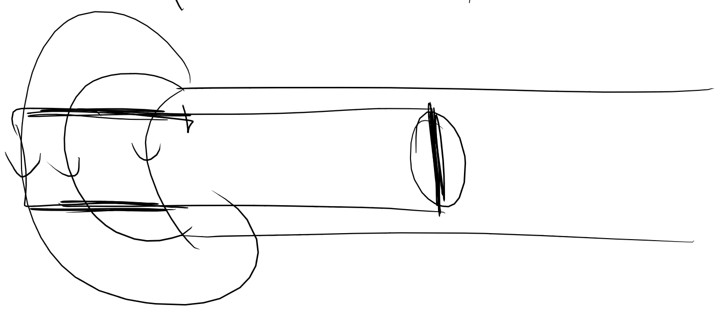
$$\boxed{\vec f = \rho \vec E - (\,)\vec E^2 (\vec\nabla \varepsilon)}$$

if $\rho = 0$

$$\vec f \propto -(\vec\nabla \varepsilon)\vec E^2$$



$$U = \int \frac{1}{2}\vec E\cdot\vec D \, d^3r$$

$\vec E, \vec D$: $\vec E = 0$ inside conductor

$\vec E$ perpendicular to the surface of the conductor

$$\vec D = \varepsilon \vec E$$

$$\vec\nabla\cdot\vec D = 4\pi\rho$$

$$\vec\nabla\times\vec E = 0$$

$\vec E', \vec D'$ satisfy the same conditions

$$\Delta U = \int\left[\frac{1}{2}\vec E'\cdot\vec D' - \frac{1}{2}\vec E\cdot\vec D\right]d^3r \geq 0$$

$$\Delta U' = \int\left[\frac{1}{2}\vec E\cdot\vec D - \frac{1}{2}\vec E'\cdot\vec D'\right]d^3r \geq 0$$

$$\Rightarrow \Delta U = 0$$

$$\Delta U \propto \int (\vec{E}'')^2 = 0$$

$$\Rightarrow \vec{E}'' = \vec{E}' - \vec{E} = 0$$

$\vec{E}'$ & $\vec{D}'$: $\vec{E}$ zero inside conductor
perpendicular to the
surface of conductors

$\vec{r}$: observation point (no primes)

$\vec{r}'$: position of the charge creating $\vec{E}$, ϕ, etc. (single prime)

$\vec{r}''$: averaging coordinate.

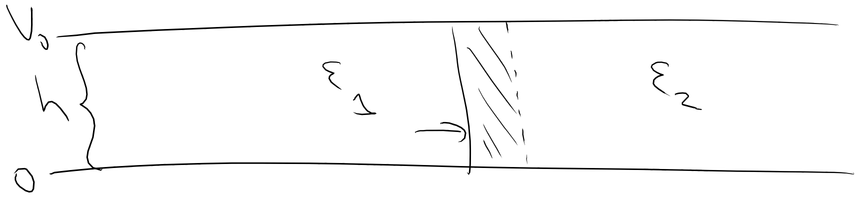
$$\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}$$

$$\vec{E}(\vec{r};\vec{r}') = \frac{q}{|\vec{r} - \vec{r}'|^2} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|} = q \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r};\vec{r}') = \frac{\partial E_x}{\partial x} + \dots$$

$$\vec{\nabla}_{\vec{r}'} \cdot \vec{E}(\vec{r};\vec{r}') = \vec{\nabla}' \cdot \vec{E}(\vec{r};\vec{r}') \equiv \frac{\partial E_x}{\partial x'} + \dots$$

$$\vec{\nabla} f(\vec{r} - \vec{r}') = -\vec{\nabla}_{\vec{r}'} f(\vec{r} - \vec{r}')$$



$$\boxed{V = \frac{V_0}{h} z}$$

$$U_i = \int \frac{1}{2} \vec{E}_1 \cdot \vec{D}_1 = \int \frac{1}{2} \varepsilon_1 \vec{E}^2$$

$$U_f = \int \frac{1}{2} \vec{E}_2 \cdot \vec{D}_2 = \int \frac{1}{2} \varepsilon_2 \vec{E}^2$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\Delta U = \int \frac{1}{2} (\varepsilon_1 - \varepsilon_2) \vec{E}^2$$

$$\boxed{\vec{k} = (\rho_f + \rho_b) \vec{E}}$$

$$= \left[\rho_f \cancel{+} (\vec{\nabla} \cdot \vec{P})\right] \vec{E}$$

$$= \left[\rho_f - \vec{\nabla} \cdot \left((\varepsilon - 1) \vec{E} \right)\right] \vec{E}$$

$$= \left[\rho_f - (\vec{\nabla} \varepsilon) \cdot \vec{E} - (\varepsilon - 1)(\vec{\nabla} \cdot \vec{E})\right] \vec{E}$$

$$\vec{k} = \rho_f \vec{E} - \left[(\vec{\nabla} \varepsilon) \cdot \vec{E}\right] \vec{E} - (\varepsilon - 1)(\vec{\nabla} \cdot \vec{E}) \vec{E}$$

$$\int (\vec{k} \cdot \delta \vec{r}) \, dV = \int \left[\rho_f \vec{E} - \vec{E}^2 \nabla \varepsilon\right] \cdot \delta \vec{r} \, dV$$

+ additional term

$$\int (\text{additional term}) \cdot \delta \vec{r} \, dV = 0$$

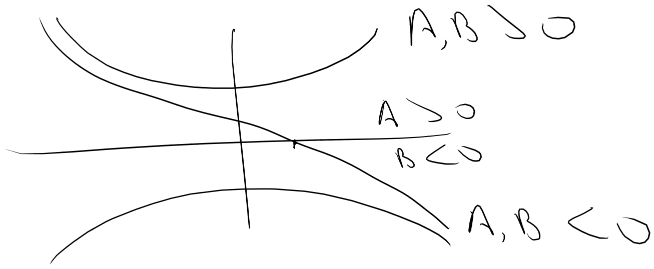
$$\rho_f \vec{E} - \vec{E}^2 \nabla \varepsilon + \text{additional term} \parallel \vec{E}$$

$$f'' + C f = 0$$

$C < 0$: f can be zero at most once

$C = -k^2$

$$f = A e^{kx} + B e^{-kx}$$



$f > 0 \quad \forall x$ if $A, B > 0$

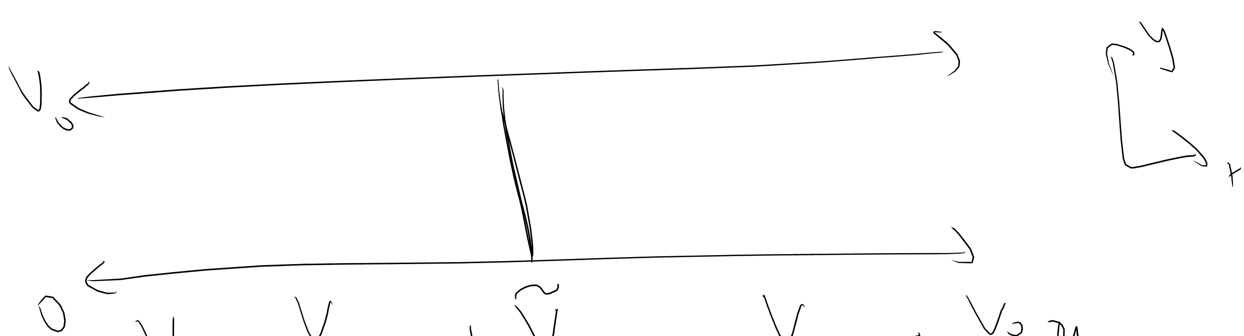
$f < 0 \quad \forall x$ if $A, B < 0$

f is never periodic

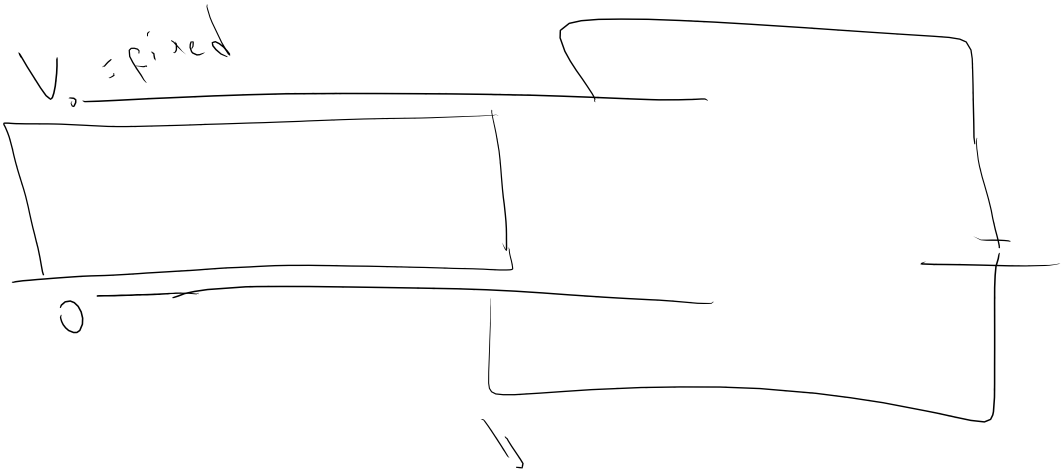
If $C > 0$

f is always periodic

never goes to zero at infinity!

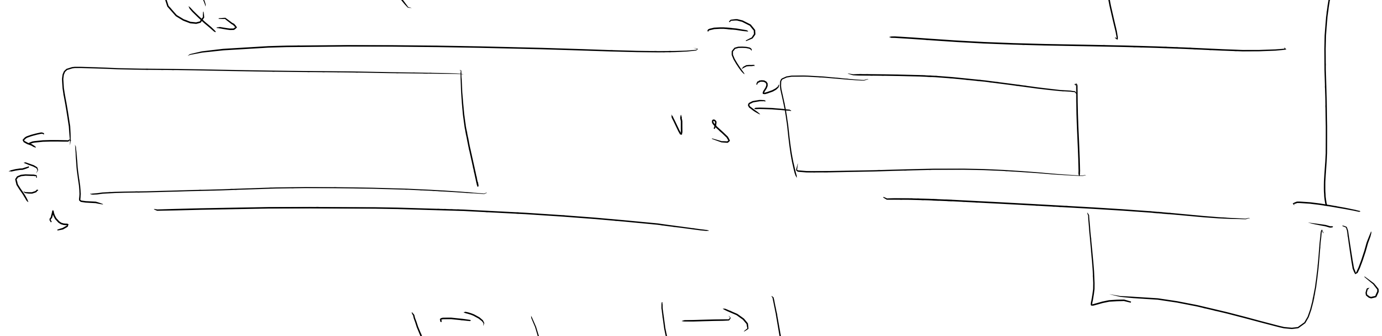


$V = V_{asyn} + \tilde{V}$ $\qquad V_{asyn} = \frac{V_0}{h} y$



$$\Rightarrow \Delta W + \Delta Q\, V_0 = \Delta U$$

$$\frac{\Delta W}{\Delta x} \to F \qquad \boxed{Q_0 = C V_0}$$



$$|\vec{F}_1| = |\vec{F}_2|$$

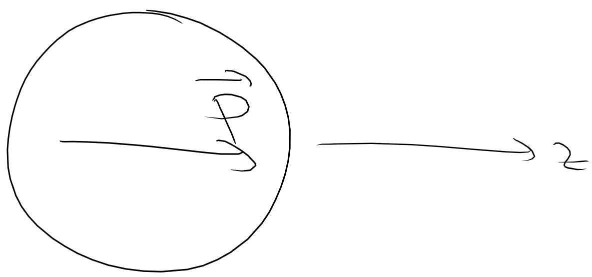
$$U = \frac{1}{2}\frac{Q^2}{C} \qquad U = \frac{1}{2} C V_0^2$$

$$\frac{dU}{dx} = -\frac{1}{2}\frac{Q^2}{C^2}\frac{dC}{dx} \qquad \frac{dU}{dx} = \frac{1}{2} V_0^2 \frac{dC}{dx}$$

$$\frac{dU}{dx} = +\frac{1}{2}\left(\frac{Q}{C}\right)^2 \frac{dC}{dx}$$

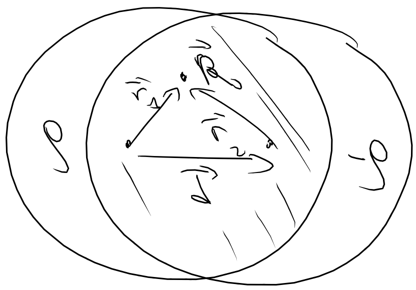
$$\left(\frac{\Delta Q}{\Delta x}\right) V_0 = \left(\frac{\Delta C}{dx}\right) V_0^2 = \frac{dC}{dx} V_0^2$$

$$|\vec{F}| = \frac{dU}{dx} - \left(\frac{dQ}{dx}\right) V_0 = -\frac{1}{2}\frac{Q^2}{C^2}\frac{dC}{dx}$$



$$\rho_b = -\vec\nabla\cdot\vec P = 0$$

$$\sigma_b = \vec P\cdot\hat n = P\,\hat z\cdot\hat n = \boxed{P\cos\Theta = \sigma_b}$$



$$\vec E = \frac{Q_{enc}}{r^2}\hat r$$

$$= \frac{4}{3}\pi r^3\rho\,\frac{1}{\cancel{r^2}}\hat r$$

$$\vec E = \frac{4}{3}\pi\rho\,\vec r$$

$$\vec E(\vec R_0) = \frac{4}{3}\pi(+\rho)\vec r_1 + \frac{4}{3}\pi(-\rho)\vec r_2$$

$$= \frac{4}{3}\pi\rho(\vec r_1 - \vec r_2)$$

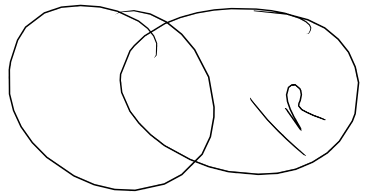
$$\vec E(\vec R_0) = \frac{4}{3}\pi\rho\,\vec d = \frac{4}{3}\pi\vec P$$

$$V^{\text{outside}} = \frac{\vec P\cdot\hat r}{r^2}$$

outside point $\cdot\vec{R}$

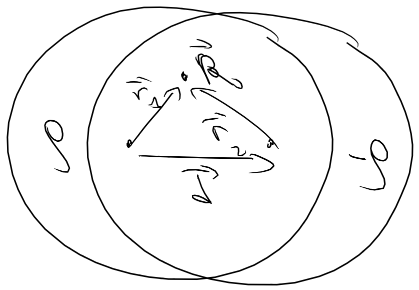
$\cdot\vec{R}$

$p = Qd = V\rho d \Rightarrow \frac{p}{V} = P$



$=$ $+Q \xrightarrow{d} -Q$

$Q = \frac{4}{3}\pi R^3$

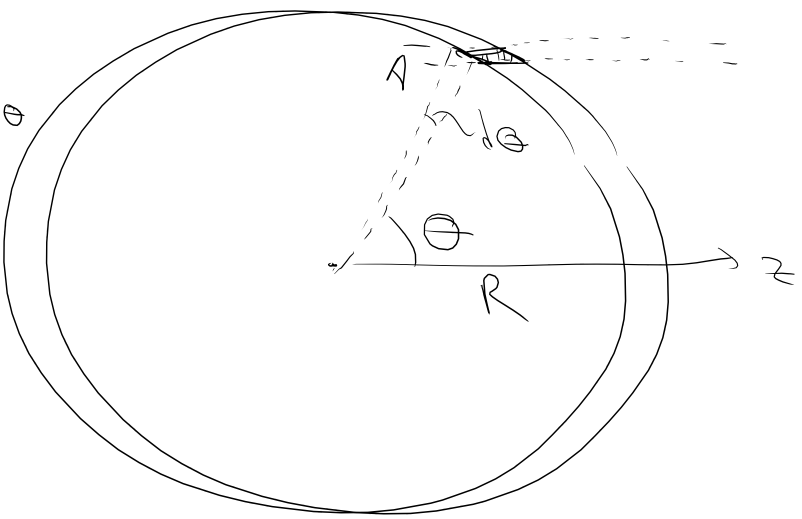


$d \to 0$

$\rho \to \infty$

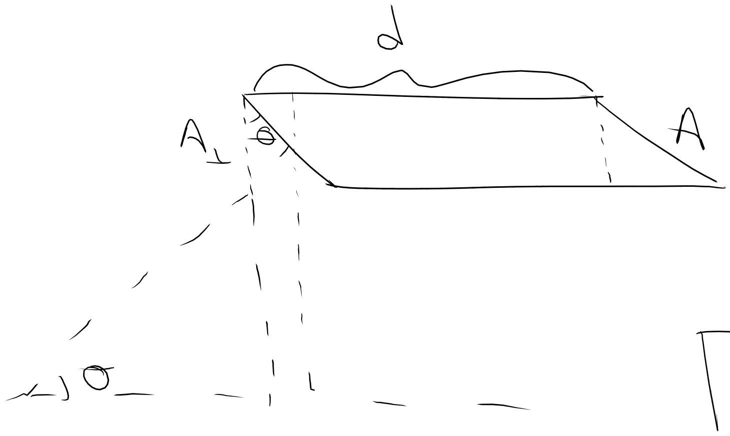
$\rho d \equiv P = \text{constant}$

$r\sin\theta\, d\theta$



$\Delta q \equiv \sigma A$

$\Delta q =$



$\Delta V = d A_\perp$

$= d A \cos\theta$

$\sigma \cancel{A} = \rho d \cancel{A} \cos\theta$

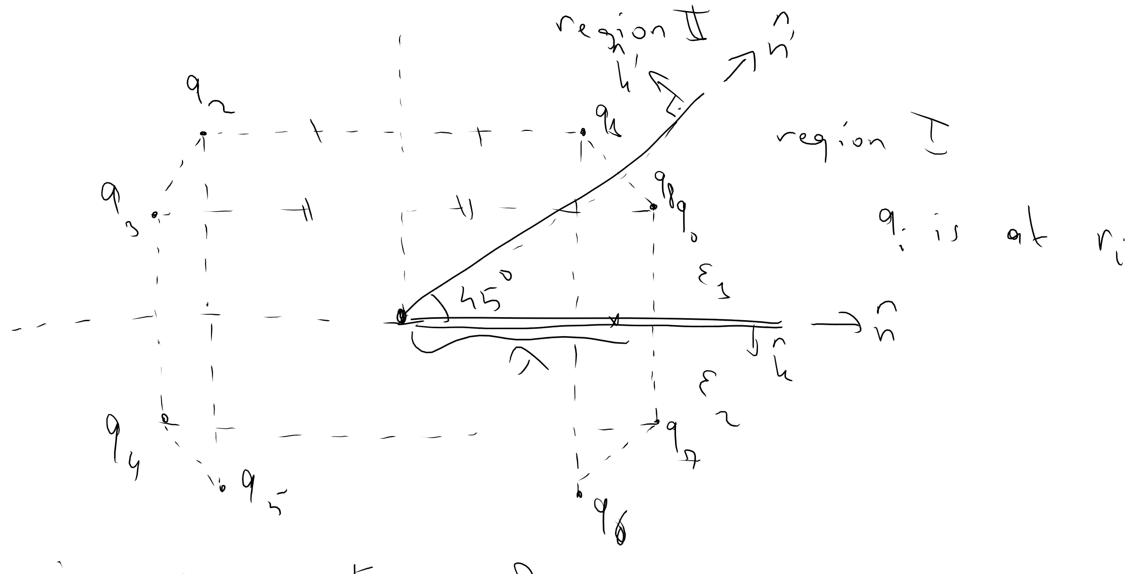
$\boxed{\sigma = P\cos\theta}$

$$\sigma = P\cos\theta$$
$$= P\, P_1(\cos\theta)$$
$$\propto P\, Y_{10}(\Omega)$$

$$q_{lm} = \int \sigma(\vec{r}) Y^*_{lm}\, r^l\, d^3r$$

$$\int Y^*_{lm}(\Omega) Y_{10}(\Omega)\, d\Omega = \delta_{l1}\delta_{m0}$$

q_{10} is non-zero



region I $\quad V^{I}(\vec{r}) = \sum_{i=0}^{7} \frac{q_i}{|\vec{r}-\vec{r}_i|}$

region II $\quad V^{II}(\vec{r}) = \frac{q_8}{|\vec{r}-\vec{r}_8|}$

$$V^{I}(\vec{r}=\lambda\hat{n}) = V^{II}(\vec{r}=\lambda\hat{n}) \qquad \forall \lambda > 0 \qquad (1)$$
$$V^{I}(\vec{r}=\lambda\hat{n}') = V^{II}(\vec{r}=\lambda\hat{n}') \qquad (2)$$

$$\vec{D} = \varepsilon \vec{E} = \varepsilon \vec{\nabla} V$$

$$\varepsilon_1 \frac{\partial V^{I}}{\partial u}\Big|_{\vec{r} = a\hat{n}} = \varepsilon_2 \frac{\partial V}{\partial u}\Big|_{\vec{r} = a\hat{n}} \qquad (3)$$

$$\varepsilon_1 \frac{\partial V^{I}}{\partial u'}\Big|_{\vec{r} = a\hat{n}'} = \varepsilon_2 \frac{\partial V^{II}}{\partial u'}\Big|_{\vec{r} = a\hat{n}'} \qquad (4)$$