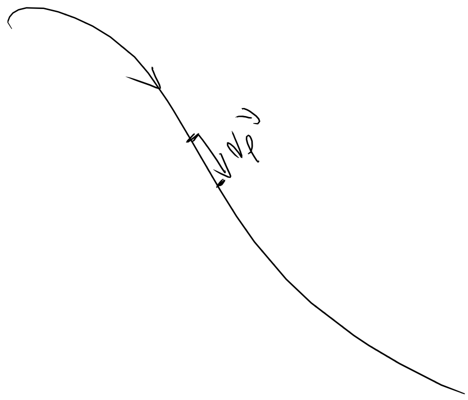
$$\vec B = \frac{\vec m - (\vec m\cdot\hat r)\hat r}{r^3} \qquad \vec \tau = \vec m \times \vec B$$




$\cdot\vec r$

$$\boxed{d\vec B = \frac{1}{c}\, I\, d\vec \ell \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}}$$

$$\frac{\partial \rho}{\partial t} + \vec\nabla\cdot\vec J = 0$$

continuity eqn.

in statics $\frac{\partial}{\partial t} \to 0 \Rightarrow \vec\nabla\cdot\vec J = 0$

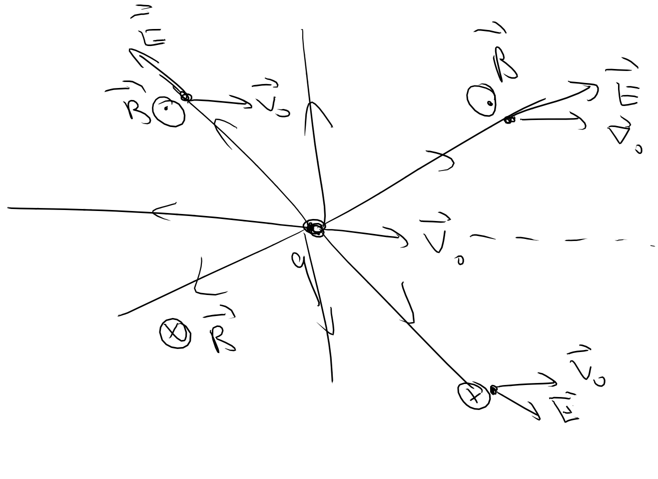
$$\vec B = \frac{1}{c}\int \vec J(\vec r')\times\frac{\vec r - \vec r'}{|\vec r - \vec r'|^3}\, d^3 r'$$

$$\vec J = \rho \vec v$$

for a point charge $\rho = q\,\delta(\vec r - \vec r_0)$

$$\Rightarrow \boxed{\vec B = \frac{1}{c}\, q\, \vec v_0 \times \frac{(\vec r - \vec r_0)}{|\vec r - \vec r_0|^3}} = \frac{\vec v_0}{c}\times \vec E(\vec r)$$

$$\rho = q\,\delta(\vec r - \vec r_0) \Rightarrow \frac{\partial \rho}{\partial t} = q\vec\nabla\delta(\vec r - \vec r_0)\left(-\frac{d\vec r_0}{dt}\right)$$




$$= -q\,\vec v_0\cdot\vec\nabla\delta(\vec r - \vec r_0) \neq 0$$

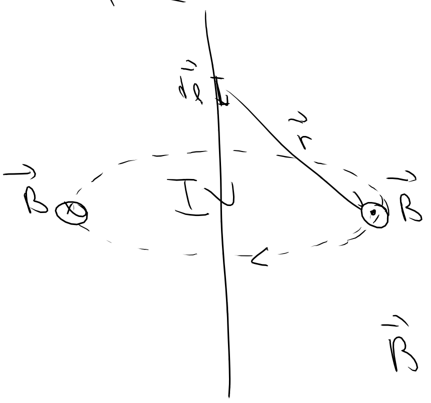
$$\vec B \parallel \vec v_0 \times \vec E$$

$$\vec{\nabla}\cdot\vec{B} = 0 \implies \oint \vec{B}\cdot d\vec{S} = 0$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} \implies \oint_{\partial A} \vec{B}\cdot d\vec{l} = \frac{4\pi}{c}\int_A \vec{J}\cdot d\vec{A}$$

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} I_{enc}}$$

Example




$$\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} I_{enc}$$

$$d\vec{B} = \frac{1}{c} I \frac{d\vec{l}\times\vec{r}}{r^3}$$

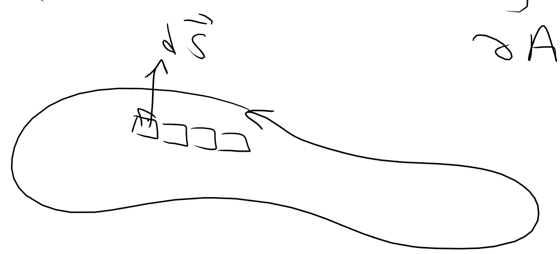
$$\vec{B} = B(s,\phi,z)\hat{\phi} = B(s)\hat{\phi}$$

$$\oint \vec{B}\cdot d\vec{l} = \oint B(s)\,dl = B(s)\,2\pi s = \frac{4\pi}{c} I$$

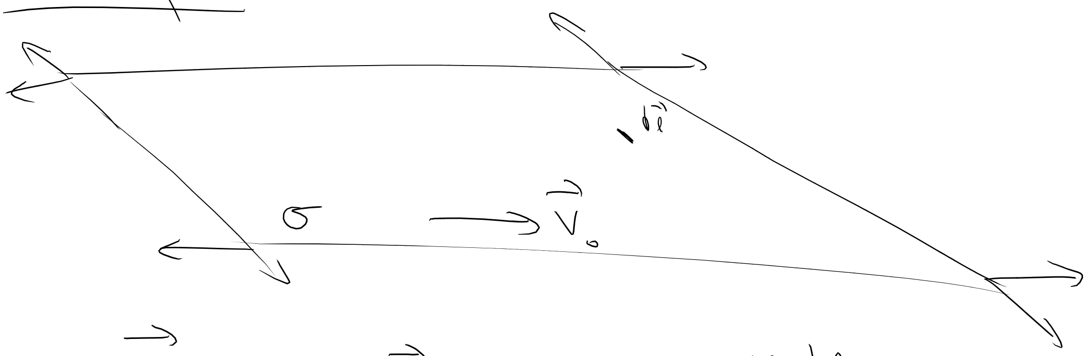
$$B(s) = \frac{4\pi}{c} I \frac{1}{2\pi s} = \boxed{\frac{2}{c}\frac{I}{s} = B(s)}$$

$$\boxed{\vec{B}(\vec{r}) = \frac{2I}{c}\frac{1}{s}\hat{\phi}}$$

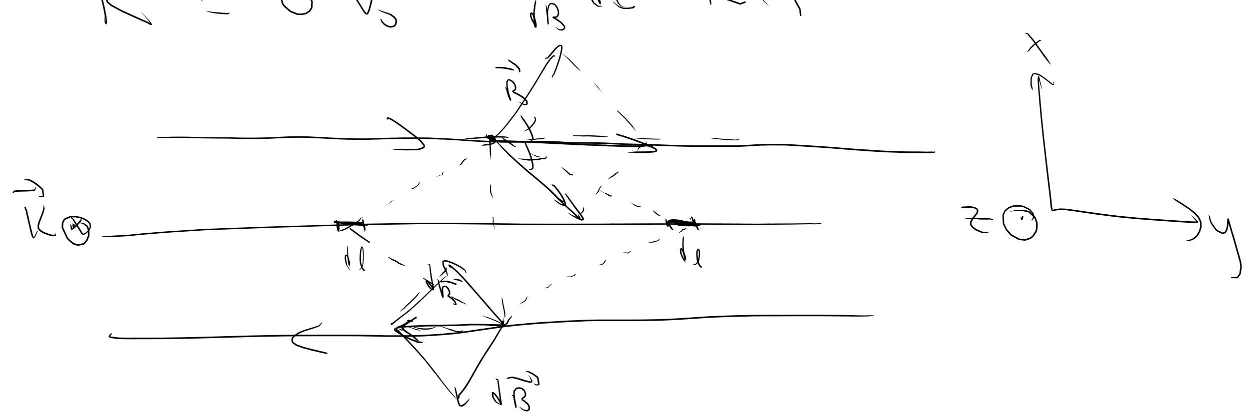
$$\int_A (\vec{\nabla}\times\vec{C})\cdot d\vec{S} = \int_{\partial A} \vec{C}\cdot d\vec{l}$$



# Example

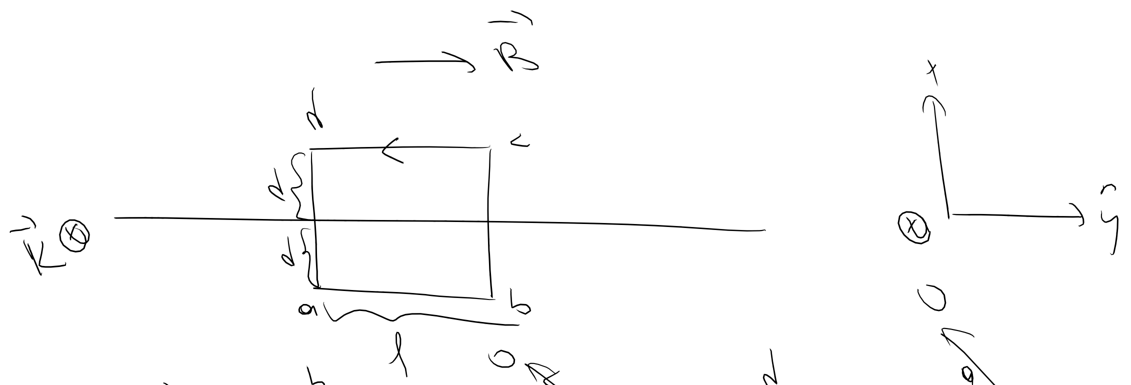



$\vec{K} = \sigma \vec{v}_0 \qquad dI = K\,dl$




$\vec{K} = \sigma v_0 \hat{z}$

$$\vec{B} = \left( \begin{cases} \hat{y} & \text{if } x > 0 \\ -\hat{y} & \text{if } x < 0 \end{cases} \right) B(x, \cancel{y}, \cancel{z})$$




$$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \overset{0}{\cancel{\int_b^c \vec{B}\cdot d\vec{l}}} + \int_c^d \vec{B}\cdot d\vec{l} + \overset{0}{\cancel{\int_d^a \vec{B}\cdot d\vec{l}}}$$

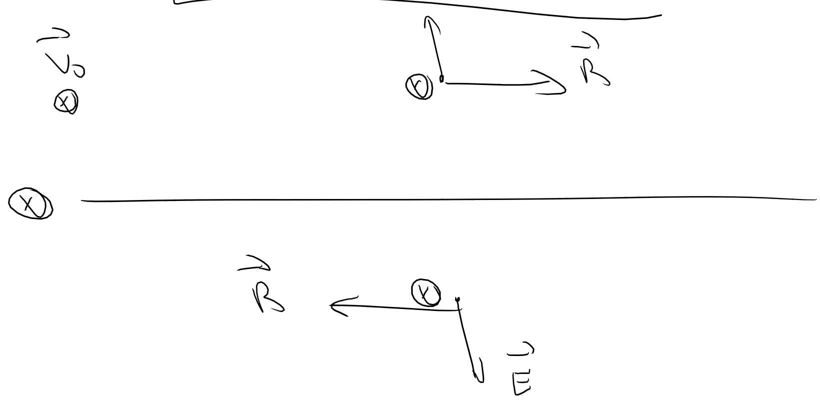
$$\vec{B} = B(x)\,\hat{y} \qquad = B(-d)\,l + B(d)(-l) = \frac{4\pi}{c}(-Kl)$$

$$\underset{-B(d)}{\underset{\shortparallel}{B(-d)}} - B(d) = -\frac{4\pi}{c}\sigma v_0$$

$B(-d) = B(d)$

$+\cancel{2}B(d) = +\frac{\cancel{4}^{2}\pi}{c}\sigma V_0$

$$\boxed{B(d) = \frac{2\pi}{c}\sigma V_0} = \left|\frac{\vec{V}_0}{c}\times\vec{E}\right|$$

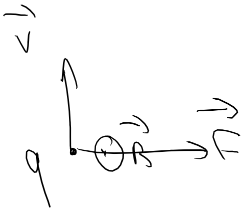


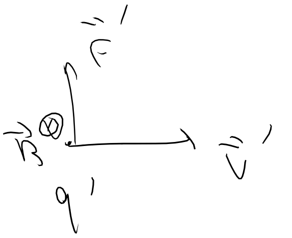

$$\vec{B} = \frac{\vec{V}_0}{c}\times\vec{E}$$

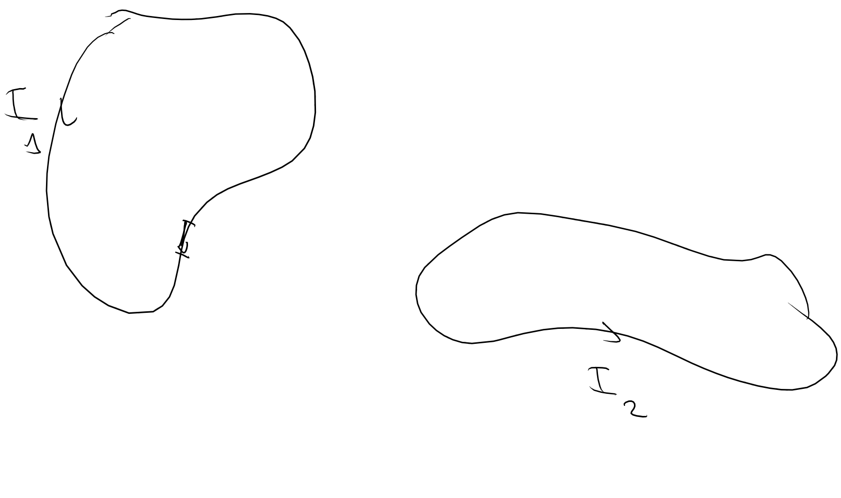
$|\vec{E}| = 2\pi\sigma$

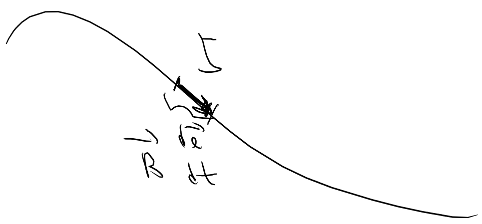
## Lorentz Force

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$$






$$I = |\lambda \vec{v}|$$

$$\vec{F} = q\frac{\vec{v}}{c} \times \vec{B}$$

$$I = \frac{dq}{dt} = \frac{dq}{dl}\frac{dt}{dl} = \lambda v$$

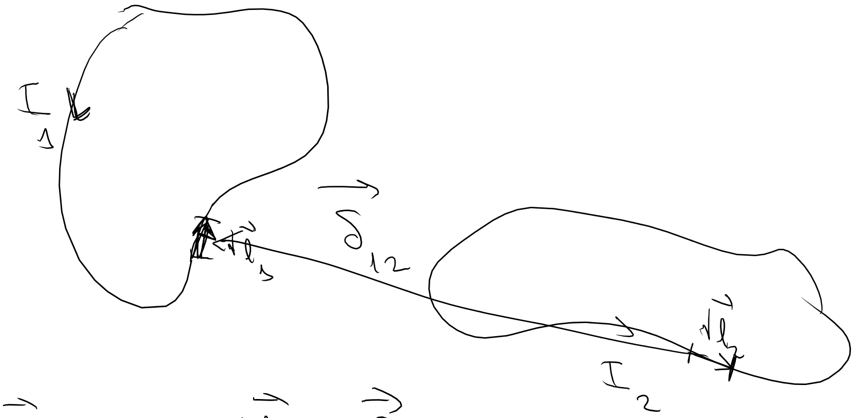
$$\vec{J} = (\lambda_- \vec{v}_e + \lambda_+ \vec{v}_p)\,\delta(\ )$$

$$d\vec{F} = \sum_i q_i \vec{v} \times \vec{B}$$

$$= \sum_i \vec{J}_i \times \vec{B}$$

$$= \Big(\sum_i \vec{J}_i\Big) \times \vec{B} = \boxed{\frac{I}{c}\, d\vec{l} \times \vec{B} = d\vec{F}}$$

since $\sum_i q_i \vec{v} = I\, d\vec{l}$




$$d\vec{F}_{12} = \frac{1}{c} I_1 \, d\vec{l}_1 \times \vec{B}_2$$

$$\vec{F}_{12} = \frac{1}{c} \oint I_1 \, d\vec{l}_1 \times \vec{B}_2$$

$$d\vec{B}_2 = \frac{1}{c} I_2 \, d\vec{l}_2 \times \frac{\hat{\delta}_{12}}{\delta_{12}^2} \qquad \vec{\delta}_{12} = \vec{r}_1 - \vec{r}_2$$

$$\vec{F}_{12} = \frac{1}{c^2} I_1 I_2 \oint\oint d\vec{l}_1 \times \left( d\vec{l}_2 \times \frac{\hat{\delta}_{12}}{\delta_{12}^2} \right)$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\vec{F}_{12} = \frac{1}{c^2} I_1 I_2 \left[ \iint d\vec{l}_2 \frac{(d\vec{l}_1 \cdot \hat{\delta}_{12})}{\delta_{12}^2} - \iint \frac{\hat{\delta}_{12}}{\delta_{12}^2} \, d\vec{l}_1 \cdot d\vec{l}_2 \right]$$

$$\vec{F}_{12} = -\frac{1}{c^2} I_1 I_2 \oint\oint d\vec{l}_1 \cdot d\vec{l}_2 \frac{\hat{\delta}_{12}}{\delta_{12}^2} + \frac{1}{c} I_1 I_2 \oint\oint d\vec{l}_2 \left( d\vec{l}_1 \cdot \frac{\hat{\delta}_{12}}{\delta_{12}^2} \right)$$

$$\oint d\vec{l}_1 \cdot \frac{\hat{\delta}_{12}}{\delta_{12}^2} = \int d\vec{A} \cdot \underbrace{\vec{\nabla} \times \left( \frac{\hat{\delta}_{12}}{\delta_{12}^2} \right)}_{0} = 0$$

$$\boxed{\vec{F}_{12} = -\frac{1}{c^2} I_1 I_2 \oint\oint d\vec{l}_1 \cdot d\vec{l}_2 \, \frac{\hat{\delta}_{12}}{\delta_{12}^2}}$$

# Work Done By Magnetic Force

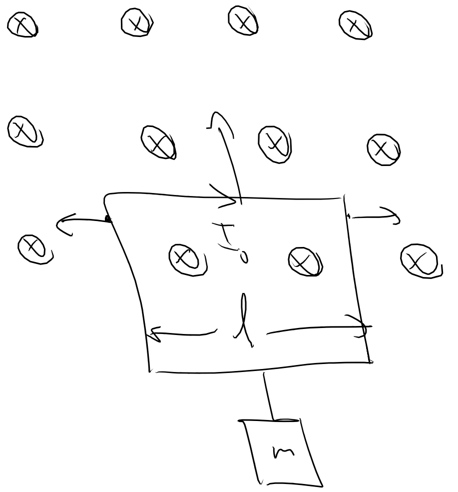
$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$dW = \vec{F} \cdot d\vec{l} = q\,(\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= q\left(\frac{d\vec{l}}{dt} \times \vec{B}\right) \cdot d\vec{l} = 0$$

$$\boxed{\text{Magnetic force does not do any work!}}$$

Example for what value of $I_0$, the mass is at rest?




$$d\vec{F} = \frac{I}{c}\, d\vec{l} \times \vec{B}$$

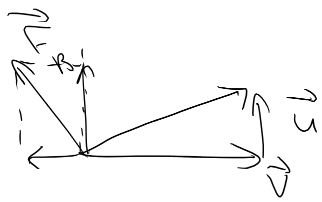
$$\left[\frac{I}{c}\, d\vec{l} \times \vec{B}\right] = \frac{[q]}{[t]} \frac{[l]}{[c]} [\vec{B}]$$

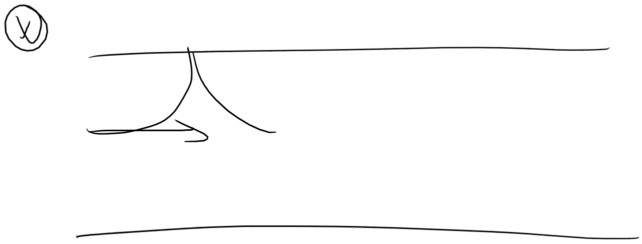
$$[F] = \boxed{\frac{l}{[ct]}}\,[F] \;\checkmark$$

$$\frac{I_0}{c}\, l B = mg \rightarrow \boxed{I_0 = \frac{mgc}{Bl}}$$

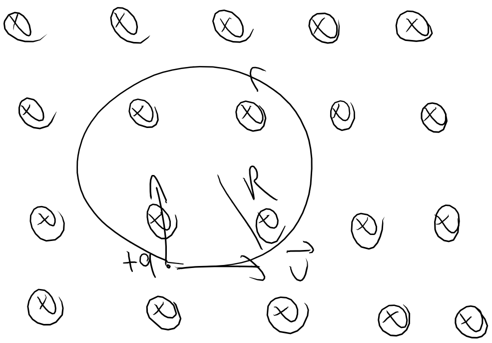
For $I > I_0$, the wire moves upward.

$$W \neq 0$$



Ex Point charge in a uniform magnetic field

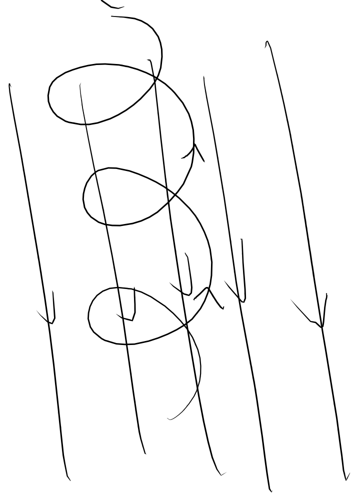



uniform circular motion!

$$\boxed{\frac{q\cancel{v}}{c} B = \frac{m v^{\cancel{2}}}{R}}$$

$$\boxed{\frac{q}{m} = \frac{v c}{R B}}$$

$$\vec{F} = q \frac{\vec{v}}{c} \times \vec{B}$$



magnetic confinement