$$\boxed{Q_1 = -\frac{2\sinh(x)}{1-e^{x/J}}}$$

$J x^b \longrightarrow x^{non}$

$x = \mu_B g J B \frac{1}{kT}$

$$x = \mu_B g B \beta J = \frac{\mu_B g J B}{kT}$$

$$\underline{g = \frac{3}{2} - \frac{l(l+1) - s(s+1)}{2J(J+1)}} \quad |l-s| < J < |l+s|$$

$$S = ? = -\frac{\partial A}{\partial T} = -\frac{\partial}{\partial T}\left(-kT \ln Q_1\right)$$

$$= k\frac{\partial}{\partial T}\left(T \ln Q_1\right)$$

$$= k \ln Q_1 + kT\frac{1}{Q_1}\frac{\partial Q_1}{\partial T}$$

$$= k \ln Q_1 + kT\frac{1}{Q_1}\frac{\partial Q_1}{\partial x}\frac{\partial x}{\partial T}$$

$$= k \ln\left(\frac{2\sinh x}{e^{x/J}-1}\right)$$

$$+ \frac{k\cancel{T}}{\cancel{2}\sinh x}\cancel{(e^{x/J}-1)}\left[\frac{\cancel{2}\cosh x}{\cancel{e^{x/J}-1}} - \frac{2\sinh x}{(e^{x/J}-1)^2}\left(\frac{1}{J}e^{x/J}\right)\right]\left(\frac{-x}{\cancel{T}}\right)$$

$$= k \ln\left(\frac{2\sinh x}{e^{x/J} - 1}\right)$$

$$= \frac{kx}{\sinh x}\left[\cosh x - \frac{\sinh x}{J\left(1 - e^{-x/J}\right)}\right] = S$$

<u>as $T \to \infty$ $(x \to 0)$</u>

$$S \simeq k \ln\left(\frac{2\cancel{x}J}{\cancel{x}}\right) - \frac{k\cancel{x}}{\cancel{x}}\cancel{\left(1 - \frac{x}{J\frac{x}{J}}\right)}$$

$$S = k \ln(2J)$$

$$\sinh x = \frac{e^{x} - e^{-x}}{2} \simeq \frac{\cancel{2}(1+x) - (\cancel{1}-x)}{\cancel{2}} \simeq x$$

$$e^{x/J} - 1 \simeq \left(1 + \frac{x}{J}\right) - 1 = \frac{x}{J}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \simeq 1$$

$$1 - e^{-\frac{x}{J}} \simeq (1) - \left(1 - \frac{x}{J}\right) = \frac{x}{J}$$

$$Q_1 = \frac{\sinh\left(x\left(1+\frac{1}{2J}\right)\right)}{\sinh\left(\frac{x}{2J}\right)} \Leftarrow \text{correct result}$$

$$S = k \ln Q_1 + kT \frac{1}{Q_1} \frac{\partial Q_1}{\partial x} \frac{\partial x}{\partial T}$$

$$= k \ln \frac{\sinh\left(x\left(1+\frac{1}{2J}\right)\right)}{\sinh\left(\frac{x}{2J}\right)} +$$

$$+ k\cancel{T} \frac{(-x)\cancel{\sinh\left(\frac{x}{2J}\right)}}{\cancel{T} \sinh\left(x\left(1+\frac{1}{2J}\right)\right)} \left[\left(1+\frac{1}{2J}\right) \frac{\cosh\left(x\left(1+\frac{1}{2J}\right)\right)}{\cancel{\sinh\left(\frac{x}{2J}\right)}} - \frac{\sinh\left(x\left(1+\frac{1}{2J}\right)\right) \overset{\cosh\left(\frac{x}{2J}\right)}{\checkmark}}{\sinh^{2}\left(\frac{x}{2J}\right)} \frac{1}{2J}\right]$$

$$S = k \ln \frac{\sinh\left(x\left(1+\frac{1}{2J}\right)\right)}{\sinh\left(\frac{x}{2J}\right)}$$

$$- xk \frac{1}{\sinh\left(x\left(1+\frac{1}{2J}\right)\right)} \left[\left(1+\frac{1}{2J}\right) \cosh\left(x\left(1+\frac{1}{2J}\right)\right) - \frac{\overset{\cosh\left(\frac{x}{2J}\right)}{\checkmark} \sinh\left(x\left(1+\frac{1}{2J}\right)\right)}{\sinh\left(\frac{x}{2J}\right) 2J}\right]$$

Large T (small x)

$$S = k \ln \left(\frac{\cancel{x}\left(1+\frac{1}{2J}\right)}{\frac{\cancel{x}}{2J}}\right)$$

$$\cancel{-kx \frac{1}{x\left(1+\frac{1}{2J}\right)} \left[\left(1+\frac{1}{2J}\right) - \frac{\cancel{x}\left(1+\frac{1}{2J}\right)}{\frac{\cancel{x}}{\cancel{2J}} \cdot \cancel{2J}}\right]} \; 0$$

$$S \simeq k \ln(2J+1)$$

<u>Small T limit ($x \to \infty$)</u>

$$\sinh(x\alpha) \to \frac{e^{x\alpha}}{2} \qquad (\alpha > 0)$$

$$\cosh(x\alpha) \to \frac{e^{x\alpha}}{2}$$

$$S = k \ln\left(\frac{e^{x\left(1+\cancel{\frac{1}{2J}}\right)}}{\cancel{e^{x/2J}}}\right)$$

$$- xk \frac{\cancel{2} \cdot 1}{\cancel{e^{x\left(1+\frac{1}{2J}\right)}}} \left[\left(1+\frac{1}{2J}\right) \frac{\cancel{e^{x\left(1+\frac{1}{2J}\right)}}}{\cancel{2}} - \frac{\cancel{e^{\frac{x}{2J}}} \frac{1}{2}}{\cancel{2J} e^{\frac{x}{2J}}}\right]$$

$$S = \cancel{xk} - xk\left[\left(\cancel{1}+\frac{1}{2J}\right) - \frac{1}{2J}\right]$$

$$S \simeq 0$$

$$S = -\frac{A}{T} - xk\left[\left(1+\frac{1}{2J}\right)\coth\left(x\left(1+\frac{1}{2J}\right)\right) - \frac{1}{2J}\coth\left(\frac{x}{2J}\right)\right]$$

$$U = A + TS = T\left(\frac{A}{T} + S\right)$$

$$U = -x(kT)\left[\left(1+\frac{1}{2J}\right)\coth\left(x\left(1+\frac{1}{2J}\right)\right) - \frac{1}{2J}\coth\left(\frac{x}{2J}\right)\right]$$

$$x = \frac{\mu_B g \bar{J} B}{kT}$$

$$\boxed{U = \mu_B g J B\left[\frac{1}{2J}\coth\left(\frac{x}{2J}\right) - \left(1+\frac{1}{2J}\right)\coth\left(x\left(1+\frac{1}{2J}\right)\right)\right]}$$

$$C = \frac{\partial U}{\partial T} = \frac{\partial U}{\partial x}\left(\frac{-x}{T}\right)$$

$$= \ldots\ldots$$

$$U \xrightarrow[(x\to 0)]{T\to\infty} \mu_B g J B\left[\cancel{\frac{1}{2J}}\frac{1}{\frac{x}{\cancel{2J}}} - \left(1+\frac{1}{2J}\right)\frac{1}{x\left(\cancel{1+\frac{1}{2J}}\right)}\right]$$

$$= 0 + O(x^2)$$

$$\coth(\alpha) = \frac{e^{\alpha}+e^{-\alpha}}{e^{\alpha}-e^{-\alpha}} = \frac{\cancel{2}\left(\cancel{1}+\cancel{\alpha}+\frac{\alpha^2}{2}\right) + \cancel{\left(1-\alpha+\frac{\alpha^2}{2}\right)}}{\cancel{2}\left(\cancel{1}+\alpha+\cancel{\frac{\alpha^2}{2}}\right) - \left(\cancel{1}-\alpha+\cancel{\frac{\alpha^2}{2}}\right)} = \frac{1}{\alpha} + \frac{\alpha}{2}$$

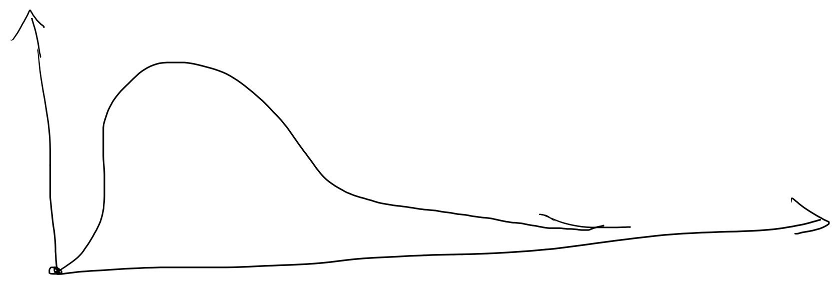
$$U \simeq \mu_B g J B \left[\frac{1}{2J}\left(\cancel{\frac{2J}{x}} + \frac{x}{2J} \right) - \left(1+\frac{1}{2J}\right)\left(\cancel{\frac{1}{x\left(1+\frac{1}{2J}\right)}} + x\left(1+\frac{1}{2J}\right) \right) \right]$$

$$= \mu_B g J B \left[\frac{1}{2J}\frac{x}{2J} - \left(1+\frac{1}{2J}\right)^2 x \right]$$

$$U = \mu_B g J B \left[\left(\frac{1}{2J}\right)^2 - \frac{1}{\left(1+\frac{1}{2J}\right)^2} \right] x$$

$$C_V = \frac{\partial U}{\partial T} = \frac{\partial U}{\partial x}\frac{\partial x}{\partial T} \xrightarrow[T\to\infty]{} 0$$

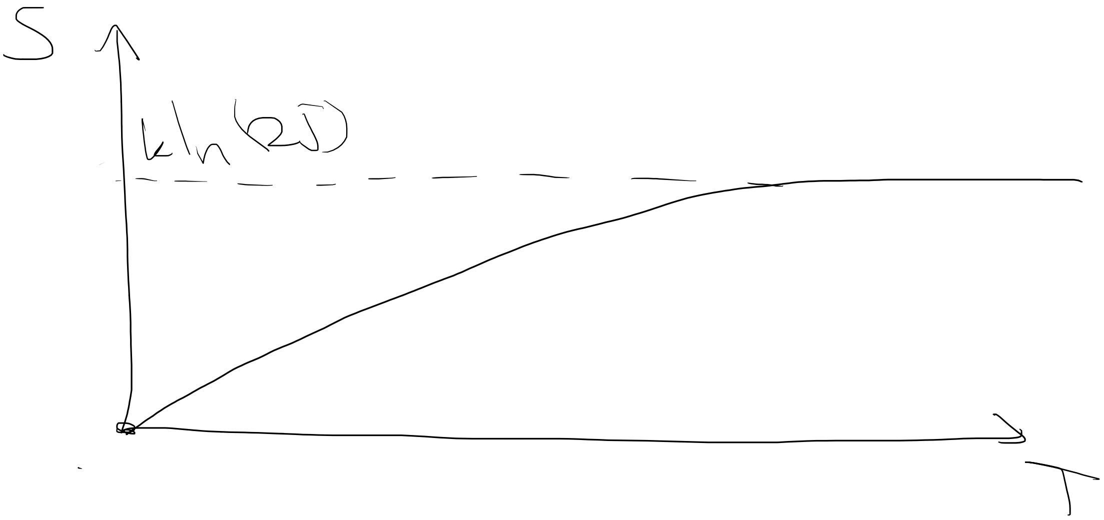
$$= \mu_B g J B \left[\left(\frac{1}{2J}\right)^2 - \left(\frac{1}{1+\frac{1}{2J}}\right)^2 \right]\left(-\frac{x}{T} \right)$$

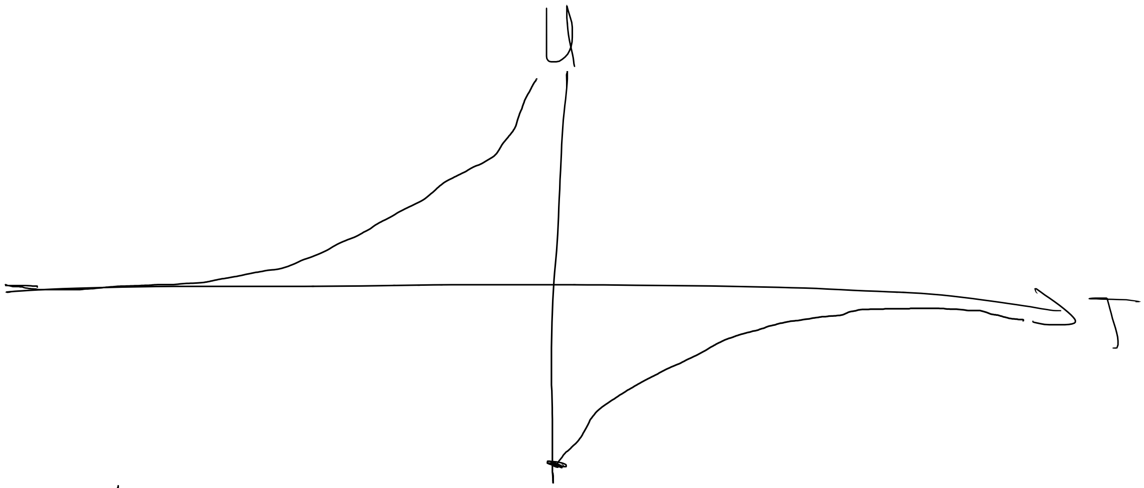


Schotsky anomaly

$$U \xrightarrow{T\to\infty} 0$$

$$U \xrightarrow{T\to 0} -\mu_B g J B$$





$J = \frac{1}{2}$

$$Q_1 = \frac{\sinh x\left(1+\frac{1}{2\frac{1}{2}}\right)}{\sinh\frac{x}{2\frac{1}{2}}} = \frac{\sinh(2x)}{\sinh x}$$

$$A = -kT\ln Q_1 = -kT\left[\ln\sinh(2x) - \ln\sinh x\right]$$

$$S = -\frac{A}{T} + kT\left[2\frac{\cosh(2x)}{\sinh(2x)} - \frac{\cosh x}{\sinh x}\right]\left(\frac{-x}{T}\right)$$

$$U = -kx\left[2\frac{\cosh 2x}{\sinh(2x)} - \frac{\cosh x}{\sinh x}\right]$$

$$T = T(U) \Rightarrow x = x(U)$$

$$\frac{1}{T} = \frac{\partial S}{\partial U}$$