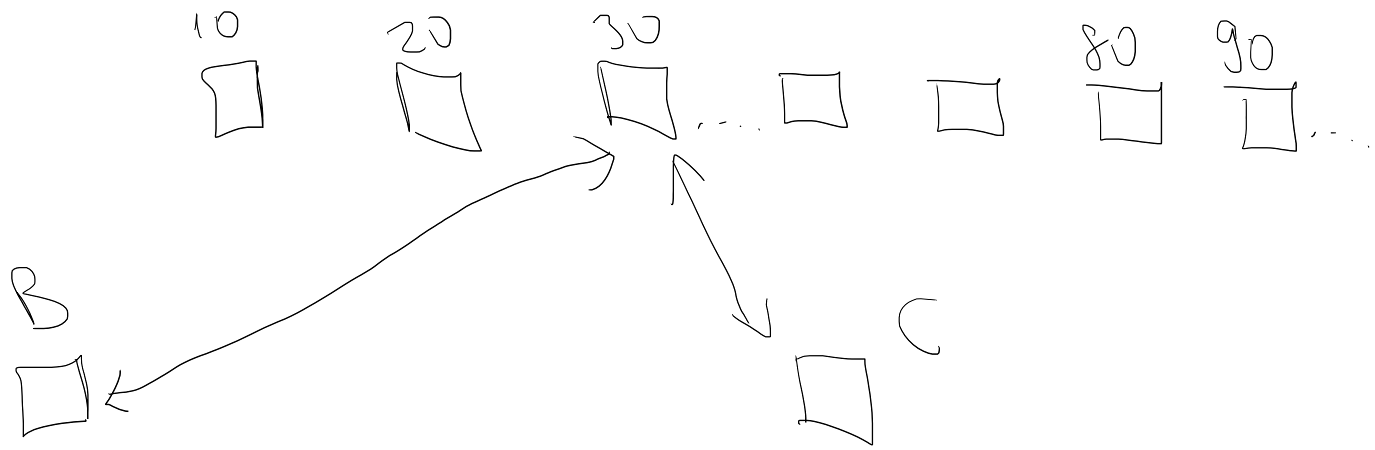
Eric Mazur "Confessions of a Converted lecturer" (YouTube)

Review of Thermodynamics

0^{th} Law: If systems A & B are in equilibrium & systems A & C are in equilibrium, then systems B & C are also in equilibrium.



1^{st} Law $\Delta U = \Delta Q - \Delta W$

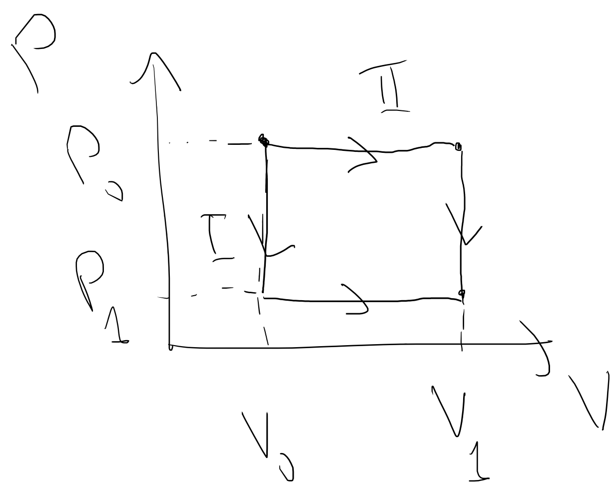
$\Delta W > 0$ if the system does work

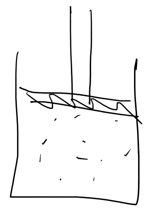
$\Delta W < 0$ if work is done on the system

$\Delta Q > 0$ if heat is given to the system

$\Delta Q < 0$ if heat is extracted from the system

<u>Example</u>





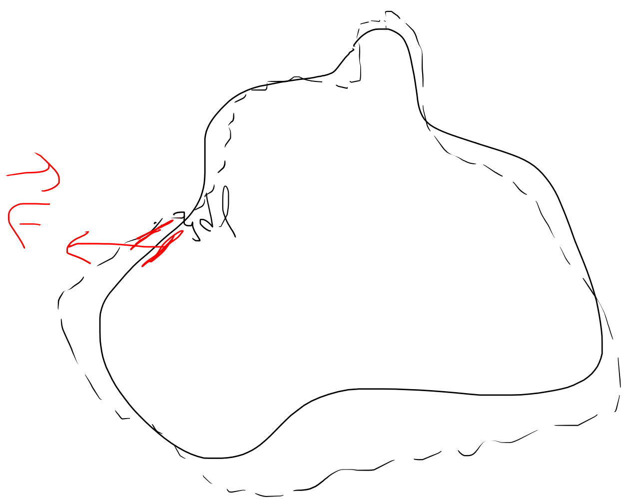
$(P_0, V_0) \to (P_1, V_1)$

$W = ?$

$$dW = \vec{F}\cdot d\vec{l}$$

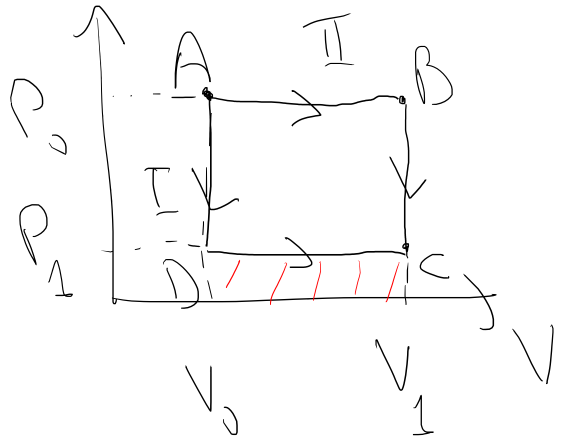
$$= \vec{F}\cdot(dl\,\hat{n})$$

$$= \frac{\vec{F}\cdot\hat{n}}{dA}\underbrace{dA\,dl}_{dV}$$



$$dW = P\,dV$$

$$dW \leq P\,dV$$



$$W = \int P\,dV$$

Work done during I

$$W_{A\to D\to C} = \underbrace{W_{A\to D}}_{\substack{\parallel \\ 0 \text{ since } dV=0}} + W_{D\to C}$$

$$= W_{D\to C} = \int P\,dV = P_1(V_1 - V_0)$$

$$W_I = P_1(V_1 - V_0)$$

Work done during II

$$W_{II} = P_0(V_1 - V_0) \neq W_I$$

Ideal Gas

$$PV = NkT$$
$$= nRT$$

k: Boltzman const.

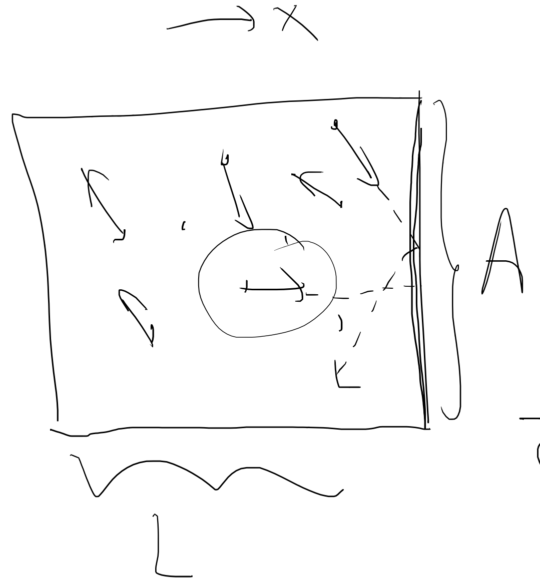
N: # of particles

R:

n: mole #

$U = ?$

Kinetic Theory of the Ideal Gas

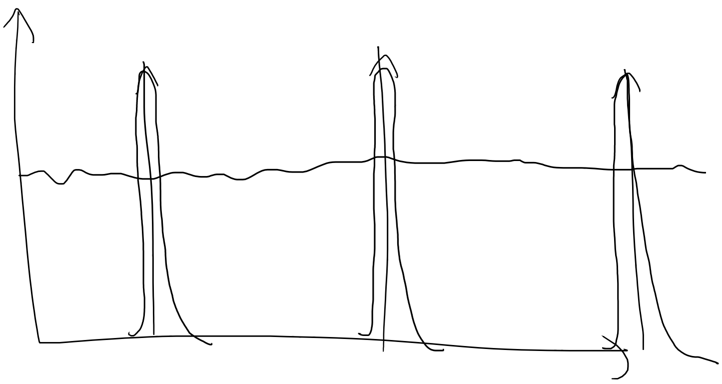


$v_i = (v_x, v_y, v_z)$

$v_f = (-v_x, v_y, v_z)$

$\Delta p_x = 2 m v_x$

$$\bar{F}_1 = \frac{\Delta p}{\Delta t} = \frac{2 m v_x}{\frac{2L}{v_x}} = \frac{m}{L} v_x^2$$



$N \sim 10^{23}$

$$F_x = \langle F_x \rangle = \sum_i \langle F_{xi} \rangle$$

$$= \sum_i \frac{m_i}{L} v_{ix}^2 = \frac{1}{L} N \left(\frac{1}{N} \sum_i m_i v_{ix}^2 \right)$$

$$F_x = \frac{1}{L} N \langle m v_x^2 \rangle$$

$$P = \frac{F_x}{A} = \frac{1}{\underbrace{AL}_{V}} N \langle m v_x^2 \rangle$$

$$\boxed{PV = N \langle m v_x^2 \rangle}$$

$$\langle v_y^2 \rangle = \langle v_x^2 \rangle = \langle v_z^2 \rangle = \frac{\langle v^2 \rangle}{3}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

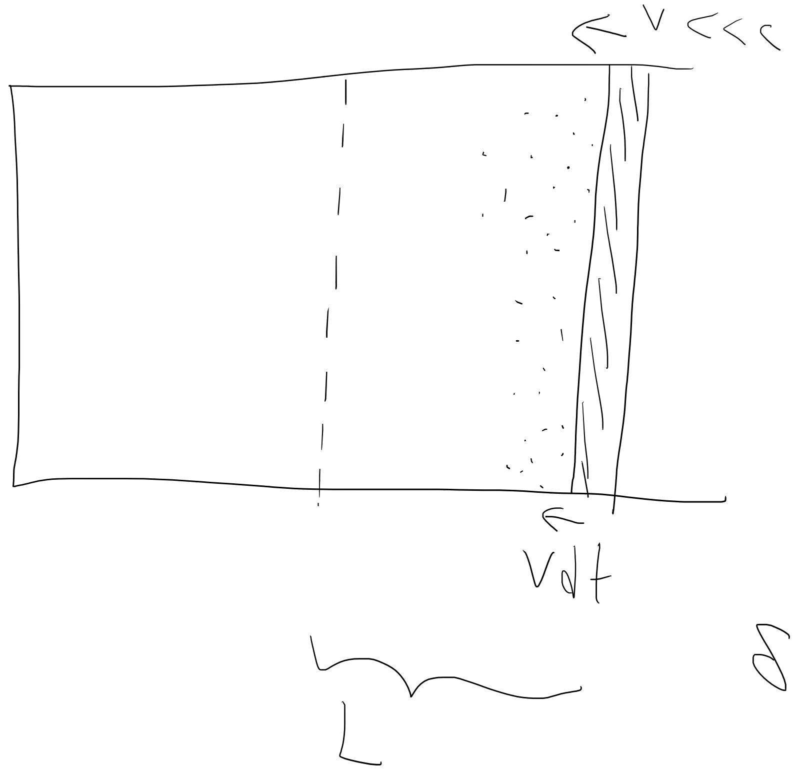
$$PV = N \left\langle \frac{mv^2}{2} \right\rangle \frac{2}{3}$$

$$= N \langle \varepsilon \rangle \frac{2}{3}$$

$$\boxed{PV = \frac{2}{3} U} \Rightarrow U = \frac{3}{2} PV = \boxed{\frac{3}{2} NkT = U}$$

<u>Example</u> Assume an isolated system of ideal gas at temperature T_0, V_0

If you compress the system,



dt

$\frac{dU}{dt}$

$\delta U \sim v$

$$U = U_0 + \int_{t=0}^{t} \frac{dU}{dt}\,dt$$

Adiabatic process $\Delta Q = 0$

$$\Delta U = -P\Delta V$$

$$U = \frac{3}{2}NkT \; ; \; U = \frac{3}{2}PV$$

$$\Delta U = \frac{3}{2}(\Delta P)V + \frac{3}{2}P(\Delta V) = -P\Delta V$$

$$\left(\frac{3}{2}(\Delta P)V + \frac{5}{2}P\Delta V = 0\right)\frac{1}{PV}$$

$$\frac{3}{2}\frac{\Delta P}{P} + \frac{5}{2}\frac{\Delta V}{V} = 0$$

$$\Delta\left(\frac{3}{\cancel{2}}\ln\left(\frac{P}{P_0}\right) + \frac{5}{\cancel{2}}\ln\left(\frac{V}{V_0}\right)\right) = 0$$

$$\ln P + \frac{5}{3}\ln V = const$$

$$\boxed{PV^{5/3} = const}$$ adiabatic law for ideal gas

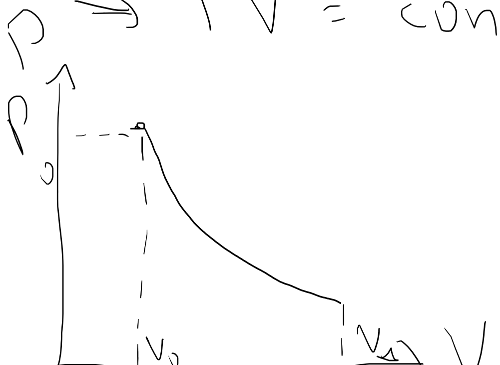
$$PV^{5/3} = (PV)V^{2/3} = NkTV^{2/3} = const$$

$$\Rightarrow \boxed{TV^{2/3} = const}$$

Isothermal Process

$T = const$

$\Rightarrow PV = const$



$$W = \int_{V_0}^{V_1} P\,dV = NkT\int_{V_0}^{V_1}\frac{dV}{V}$$

$$W = NkT\ln\left(\frac{V_1}{V_0}\right)$$

$\Delta Q = ?$ $\qquad \Delta U = \Delta Q - \Delta W$

ΔU: change in U

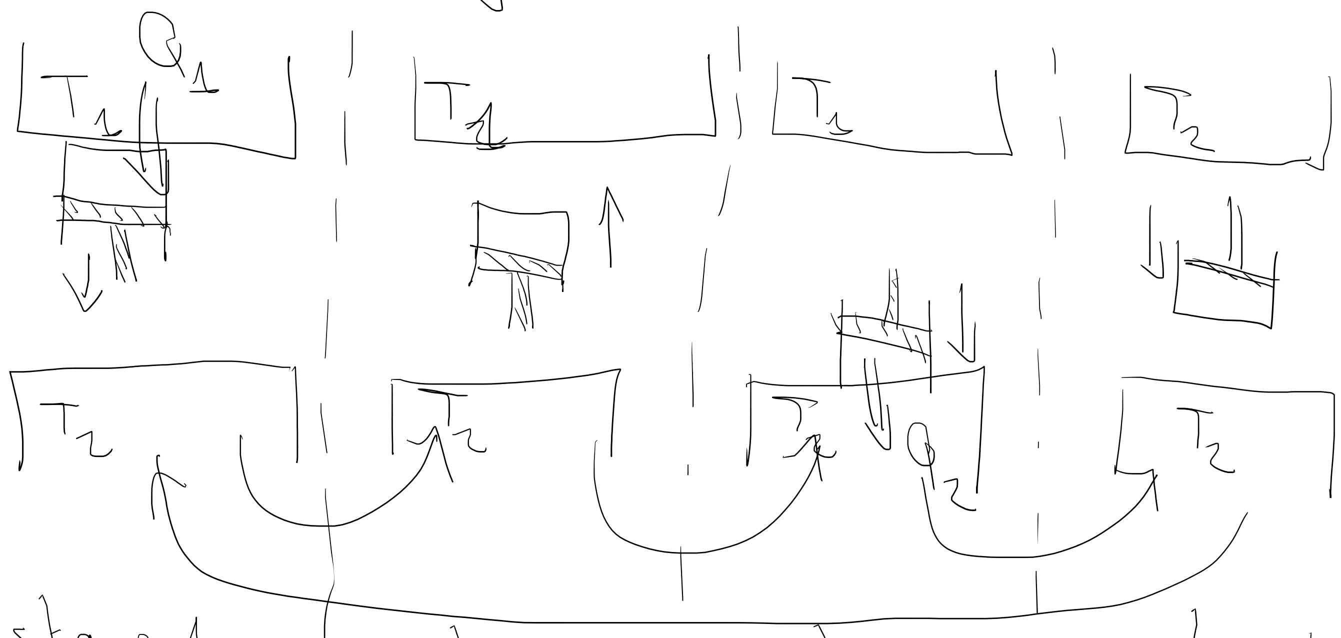
$\Delta Q \neq$ change in Q

$\Delta W \neq$ change in W

$$\Delta W = NkT \ln\left(\frac{V_1}{V_0}\right)$$

$$\Delta U = 0 \Rightarrow \Delta Q = \Delta W = NkT \ln\left(\frac{V_1}{V_0}\right)$$

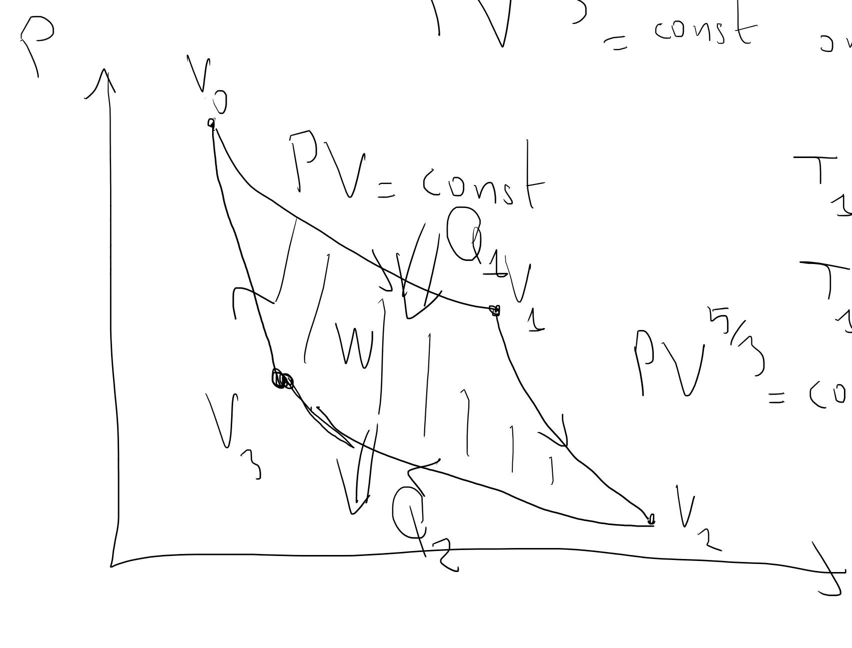
Carnot Cycle



stage 1 $\qquad$ stage 2 $\qquad$ stage 3 $\qquad$ stage 4

$T_2 < T_1$

$T V^{2/3} = \text{const}$ on an adiabatic curve.



$$T_1 V_0^{2/3} = T_2 V_3^{2/3}$$

$$T_1 V_1^{2/3} = T_2 V_2^{2/3}$$

$$\Downarrow$$

$$\frac{V_0}{V_1} = \frac{V_3}{V_2}$$

$$Q_1 = W + Q_2 \Rightarrow W = Q_1 - Q_2$$

$$W = NkT_1 \ln\left(\frac{V_1}{V_0}\right) - NkT_2 \ln\left(\frac{V_2}{V_3}\right)$$

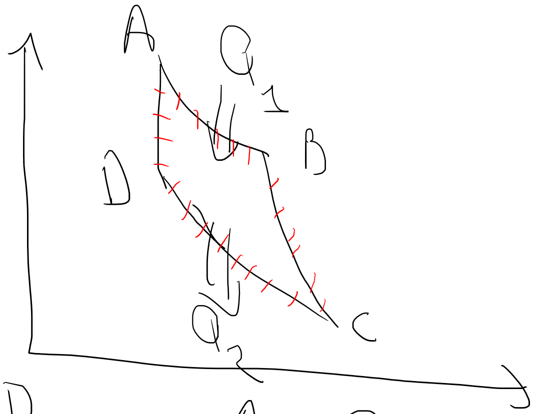
$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \ln\left(\frac{V_2}{V_3}\right) / \ln\left(\frac{V_1}{V_0}\right)$$

$$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

$$\frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = 0$$

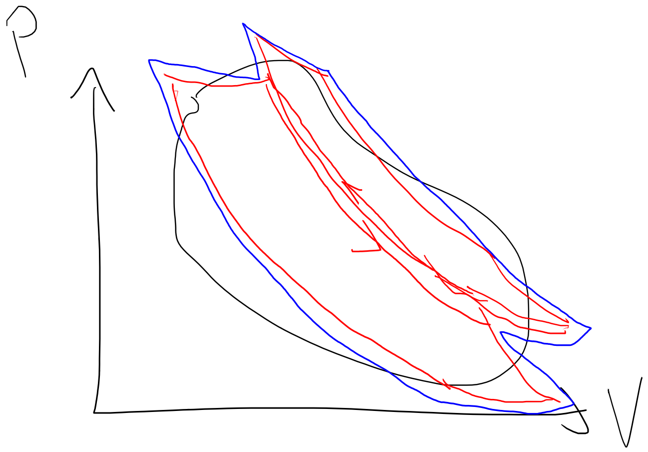
$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$



$$\oint \frac{dQ}{T} = \int_A^B \frac{dQ}{T} + \int_B^C \frac{dQ}{T} + \int_C^D \frac{dQ}{T} + \int_D^A \frac{dQ}{T}$$

$$(dQ = 0 \text{ on } B \to C \text{ and } D \to A)$$

$$= \frac{Q_1}{T_1} + 0 + \frac{(-Q_2)}{T_2} + 0 = 0$$



$$\oint \frac{dQ}{T} = \sum \oint \frac{dQ}{T}$$

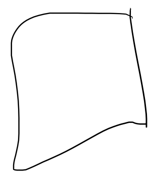
$$\oint \frac{dQ}{T} = 0$$

$$\text{reversible process} \int_A^B \frac{dQ}{T} = S(B) - S(A)$$

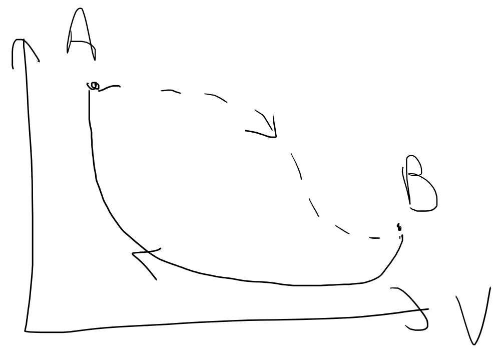
$$\frac{dQ}{T} = dS$$

2nd Law of Thermodynamics

$$\oint \frac{dQ}{T} \leq 0$$



isolated system



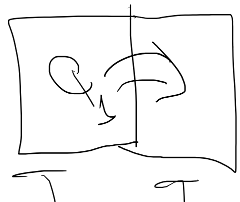
$$\oint \frac{dQ}{T} \leq 0$$

$$\underbrace{(irr)\int_A^B \frac{dQ}{T}}_{=0} + (rev)\int_B^A \frac{dQ}{T} \leq 0$$

$$S(A) - S(B) \leq 0$$

$$\Rightarrow \boxed{S(B) \geq S(A)}$$

Example



T_1 T_2

$$\Delta S = \frac{-Q_1}{T_1} + \frac{Q_1}{T_2} = Q_1\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\Delta S \geq 0$$

$$T_1 > T_2 > 0 \Rightarrow \frac{1}{T_1} < \frac{1}{T_2} \Rightarrow \frac{1}{T_2} - \frac{1}{T_1} > 0 \Rightarrow Q_1 \geq 0$$

$$T_2 > T_1 > 0 \Rightarrow \frac{1}{T_1} > \frac{1}{T_2} \Rightarrow \frac{1}{T_2} - \frac{1}{T_2} < 0 \Rightarrow Q_1 \leq 0$$

$$T_1 > 0 > T_2 \qquad T_2 < 0 \Rightarrow \frac{1}{T_2} < 0$$

$$\frac{1}{T_1} > 0$$

$$\frac{1}{T_2} - \frac{1}{T_1} < 0 \Rightarrow Q_1 < 0$$

$$dU = T\,dS - P\,dV$$

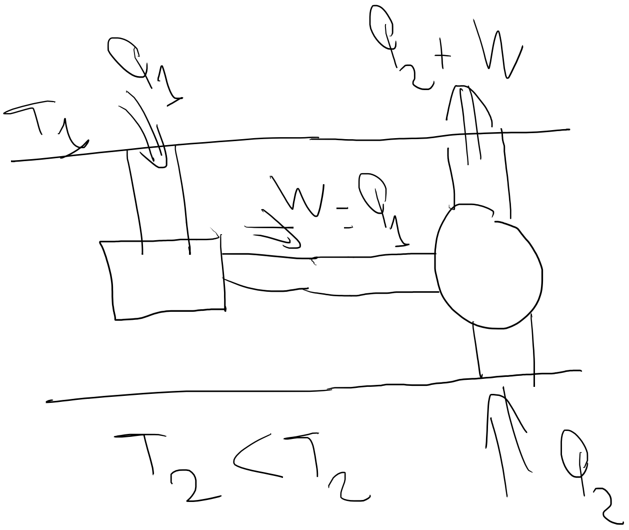
$$\Rightarrow dS = \frac{1}{T}\,dU + \frac{P}{T}\,dV$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V$$

2nd Law Heat can not flow spontaneously from a colder reservoir to a hotter one.

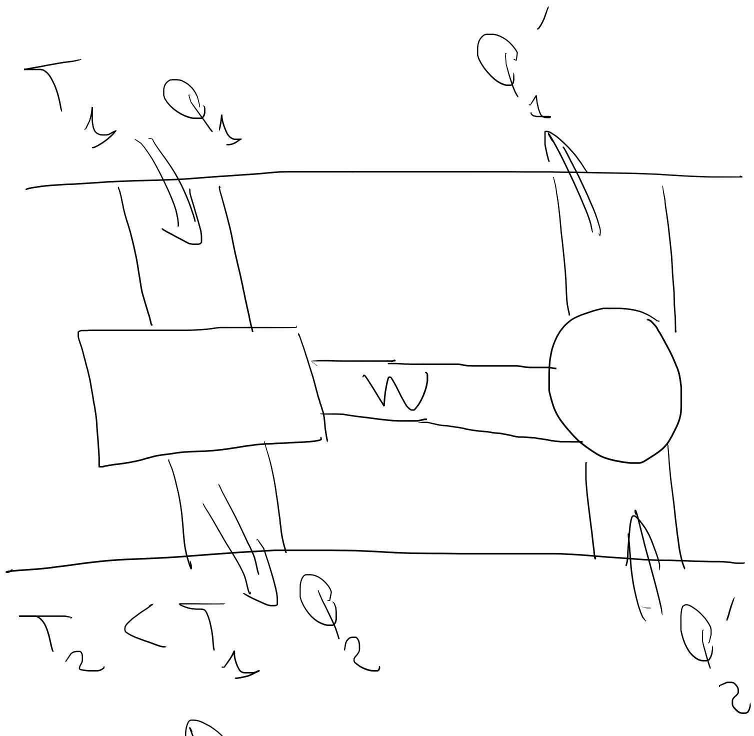
2nd Law No machine can convert heat completely into work

Assume $\exists$ a machine that can convert heat completely into work.



can not exist!

No machine can be more efficient than a Carnot engine



$$\frac{W}{Q_1'} = 1 - \frac{T_2}{T_1}$$

$$\frac{W}{Q_1} > \frac{W}{Q_1'}$$

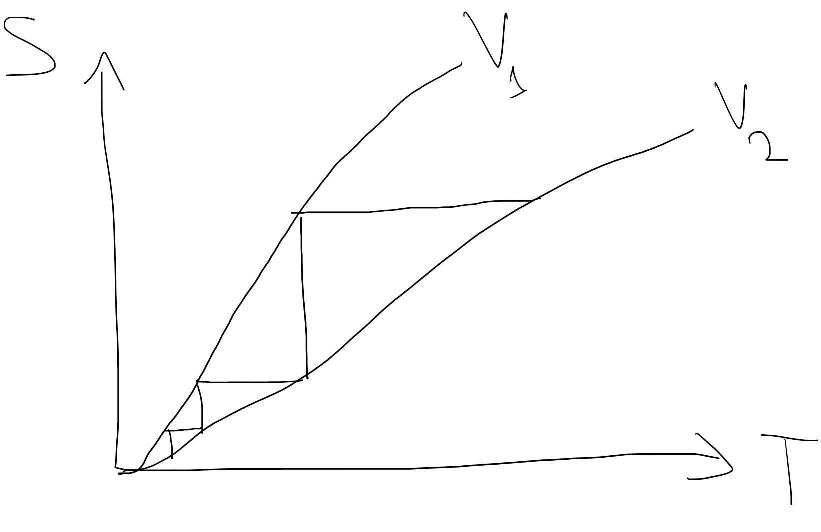
$$\Rightarrow Q_1 < Q_1'$$

more efficient than Carnot engine $\frac{W}{Q_1} > 1 - \frac{T_2}{T_1}$

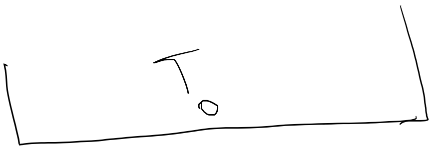
This hypothetical system takes heat from the colder reservoir and dumps it into the hotter reservoir. This can not be!

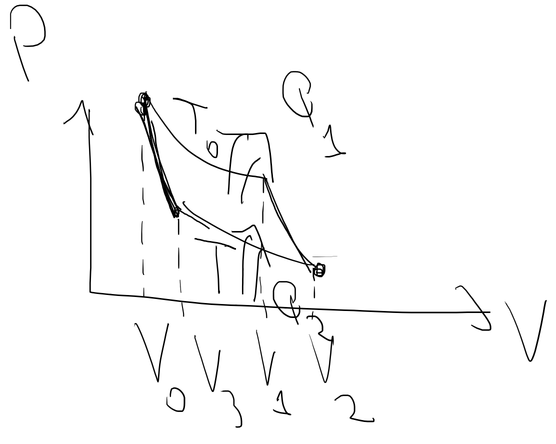
3^{rd} Law of Thermodynamics

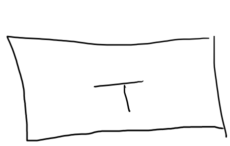
$$S(V, T=0) = 0$$



Example







$T < T_0$

$$\frac{V_0}{V_3} = \frac{V_1}{V_2} \qquad V_1, V_3, Q_1, Q_2$$

$$\frac{Q_1}{Q_2} = \frac{T_0}{T} \qquad V_1 = \frac{V_2}{V_3} V_0 = (V_2 V_0) \frac{1}{V_3}$$

$$Q_1 = Nk T_0 \ln\left(\frac{V_1}{V_0}\right) \qquad V_1(V_0, V_2, T, T_0)$$

$$Q_2 = NkT \ln\left(\frac{V_2}{V_3}\right) \qquad V_3(V_0, V_2, T, T_0)$$

$$T_0 V_0^{2/3} = T V_3^{2/3} \qquad V_3^{2/3} = \frac{T_0}{T} V_0^{2/3}$$

$$Q_2 = NkT \ln\left(\frac{V_2}{\left(\frac{T_0}{T}\right)^{3/2} V_0}\right)$$

$$= NkT \ln\left(\left(\frac{T}{T_0}\right)^{3/2} \frac{V_2}{V_0}\right) \to 0$$

<u>Heat capacity</u>

$$C = \frac{\Delta Q}{\Delta T} = T \frac{\Delta S}{\Delta T}$$

$$C_x = T\left(\frac{\partial S}{\partial T}\right)_x \Rightarrow \Delta Q = C \Delta T$$

$$S(P,T) \overset{T\to 0}{=} d(P)\,T^{\alpha} + \mathcal{O}(T^{\beta>\alpha}) \qquad \alpha>0$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p \overset{T\to 0}{=} \alpha\, T\, d(P)\, T^{\alpha-1}$$

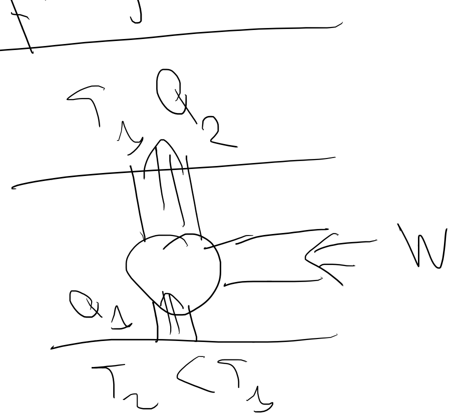
$$C_p = \alpha\, d(P)\, T^{\alpha} = \alpha S$$

$$\Delta Q = N k T \ln\left(\left(\frac{T}{T_0}\right)^{3/2} \frac{V_2}{V_0}\right)$$

$$\Delta T = \frac{\Delta Q}{C} = \boxed{\frac{N k T \ln\left(\left(\frac{T}{T_0}\right)^{3/2} \frac{V_2}{V_0}\right)}{\alpha S} = \Delta T}$$

$$\eta = 1 - \frac{T_1}{T_2} =$$

Refrigerator



$$\eta = \frac{Q_1}{W} =$$

$$\frac{Q_1}{Q_2} = \frac{T_2}{T_1}$$

$$\eta^{-1} = \frac{W}{Q_1} = \frac{Q_2 - Q_1}{Q_1} = \frac{Q_2}{Q_1} - 1$$

$$\eta^{-1} = \frac{T_1}{T_2} - 1 = \frac{T_1 - T_2}{T_2}$$

$$\eta = \frac{T_2}{T_1 - T_2}$$

<u>Heater</u>

$$\eta = \frac{Q_2}{W} > 1$$