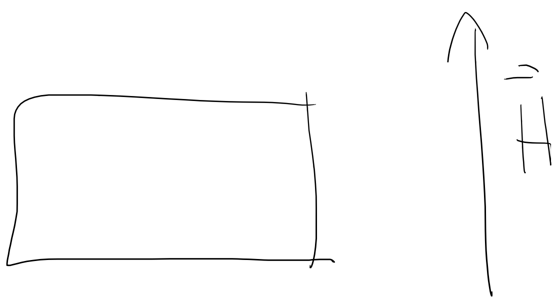
$$\langle \alpha; j, m | T^{(k)}_q | \alpha'; j' m' \rangle = \frac{1}{\sqrt{2j+1}} \underbrace{\langle \alpha; j \| T^{(k)} \| \alpha' j' \rangle}_{\text{reduced matrix element}}$$

$$\langle j' k; m', q | j, m \rangle$$

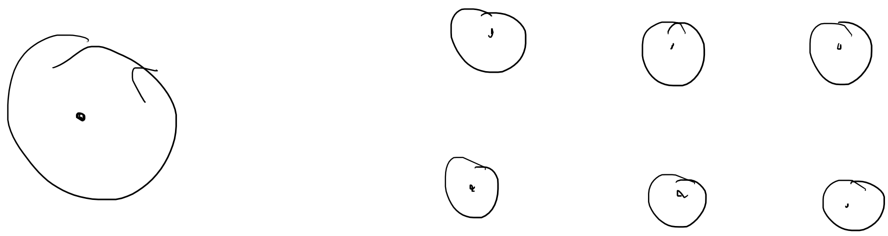
$$\langle \alpha; JM | K_z | \alpha JM \rangle = \frac{\langle \alpha JM | J_z | \alpha JM \rangle}{\hbar^2 J(J+1)} \langle \alpha JM | \vec{J} \cdot \vec{K} | \alpha JM \rangle$$

Paramagnetism



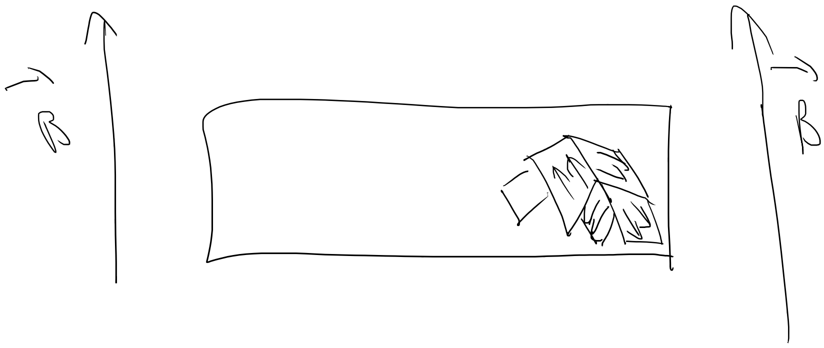
$$U = -\vec{\mu} \cdot \vec{H}$$

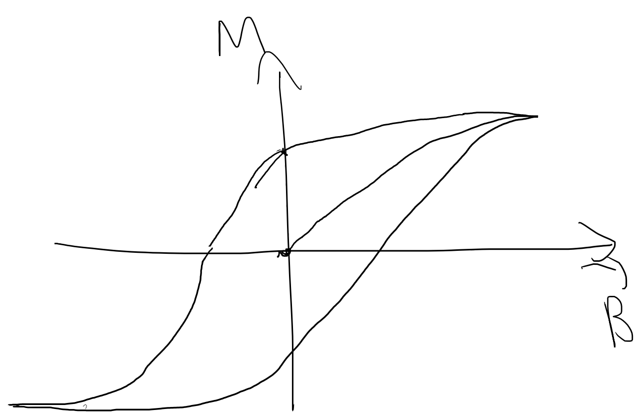
- Paramagnetic materials ←
- Diamagnetic materials
- ferromagnetic materials



$$\vec{\mu} \to \vec{\mu} + \Delta\vec{\mu}$$

$\Delta\vec{\mu}$ is in the opposite direction of external field





histerisis curve

$$E = \sum(-\vec{\mu}_i \cdot \vec{H}) = -\underbrace{\left(\sum \vec{\mu}_i\right)} \cdot \vec{B}$$

$$\vec{M} = \left\langle \sum \vec{\mu}_i \right\rangle = -\nabla_B E$$

$$Q_N = [Q_1]^N \frac{1}{N!}$$

if indistinguishable particles.

$$E = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta$$

$$Q_1 = \sum_{\theta,\phi} e^{-\beta(-\mu B\cos\theta)}$$

$$= \int e^{\beta\mu B\cos\theta} \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi}{\beta\mu B}\left(-e^{\beta\mu B\cos\theta}\Big|_{\theta=0}^{\theta=\pi}\right.$$

$$= \frac{-2\pi}{\beta\mu B}\left(-e^{-\beta\mu B} + e^{\beta\mu B}\right)$$

$$Q_1 = \frac{4\pi}{\beta\mu B}\sinh(\beta\mu B)$$

$$= \frac{4\pi}{x}\sinh x \qquad x = \beta\mu B = \frac{\mu B}{kT}$$

$$Q_N = \left(\frac{4\pi}{x}\sinh x\right)^N$$

$$A = -kT\ln Q_N = -NkT\ln\left(\frac{4\pi}{x}\sinh x\right)$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{V,N}$$

$$dU = TdS - PdV + \mu dN$$

$$d(\underbrace{U - TS}_{A}) = -SdT - PdV + \mu dN$$

$$S = -\frac{A}{T} + NkT\left[\frac{\partial}{\partial x}\ln\left(\frac{4\pi}{x}\sinh x\right)\right]\frac{d}{dT}\left(\frac{\mu B}{kT}\right)$$

$$S = -\frac{A}{T} - NkT\frac{\overbrace{\mu B}^{x}}{kT^2}\,\frac{-\frac{4\pi}{x^2}\sinh x + \frac{4\pi}{x}\cosh x}{\frac{4\pi}{x}\sinh x}$$

$$S = -\frac{A}{T} - Nk\,\cancel{x}\,\frac{\cancel{x}}{\cancel{4\pi}}\,\frac{1}{\sinh x}\left(\frac{-\cancel{4\pi}}{\cancel{x^2}}\right)(\sinh x - x\cosh x)$$

$$S = -\frac{A}{T} + Nk(1 - x\coth x)$$

$$U = A + TS = NkT(1 - x\coth x)$$

$$\boxed{U = NkT(1 - x\coth x)}$$

$$\frac{U}{NkT} = ? \qquad x = \frac{\mu B}{kT}$$

$$dU = TdS - PdV + \mu dN - \vec{M}\cdot d\vec{B}$$

$$dA = -SdT - PdV + \mu dN - \vec{M}\cdot d\vec{B}$$

$$M_z = -\left(\frac{\partial A}{\partial B_z}\right)_{T,V,N} = -\left(\frac{\partial A}{\partial B}\right)_{T,V,N}$$

$$A = -NkT\ln\left(\frac{4\pi}{x}\sinh x\right) \qquad x = \frac{\mu B}{kT}$$

$$M_z = -\left(\frac{\partial A}{\partial B}\right)_{T,V,N}$$

$$= NkT\,\frac{\partial \ln\left(\frac{4\pi}{x}\sinh x\right)}{\partial x}\,\frac{\partial x}{\partial B}$$

$$= N\cancel{kT}\,\frac{-\frac{4\pi}{x^2}\sinh x + \frac{4\pi}{x}\cosh x}{\frac{4\pi}{x}\sinh x}\left(\frac{\mu}{\cancel{kT}}\right)$$

$$= N\left(-\frac{\cancel{4\pi}}{x^{\cancel{2}}}\right)\mu\left(\frac{\sinh x - x\cosh x}{\sinh x}\right)\frac{\cancel{x}}{\cancel{4\pi}}$$

$$M = -N\mu\frac{1}{x}\left(1 - x\coth x\right)$$

$$\boxed{M = -N\mu\left(\frac{1}{x} - \coth x\right)}$$

$$\boxed{U = NkT\left(1 - x\coth x\right)}$$

Small x ($T \to \infty$ and/or $B \to 0$)

$$\coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}} \simeq \frac{(1+x) + (1-x)}{(1+x) - (1-x)} \simeq \frac{1}{x}$$

$$U \to NkT\,(1 - 1) = 0$$

$$\coth x \simeq \frac{\cancel{2}\left(1 + \cancel{x} + \frac{x^2}{2}\right) + \cancel{\left(1 - x + \frac{x^2}{2}\right)}}{\left(\cancel{1} + 2x + \cancel{\frac{x^2}{2}}\right) - \left(\cancel{1} - x + \cancel{\frac{x^2}{2}}\right)} = \frac{1}{x}\left(1 + \frac{x^2}{2}\right)$$

$$\coth x \simeq \frac{1}{x} + \frac{x}{2}$$

$$U = NkT\,(1 - x\coth x)$$

$$x \to 0 \quad \simeq \quad NkT\left(\cancel{1} - \left(\cancel{1} + \frac{x^2}{2}\right)\right)$$

$$\simeq -NkT\,\frac{x^2}{2} = -\frac{Nk\cancel{T}}{2}\,\frac{(\mu B)^2}{k^{\cancel{2}}T^{\cancel{2}}}$$

$$\boxed{U = -\left(N\frac{\mu^2 B^2}{2k}\right)\frac{1}{T}}$$

$$M = -N\mu\left(\frac{1}{x} - \coth x\right)$$

$$\simeq -N\mu\left(\cancel{\frac{1}{x}} - \left(\cancel{\frac{1}{x}} + \frac{x}{2}\right)\right)$$

$$M \simeq N\mu\frac{x}{2} = \frac{N\mu}{2}\frac{\mu B}{kT} = \left(\frac{N\mu^2 B}{2k}\right)\frac{1}{T}$$

$$\boxed{M \simeq \frac{C}{T}B}$$ Currie's Law

$$\chi = \left.\frac{\partial M}{\partial B}\right|_{B=0} \qquad x = \frac{\mu B}{kT}$$

<u>Large x behaviour</u> ($T \to 0$ and/or $B \to \infty$)

$$\left.\begin{aligned} M &= N\mu \\ U &= -N\mu B \end{aligned}\right\} \text{expected}$$

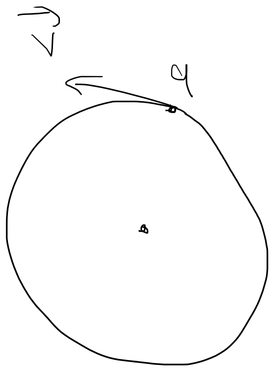
$$\boxed{M = -N\mu\left(\frac{1}{x} - \coth x\right)} \to N\mu$$

$$\boxed{U = NkT\left(1 - x\coth x\right)} \to -NkTx = -N\cancel{kT}\frac{\mu B}{\cancel{kT}}$$

$$U \overset{x\to\infty}{=} -N\mu B$$

Quantum Mechanical Treatment

$$H = -\vec{\mu}\cdot\vec{B} = -\mu_z B$$



$$\mu = IA$$

$$= \frac{q}{T}\pi r^2$$

$$= \frac{q}{\frac{2\pi r}{v}}\pi r^2$$

$$= \frac{q}{2}\, v r$$

$$= \frac{q}{2m}\overbrace{(m v r)}^{L}$$

$$\boxed{\vec{\mu}_L = \frac{q}{2m}\vec{L}}$$

$$\vec{\mu}_s = \frac{q}{2m}\Big(\overset{g}{\overset{\shortparallel}{2}}\vec{S}\Big)$$

gyromagnetic ratio

$$a = \frac{g-2}{2}$$

$$\vec{\mu} = -\frac{e}{2m}\left(\vec{L} + 2\vec{S}\right)\cdot\vec{B} = -\frac{e}{2m}\left(\vec{J} + \vec{S}\right)\cdot\vec{B}$$

$$\vec{\mu} \neq -\frac{e}{2m} g \vec{J}$$

g: Lange's g factor

$$H = -\frac{e}{2m} B \left(J_z + S_z \right)$$

$$Q_1 = \sum_{\text{microstates}} e^{-\beta E_r}$$ E_r: energy of the microstate.

$$\left\{ J^2, J_z, L^2, S^2 \right\}$$

$$\Delta E = \langle \alpha; JM | \frac{-e}{2m} B (J_z + S_z) | \alpha; JM \rangle$$

$$\Delta E = -\frac{e}{2m} B \langle \alpha; JM | J_z + S_z | \alpha; JM \rangle$$

$$\langle \alpha; JM | K_z | \alpha JM \rangle = \frac{\langle \alpha JM | J_z | \alpha JM \rangle}{\hbar^2 J(J+1)} \langle \alpha JM | \vec{J} \cdot \vec{K} | \alpha JM \rangle$$

$$\Delta E = -\frac{e}{2m} B \frac{\langle \alpha; JM | J_z | \alpha JM \rangle}{\hbar^2 J(J+1)} \langle \alpha; JM | \vec{J} \cdot (\vec{J} + \vec{S}) | \alpha JM \rangle$$

$$= -\frac{e}{2m} B \frac{\hbar M}{\hbar^2 J(J+1)} \langle \alpha; JM | \vec{J}^2 + \vec{J} \cdot \vec{S} | \alpha JM \rangle$$

$$= -\frac{e}{2m} B \frac{M}{\hbar J(J+1)} \Big[\hbar^2 J(J+1)$$

$$- \langle \alpha; JM | \frac{1}{2} \{ (\overbrace{\vec{J}-\vec{S}}^{L})^2 - J^2 - S^2 \} | \alpha JM \rangle \Big]$$

$$= -\frac{e}{2m} B \frac{M}{\hbar J(J+1)} \Big[\hbar^2 J(J+1)$$

$$- \frac{1}{2} \{ \hbar^2 l(l+1) - \hbar^2 J(J+1)$$

$$- \hbar^2 s(s+1) \} \Big]$$

$$= -\frac{e\hbar}{2m} \frac{BM}{J(J+1)} \Big[\frac{3}{2} J(J+1) - \frac{1}{2} l(l+1) + \frac{1}{2} s(s+1) \Big]$$

$$\Delta E = E = -\frac{e\hbar}{2m} BM \underbrace{\Big[\frac{3}{2} - \frac{l(l+1) - s(s+1)}{2J(J+1)} \Big]}_{g}$$

$$\Delta E = -\mu_B g M B \qquad \mu_B \equiv \frac{e\hbar}{2m} : \text{Bohr magneton}$$

$$Q_1 = \sum_M e^{-\beta(-\mu_B g M B)} = \sum_{M=-J}^{J} e^{(g\mu_B B\beta) M}$$

$$Q_1 = \sum_{M=-J}^{J} (e^{x})^M \qquad x = \mu_B g B \beta$$

$$Q_1 = \sum_{M=-J}^{\infty} \left(e^{x}\right)^{M} - \sum_{M=J+1}^{\infty} \left(e^{x}\right)^{M}$$

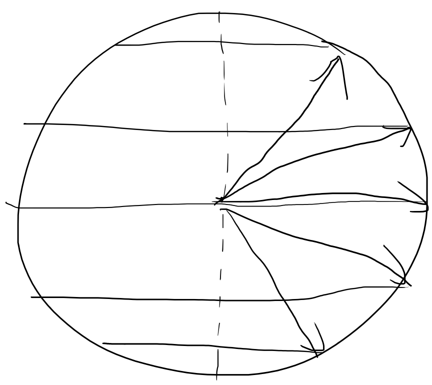
$\Leftarrow$ correction

$$= \frac{e^{-Jx}}{1-e^{+x}} - \frac{e^{x(J+1)}}{1-e^{x}} = e^{\frac{x}{2}}\, \frac{e^{-(J+\frac{1}{2})x} - e^{(J+\frac{1}{2})x}}{1-e^{x}}$$

$$\boxed{Q_1 = -\frac{2\sinh\left(x(J+\frac{1}{2})\right)}{\cancel{1-e^{x}}\; e^{-\frac{x}{2}} - e^{\frac{x}{2}}}} = \frac{\sinh\left(x(J+\frac{1}{2})\right)}{\sinh\left(\frac{x}{2}\right)}$$

$$x = \mu_B g B \beta = \frac{\mu_B g B}{kT}$$

$$g = \frac{3}{2} - \frac{l(l+1) - s(s+1)}{2J(J+1)} \qquad |l-s| < J < |l+s|$$



$J = 2$

$J \to \infty$ we should recover classical behavior!

$T \to \infty$ also (?)

$$\Delta E^{qm} = -\mu_B g J \frac{M}{J} B \qquad E^{cl} = -\mu B \cos\Theta$$

$J \to \infty$

$\mu_B g J \to$ finite. (identify with μ in the classical treatment)

$$\underline{x^{qm} = \mu_B g B \beta} \qquad x_J^{qm} = \underbrace{(\mu_B g J)}_{\mu^{cl}} B\beta = x^{cl}$$

$$Q_1 = - \frac{2\sinh(xJ)}{1-e^x}$$

$$= - \frac{2\sinh(x^{cl})}{1-e^{x^{qm}}}$$ ⇐

$$\to -\frac{2\cosh(x^{cl})}{\cancel{1}-(\cancel{1}+x^{qm})} = J\frac{2\sinh(x^{cl})}{x^{cl}}$$

$$Q_1^{cl} = \frac{4\pi}{x^{cl}} \sinh x^{cl}$$

⇐ correct the results.