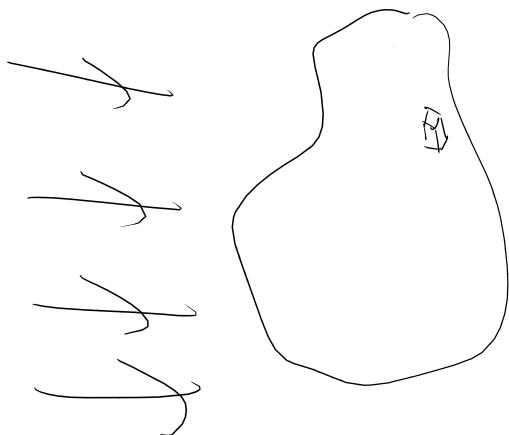


$$\vec{V} = A \hat{x} + B \hat{y} + C \hat{z}$$

$$\vec{T} = T_{xy} \hat{x} \hat{y} + T_{xz} \hat{x} \hat{z} + \dots$$

$$U = \int \frac{1}{r^2} \vec{E} \cdot \vec{D} dr$$

$$U = \int \frac{1}{r^2} \vec{E}^2 dr$$

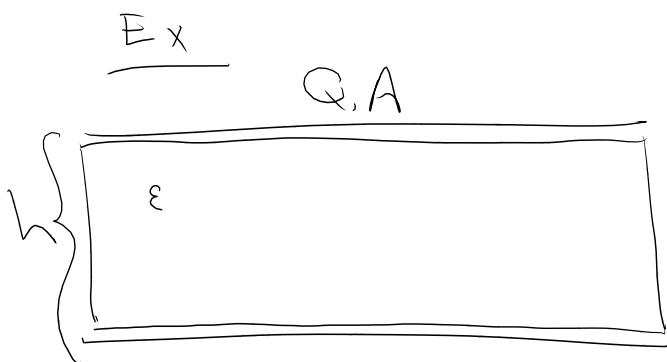


$$f = \frac{1}{4\pi} \int \vec{D} \cdot \vec{E} = g E$$

$$g_b, \sigma_b \sim \frac{1}{E}$$

$$+ (\rho_b, \sigma_b) E$$

$$f \propto g E + \epsilon E^2$$



$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} \frac{Q^2}{C}$$

$$\vec{\nabla} \cdot \vec{D} = \text{ns} g_f \Rightarrow \text{Gauss Law for } \vec{D}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{\nabla} \cdot (\epsilon \vec{E}) = \epsilon \vec{\nabla} \cdot \vec{E}$$

$$\boxed{\vec{D} \cdot \vec{E} = \frac{\text{ns}}{\epsilon} \rho_f}$$

$$|\vec{E}| = \epsilon_0 \frac{\sigma}{\epsilon} = \frac{\sigma}{\epsilon} \Rightarrow V = \frac{V^0}{\epsilon}$$

$$Q = C \Delta V$$

$$C = \frac{Q}{\Delta V} = \epsilon \frac{Q}{\Delta V^0} = \epsilon C^0$$

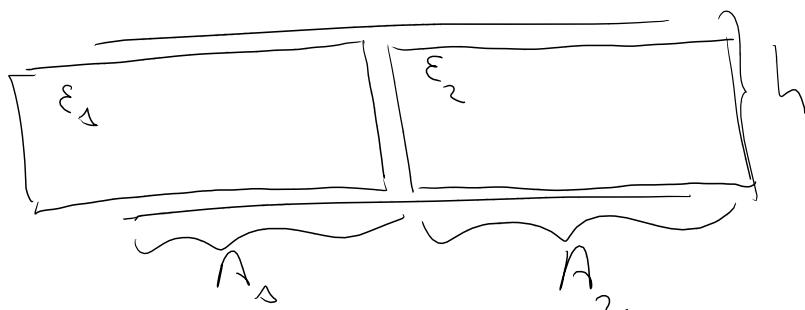
$$E = \epsilon_0 \frac{\sigma}{\epsilon} = \epsilon_0 \frac{Q}{A \epsilon} \quad D = \epsilon_0 \sigma$$

$$\Delta V = E h = \epsilon_0 \frac{Q}{A \epsilon} h = \frac{Q}{C}$$

$$C^0 = \frac{1}{\epsilon_0} \frac{A}{h}$$

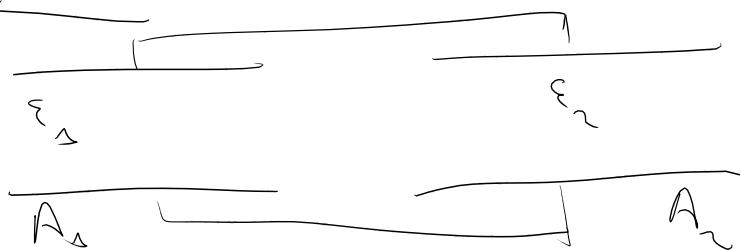
$$C = \frac{1}{\epsilon_0} \frac{A}{h} \epsilon$$

Example

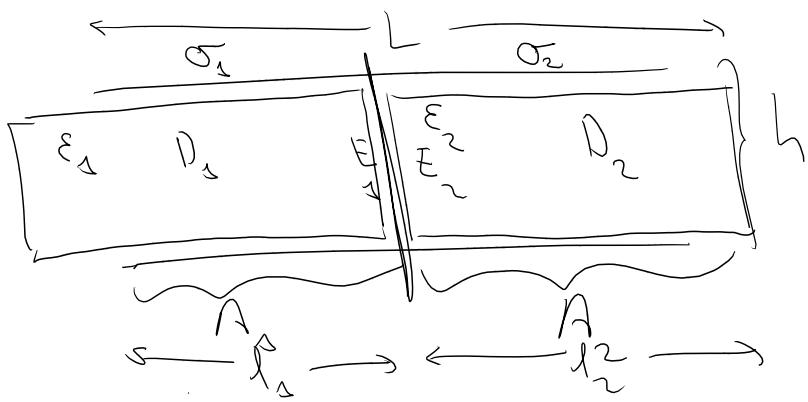


$$C = ?$$

method 1



$$\begin{aligned} C &= \epsilon_1 + \epsilon_2 \\ &= \frac{1}{\epsilon_0} h \left(\frac{\epsilon_1}{A_1} + \frac{\epsilon_2}{A_2} \right) \end{aligned}$$



$$\frac{A_1}{A_2} = \frac{l_1}{l_2}$$

$$L = l_1 + l_2$$

d = depth

$$A_1 = l_1 d$$

$$A_2 = (L - l_1)d$$

$$A_1 + A_2 = Ld$$

$$D_1 = h \sigma_1 \alpha_1$$

$$D_2 = h \sigma_2 \alpha_2$$

$$E_1 = E_2$$

$$\frac{D_1}{\varepsilon_1} = \frac{D_2}{\varepsilon_2} \Rightarrow \frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2}$$

$$\Delta V = E_1 h = h \sigma_1 \frac{\sigma_1}{\varepsilon_1} h$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{h \sigma_1 \frac{\sigma_1}{\varepsilon_1} h}$$

$$\frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2}$$

$$\begin{aligned} Q &= \sigma_1 A_1 + \sigma_2 A_2 \\ &= \sigma_1 A_1 + \frac{\varepsilon_2}{\varepsilon_1} \sigma_1 A_2 \end{aligned}$$

$$\sigma_1 = \frac{Q}{A_1 + \frac{\varepsilon_2}{\varepsilon_1} A_2}$$

$$C = \frac{h \sigma_1}{\frac{Q}{A_1 + \frac{\varepsilon_2}{\varepsilon_1} A_2}}$$

$$C = \frac{\varepsilon_1 A_1 + \varepsilon_2 A_2}{h \sigma_1} = \frac{\varepsilon_1}{h \sigma_1} \frac{A_1}{h} + \frac{\varepsilon_2}{h \sigma_1} \frac{A_2}{h}$$

$$U = \frac{1}{2} C (AV)^2 = \frac{1}{2} \frac{Q^2}{C}$$

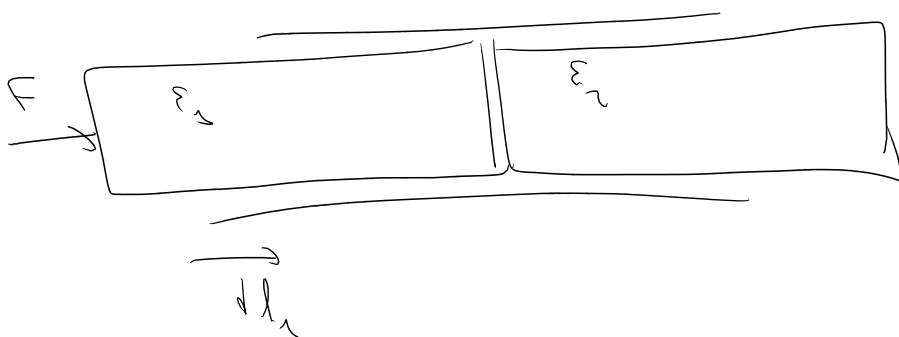
$$\frac{dU}{dl_1} = \frac{1}{2} Q^2 \left(-\frac{1}{C^2} \right) \frac{dC}{dl_1}$$

$$C = \frac{\epsilon_1}{4\pi} \frac{l_1 d}{h} + \frac{\epsilon_2}{4\pi} \frac{(L-l_1)d}{h}$$

$$\frac{dC}{dl_1} = \frac{\epsilon_1}{4\pi} \frac{d}{h} - \frac{\epsilon_2}{4\pi} \frac{d}{h}$$

$$\frac{dC}{dl_1} = \frac{(\epsilon_1 - \epsilon_2)}{4\pi} \frac{d}{h}$$

$$\boxed{\frac{dU}{dl_1} < -\frac{1}{2} \frac{Q^2}{C^2} \frac{(\epsilon_1 - \epsilon_2)}{4\pi} \frac{d}{h}}$$



Electrostatics

$$\boxed{dW = dU}$$

$$dW = F dl_1 = dU = \frac{dU}{dl_1} dl_1$$

$$F = \frac{dU}{dl_1} = -\frac{1}{2} \frac{Q^2}{C^2} \frac{(\epsilon_1 - \epsilon_2)}{4\pi} \frac{d}{h}$$

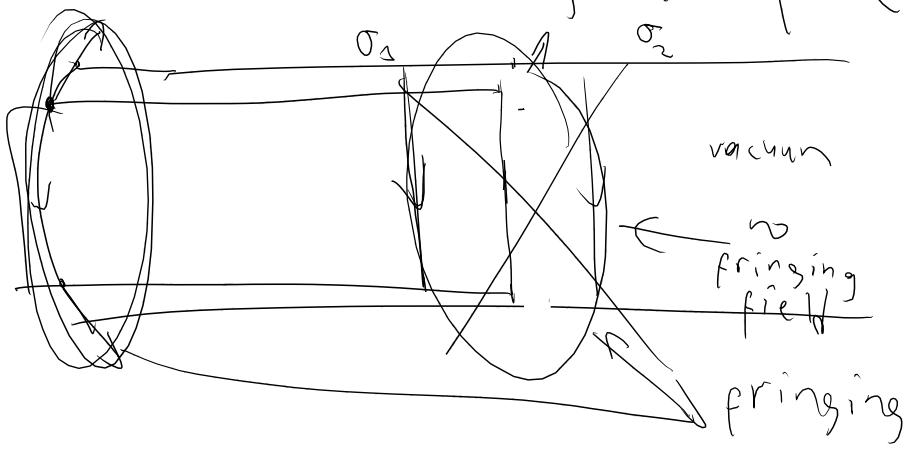
$$F_z = -\frac{1}{2} \frac{Q^2}{C^2} \frac{(\epsilon_3 - \epsilon_2)}{4\pi} \frac{d}{h}$$

$$P = \frac{F}{A} = -\frac{1}{2} \frac{Q^2}{C^2} \frac{(\epsilon_3 - \epsilon_2)}{4\pi} \frac{d}{h} \frac{1}{h}$$

$$P = -\frac{1}{8\pi} \left(\frac{\Delta V}{h} \right)^2 (\epsilon_3 - \epsilon_2)$$

$$P = -\frac{1}{8\pi} E^2 (\epsilon_3 - \epsilon_2)$$

What is creating the force?



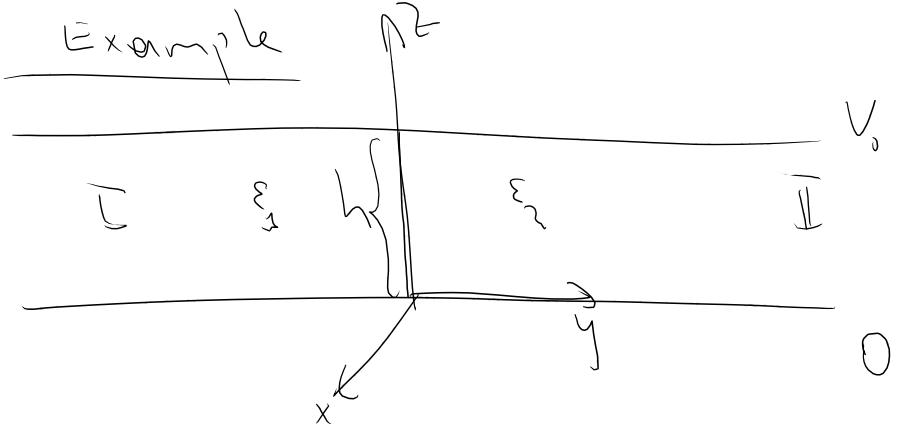
$$\frac{\sigma_2}{\epsilon_2} = \frac{\sigma_1}{\epsilon_1}$$

$$D_1 = 4\pi \sigma_1$$

$$\frac{\epsilon_3}{D_3} = \frac{E_2}{D_2}$$

$$D_3 \neq D_2$$

$$P \propto \frac{\partial \epsilon}{\partial n}$$



$$V(y, z)$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{\nabla} \cdot (\epsilon \vec{\nabla} V) = 0$$

in region I, II $\epsilon = \text{const}$

$$\vec{\nabla}^2 V = 0 = \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

~~$$V = Y(y) Z(z)$$~~

~~$$\frac{Y''}{Y} = C_1, \quad \frac{Z''}{Z} = C_2$$~~

~~$$C_1 + C_2 = 0$$~~

$$Z(z=0) = 0$$

$$V(y \rightarrow \pm\infty, z) = \frac{V_0}{h} z$$

$$\tilde{V} = V - \frac{V_0}{h} z$$

$$\nabla^2 \tilde{V} = 0$$



$$\tilde{V}(y \rightarrow \pm\infty, z) = 0 \quad \tilde{\epsilon}(z = h) = 0$$

$$\tilde{V} = Y(y) \tilde{\epsilon}(z) \quad \tilde{\epsilon}(z = 0) = 0$$

$$\frac{Y''}{Y} = C_1 = k^2 \quad \frac{Z''}{Z} = C_2 = -k^2$$

$$C_1 + C_2 = 0 \quad \tilde{\epsilon}(z = h) = 0 \quad k_n = \frac{n\pi}{h}$$

$$\tilde{\epsilon}(z) = A_n \sin(kz)$$

$$Y(y) = B_n e^{ky} + C_n e^{-ky}$$

in region I, $Y(y \rightarrow -\infty) = 0$

$$\Rightarrow Y^I(y) = B_n e^{ky}$$

in region I, $Y^I(y \rightarrow \infty) = 0$

$$Y^I(y) = C_n e^{-ky}$$

$$V^E = \sum_n A_n \sin\left(\frac{n\pi}{h}z\right) e^{\frac{n\pi}{h}y}$$

$$V^H = \sum_n B_n \sin\left(\frac{n\pi}{h}z\right) e^{-\frac{n\pi}{h}y}$$

$$V^E(y=0) = V^H(y=0)$$

$$\Rightarrow \sum_n A_n \sin\left(\frac{n\pi}{h}z\right) = \sum_n B_n \sin\left(\frac{n\pi}{h}z\right)$$

$$\Rightarrow A_n = B_n$$

$$\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \nabla D_z = 0$$

$$\Rightarrow \varepsilon_1 E_{z1} = \varepsilon_2 E_{zz}$$

$$\Rightarrow \varepsilon_1 \left. \frac{\partial V^E}{\partial y} \right|_{y=0} = \varepsilon_2 \left. \frac{\partial V^H}{\partial y} \right|_{y=0}$$

$$\sum_n \varepsilon_1 A_n \left(\frac{n\pi}{h} \right) \sin\left(\frac{n\pi}{h}z\right) = \sum_n \varepsilon_2 B_n \left(-\frac{n\pi}{h} \right) \sin\left(\frac{n\pi}{h}z\right)$$

$$\Rightarrow \begin{cases} \varepsilon_1 A_n = -\varepsilon_2 B_n \\ A_n = B_n \end{cases} \quad A_n = B_n = 0$$

$$\Rightarrow \tilde{V} = 0$$

$$V = \frac{V_0}{h} z$$

no fringing effect
at the junction of
the dielectrics!

$$\vec{E} = -\vec{\nabla} V = -\frac{V_0}{h} \hat{z}$$

$$\vec{E}_1 \parallel \vec{E}_2$$

$$O = D_{11} = D_{21}$$

$$N(D_2) \propto \sigma_f$$

$$\Delta(E_1) \propto \sigma = \sigma_f + \sigma_b$$

$$f \neq E^*(\bar{\nabla}_E) \neq 0$$

$$V = \frac{V_0}{h} z$$

$$\vec{E} = - \frac{\nabla \phi}{k}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\text{D} \stackrel{?}{=} \text{E}$$

$$\vec{D} \cdot \vec{P}_{\perp} = \vec{D} \cdot \vec{P}_{\parallel} = 0$$

$$D^{\frac{1}{2}} \in \mathcal{L}^2$$

$$V(y, z = 0) = 0$$

$$V(y, z = h) = V_0$$

$$\Delta(E_{||}(y=0)) = 0 \quad \checkmark$$

$$\Delta(D_{\gamma}(y=0)) = 0 \quad \checkmark$$

$$V(y \rightarrow_{QA_3} z) = \frac{V_0}{h} z$$

