^2\right]^{3/2}}\Bigg|_{y=0}$$

$$\varepsilon_2 \frac{\partial V^{II}}{\partial y}\bigg|_{y=0} = + \frac{\varepsilon_2}{\varepsilon_1} (q-q_3) \frac{(+d)}{\left[(x-d')^2+d^2\right]^{3/2}}$$

$$\varepsilon_1 \frac{\partial V^{I}}{\partial y}\bigg|_{y=0} = \cancel{\varepsilon_1} \frac{\partial}{\partial y} \Bigg\{$$

$$\cancel{\frac{1}{\varepsilon_1}} \frac{q}{\left[(x-d')^2+(y-d)^2\right]^{1/2}} - \cancel{\frac{1}{\varepsilon_1}} \frac{q_3}{\left[(x-d')^2+(y+d)^2\right]^{1/2}}$$

$$+ \cancel{\frac{1}{\varepsilon_1}} \frac{q_3}{\left[(x+d')^2+(y+d)^2\right]^{1/2}} - \cancel{\frac{1}{\varepsilon_1}} \frac{q_3}{\left[(x+d')^2+(y-d)^2\right]^{1/2}} \Bigg\}\Bigg|_{y=0}$$

$$= \frac{q\,d}{\left[(x-d')^2+d^2\right]^{3/2}} + \frac{q_3\,d}{\left[(x-d')^2+d^2\right]^{3/2}} - \cancel{\frac{q_3\,d}{\left[(x+d')^2+d^2\right]^{3/2}}}$$

$$+ \cancel{\frac{q_3\,d}{\left[(x+d')^2+d^2\right]^{3/2}}}$$

$$\frac{\varepsilon_2}{\varepsilon_1} (q-q_3) \frac{d}{\left[(x-d')^2+d^2\right]^{3/2}} = (q+q_3) \frac{d}{\left[\quad\right]^{3/2}}$$

$$\boxed{\frac{\varepsilon_2}{\varepsilon_1}(q - q_3) = q + q_3}$$

$$\frac{\varepsilon_2}{\varepsilon_1} q - q = \frac{\varepsilon_2}{\varepsilon_1} q_3 + q_3 \Rightarrow \boxed{q_3 = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} q}$$

$$q'' = \frac{\varepsilon_2}{\varepsilon_1}(q + q_4)$$

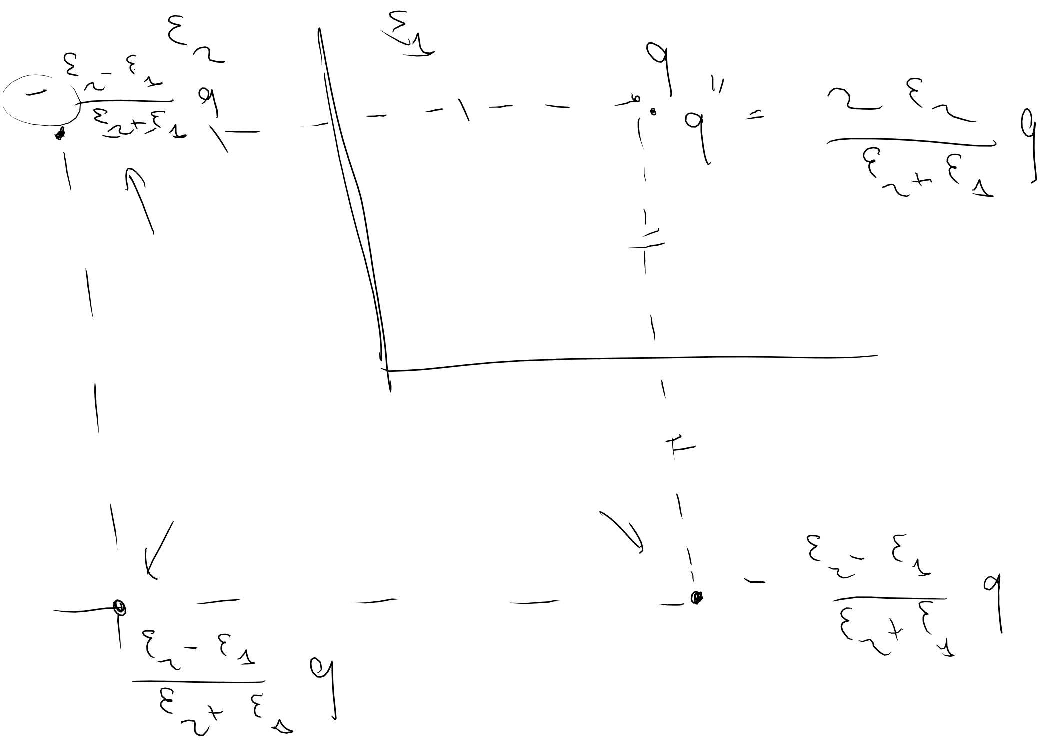
$$= \frac{\varepsilon_2}{\varepsilon_1} q \left(1 - \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}\right)$$

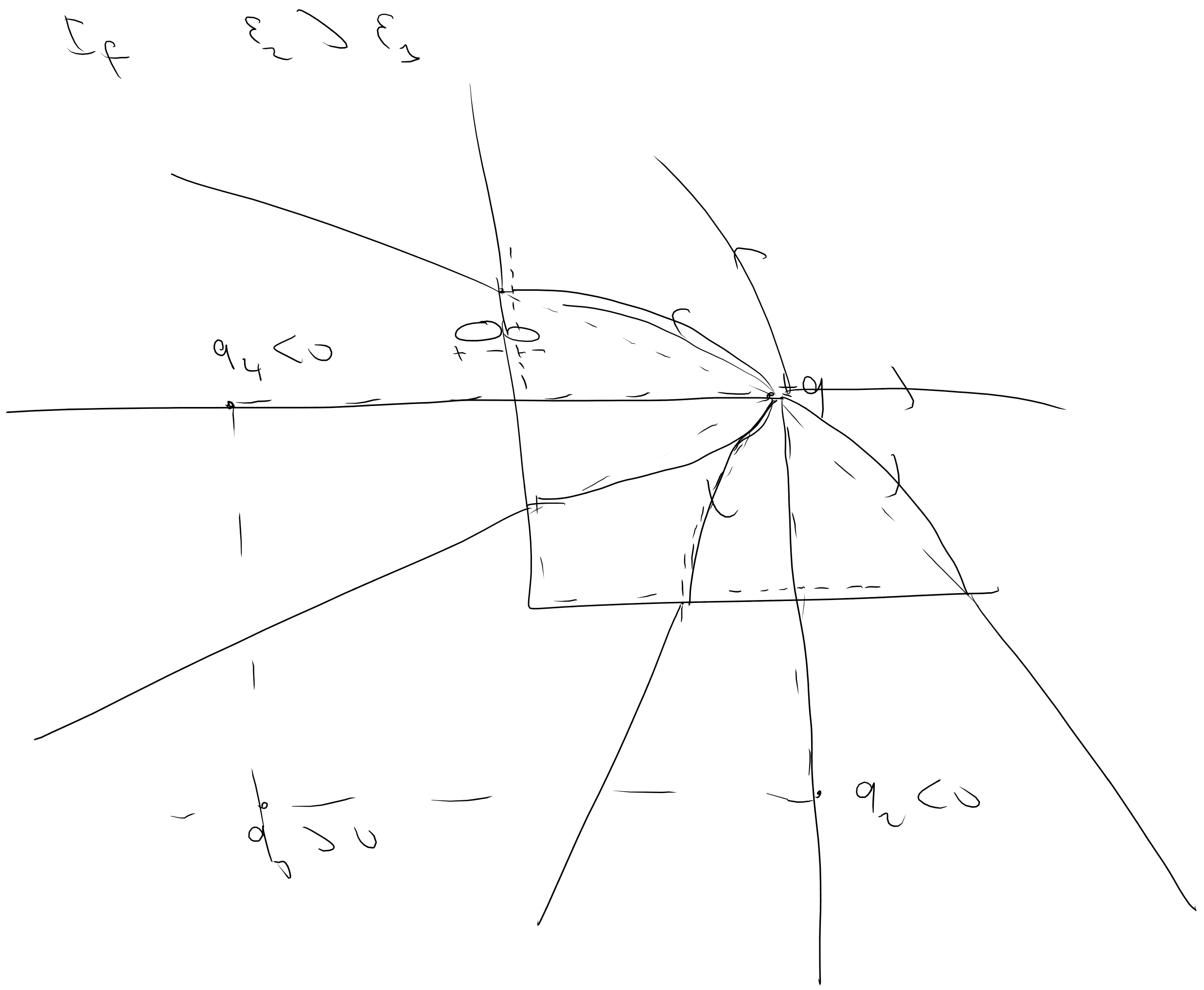
$$= \frac{\varepsilon_2}{\cancel{\varepsilon_1}} q \frac{2 \cancel{\varepsilon_1}}{\varepsilon_2 + \varepsilon_1} = \frac{2\varepsilon_2}{\varepsilon_2 + \varepsilon_1} q$$

$$q_2 = -\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} q$$

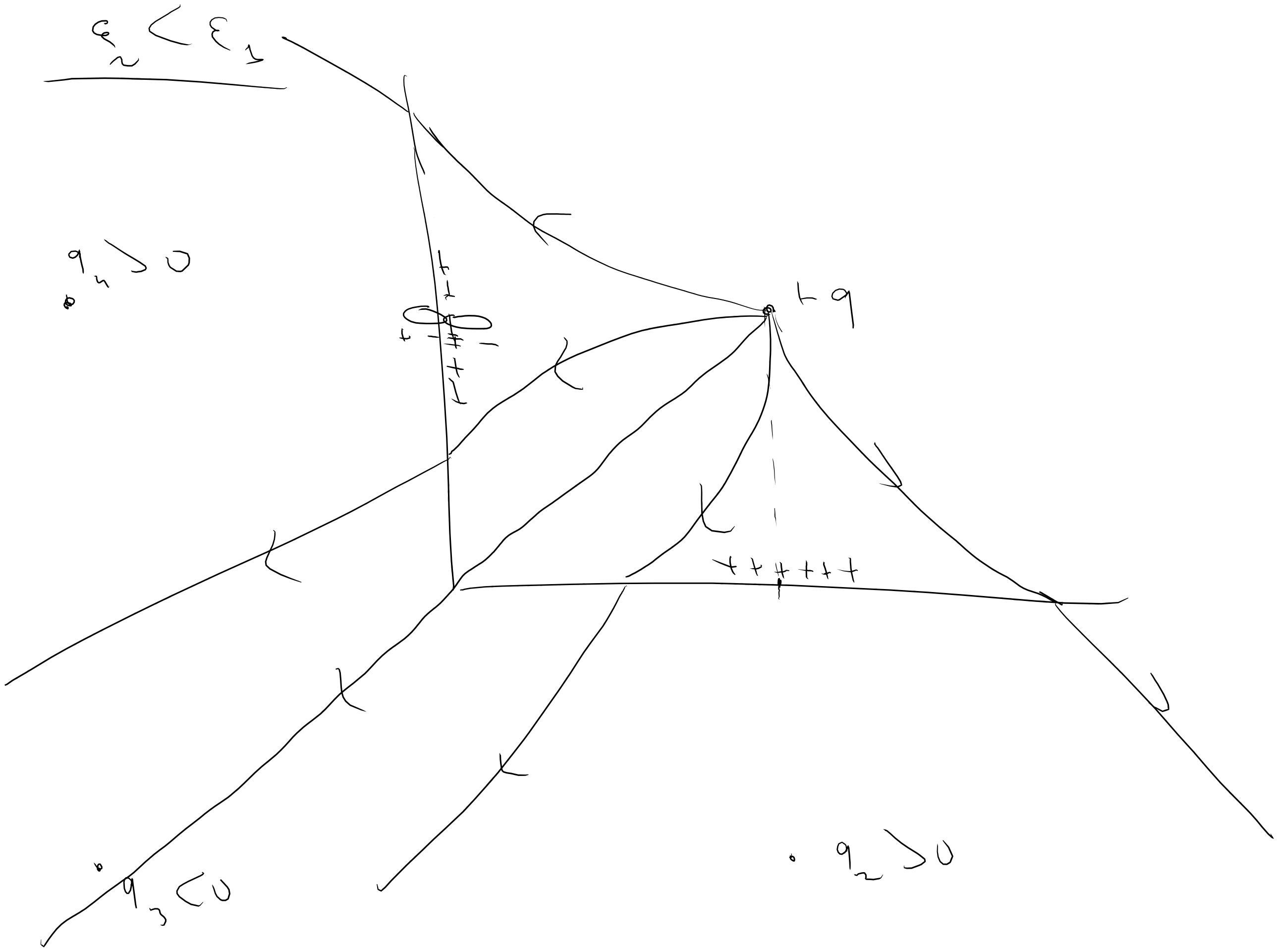
$$q_4 = \cancel{-} \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} q$$

$$q'' = \frac{2\varepsilon_2}{\varepsilon_2 - \varepsilon_1} q$$





$\varepsilon_2 > \varepsilon_1$
$a_4 < 0$
a_1
$a_2 < 0$
$a_3 > 0$



$\sigma_b = ?$ on the interface?

$$\left.\frac{\partial V^{II}}{\partial y}\right|_{y=0} = \frac{1}{\varepsilon_1}(q - q_1)\frac{(+d)}{[(x-d')^2 + d^2]^{3/2}}$$

$$\left.\frac{\partial V^{I}}{\partial y}\right|_{y=0} = \frac{1}{\varepsilon_1}\frac{(q+q_1)\,d}{[(x-d')^2 + d^2]^{3/2}}$$

$$\vec{\nabla} \times \vec{E} = 0 \Rightarrow \Delta E_{\parallel} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi(\rho_b + \rho_f) \Rightarrow \Delta E_{\perp} = 4\pi(\sigma_f + \sigma_b)$$

$$\vec{E}^{\,outside} - \vec{E}^{\,inside} = 4\pi\sigma\,\hat{n}$$

$\hat{n}$ is a unit vector pointing from "inside" the surface to the "outside"

$$\frac{1}{\varepsilon_1}\frac{\cancel{q}+q_1}{\left[(x-d')^2+d^2\right]^{3/2}} - \frac{1}{\varepsilon_1}\frac{(\cancel{q}-q_1)}{\left[(x-d')^2+d^2\right]^{3/2}}$$

$$= -4\pi\sigma$$

$$\frac{2q_1}{\varepsilon_1}\frac{1}{\left[(x-d')^2+d^2\right]^{3/2}} = -4\pi\sigma$$

$$q_1 = \frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\,q$$

$$\boxed{\sigma_b = -\frac{1}{2\pi\varepsilon_1}\left(\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\right) q\,\frac{1}{\left[(x-d')^2+d^2\right]^{3/2}}}$$

$$\vec{\nabla}\cdot\vec{D} = 4\pi\rho_{free}$$

$$\vec{\nabla}\times\vec{E} = 0 \Rightarrow \vec{E} = -\nabla\Phi$$

$$\vec{D} = \varepsilon\vec{E} = -\varepsilon\vec{\nabla}\Phi$$

$$\vec{\nabla}(\varepsilon\vec{\nabla}\Phi) = -4\pi\rho_{free}$$

$$\rho = \rho_{free} + \rho_{bound}$$

$\varepsilon = const$ inside our regions

$\rho_f = 0$

$$\nabla^2 \Phi = 0 \quad \text{or} \quad \vec{\nabla} \cdot \vec{E} = 0$$

$$\Rightarrow \rho = 0 \Rightarrow \rho_{bound} = 0$$

$$0 \overset{!}{=} \rho_{bound} = - \vec{\nabla} \cdot \vec{P}$$