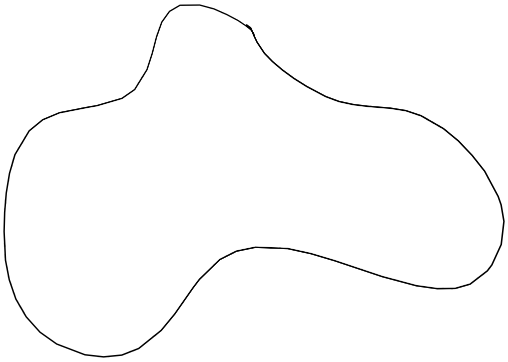


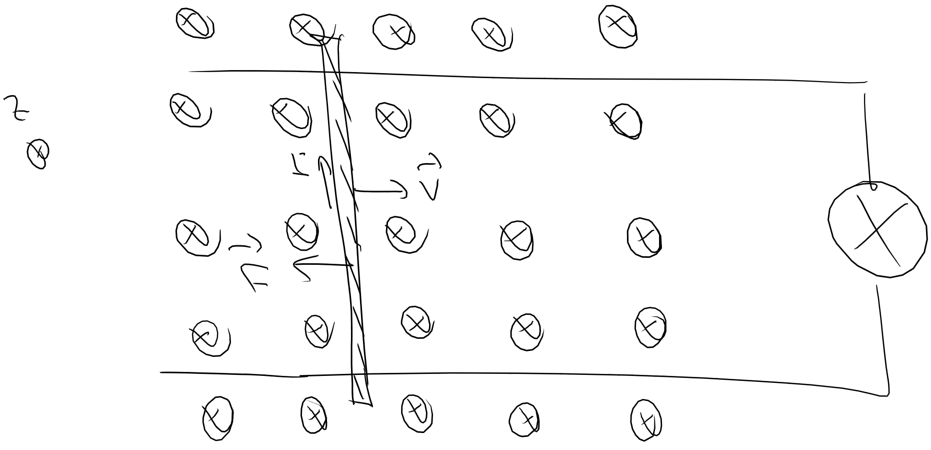
$$\mathcal{E} = -\frac{1}{c}\frac{d\Phi_B}{dt} \qquad \Phi_B = \int_S \vec{B}\cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{A}$$

$$\int (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{1}{c}\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = -\frac{1}{c}\int \frac{\partial\vec{B}}{\partial t}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}}$$

Lenz Rule: induced ~~emf~~ current is in the direction which will oppose the change that creates it.



$$\vec{F} = q\vec{v}\times\vec{B}$$

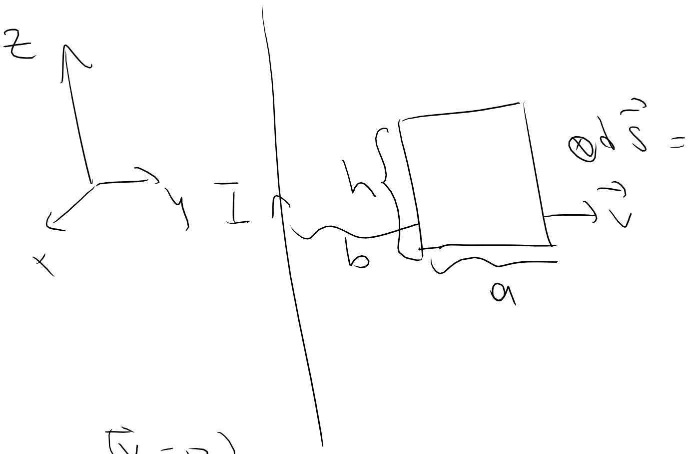
$$d\vec{F} = I\,d\vec{l}\times\vec{B}$$

$$\phi_B = \int \vec{B}\cdot d\vec{S}$$

Example

$\vec{r} = (s, \phi, z)$

$$\vec{B}(\vec{r}) = \hat{\phi}\frac{B_0 L_0}{s}\frac{\sin(\omega t)}{\omega t}$$



$\otimes d\vec{S} = dS\,\hat{\phi}$ $\qquad \phi_B = \int \vec{B}\cdot d\vec{S}$

$$\phi_B = \int B\, dS$$

$(\vec{v} = 0)$

i) at $t=0$, what is the direction of the induced current?

counter clock wise

ii) $\vec{v} = v_0\hat{y}$ $\quad v_0 > 0$ constant

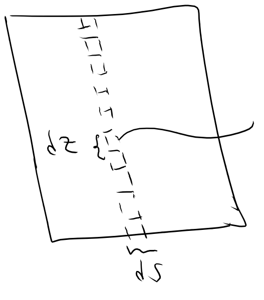
direction of the induced current $(\omega \to 0)$

$$\vec{B} = \hat{\phi}\frac{B_0 L_0}{s}$$

clock wise direction

iii) $\mathcal{E}(t) = ?$

$$\mathcal{E} = -\frac{1}{c}\frac{d\phi_B}{dt} \qquad d\vec{S} = dS\,\hat{\phi}$$



$$\int \vec{B}\cdot d\vec{S} = \int \frac{B_0 L_0}{s}\frac{\sin\omega t}{\omega t}\, ds\, dz$$

$d\vec{S} = ds\, dz\, \hat{\phi}$

$$= B_0 L_0 \frac{\sin\omega t}{\omega t}\int_0^h dz \int_b^{b+a}\frac{ds}{s}$$

$$\phi_B = B_0 L_0 \frac{\sin\omega t}{\omega t}\, h \ln\left(\frac{b+a}{b}\right)$$

$$\mathcal{E} = -\frac{1}{c}\frac{d\phi_B}{dt}$$

$$\mathcal{E} = -\frac{B_0 L_0}{c} h \ln\left(\frac{b+a}{b}\right) \frac{d}{dt} \frac{\sin \omega t}{\omega t}$$

$$- \frac{B_0 L_0 h}{c} \frac{\sin \omega t}{\omega t} \frac{d}{dt} \ln\left(1 + \frac{a}{b}\right)$$

$$= -\frac{B_0 L_0 h \ln\left(\frac{b+a}{b}\right)}{c} \frac{d}{dt} \frac{\sin \omega t}{\omega t}$$

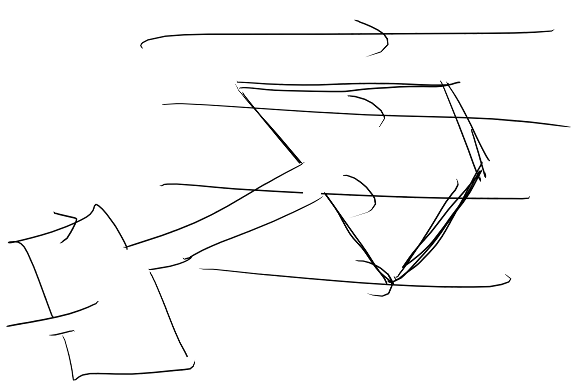
$$- \frac{B_0 L_0 h}{c} \frac{\sin \omega t}{\omega t} \left[\frac{d}{db} \ln\left(1 + \frac{a}{b}\right)\right] \underbrace{\frac{db}{dt}}_{v}$$

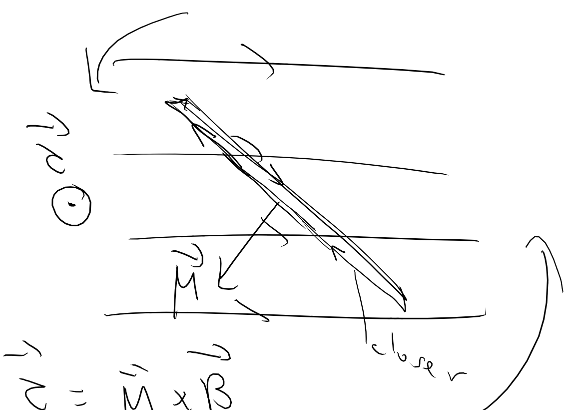
if $\mathcal{E} > 0 \Rightarrow I_{ind} > 0$ clockwise

if $\mathcal{E} < 0 \Rightarrow I_{ind} < 0$ counterclockwise

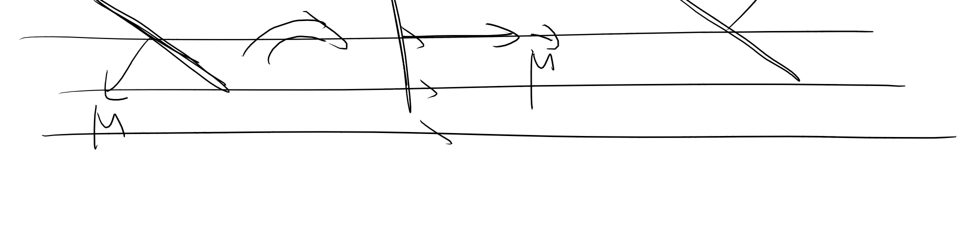
$$\boxed{\mathcal{E} = -\frac{1}{c} \frac{d \phi_B}{dt}}$$

Electric Generators & Electric Motors





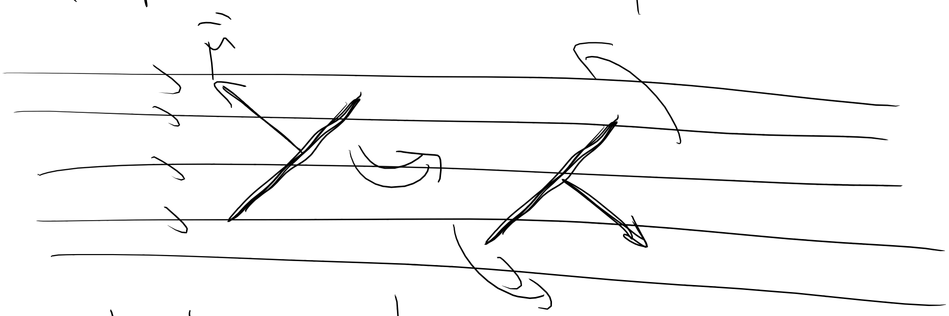
$\vec{\tau} = \vec{\mu} \times \vec{B}$



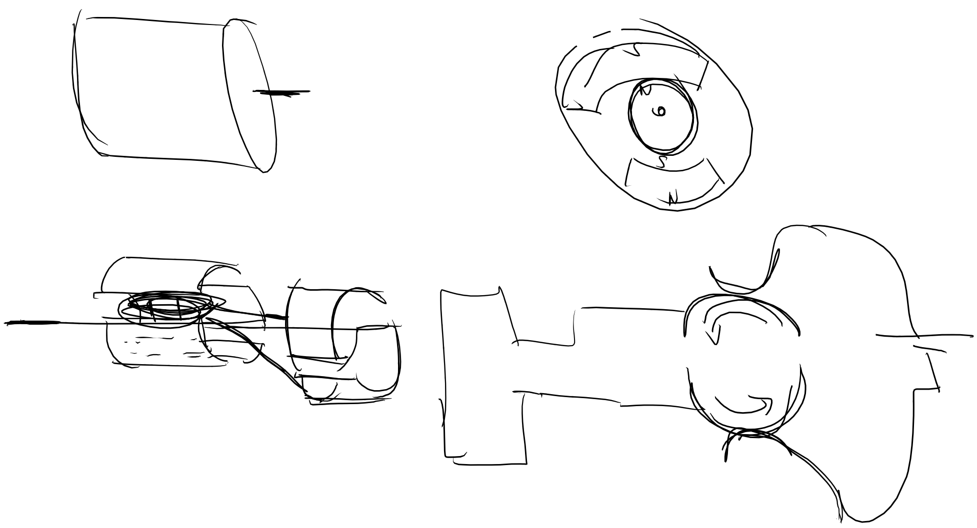
$U = -\vec{\mu} \cdot \vec{B}$

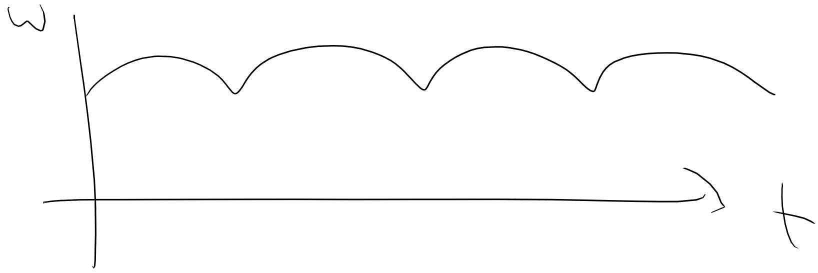
$\mu = I a N$

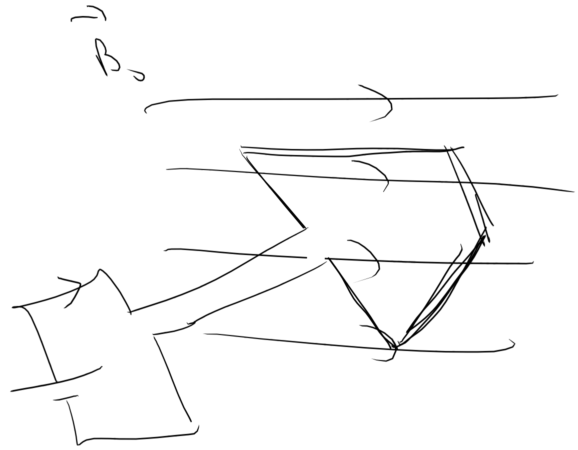
flip the direction of the current

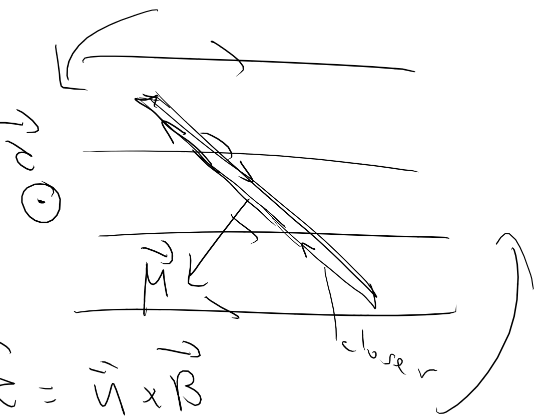


electric motor.









$\vec{\varepsilon} = \vec{v} \times \vec{B}$

$$\phi_B = \int \vec{B} \cdot d\vec{S} = \vec{B}_0 \cdot \int d\vec{S} = B_0 A \cos\theta$$

$$\mathcal{E} = -\frac{1}{c}\frac{d\phi_B}{dt} = +\frac{1}{c} B_0 A (+\sin\theta) \frac{d\theta}{dt}$$

$$\boxed{\mathcal{E} = \frac{B_0 A}{c} (\sin\theta)\, \omega(t)}$$

if $\boxed{\theta = \omega t}$ $\omega = \text{const}$

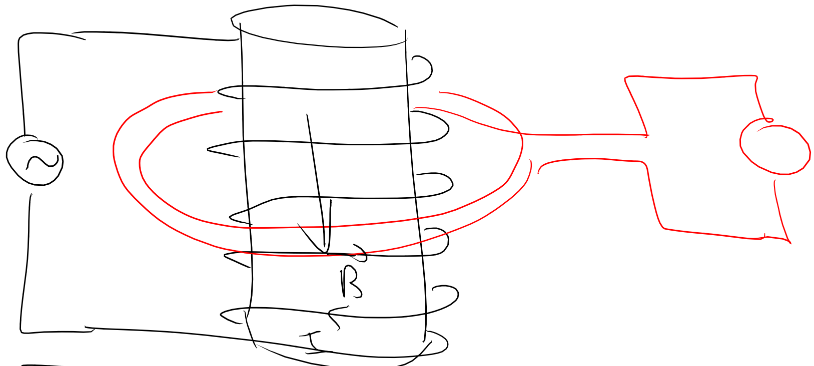
$$\boxed{\mathcal{E} = \frac{B_0 A}{c} \omega \sin\omega t}$$

$$[\mathcal{E}] = \left[\int \vec{E} \cdot d\vec{l}\right] = [\vec{E}][l]$$

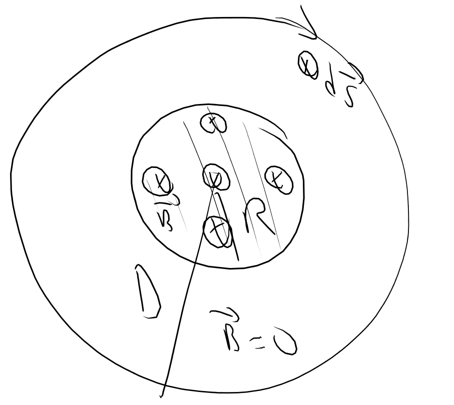
$$[\vec{E}][l] = [B_0] \frac{[A]}{c} [\omega]$$

$$[l] = \frac{[A]}{[c]} [\omega] = \frac{[A]}{[l]\cancel{[\omega]}} \cancel{[\omega]} = [l] \checkmark$$

Example



$$\boxed{\vec{B} = B_0 \sin(\omega t)}$$



$B_0 > 0$ if $t_0 > 0$

$$\Phi_B \overset{(2)}{=} B_0 \sin(\omega t)\, \pi R^2$$

$$\mathcal{E} = -\frac{1}{c}\left(\frac{d\Phi_B}{dt}\right) = -\frac{B_0 \pi R^2 \omega}{c}\cos(\omega t) = \mathcal{E}$$

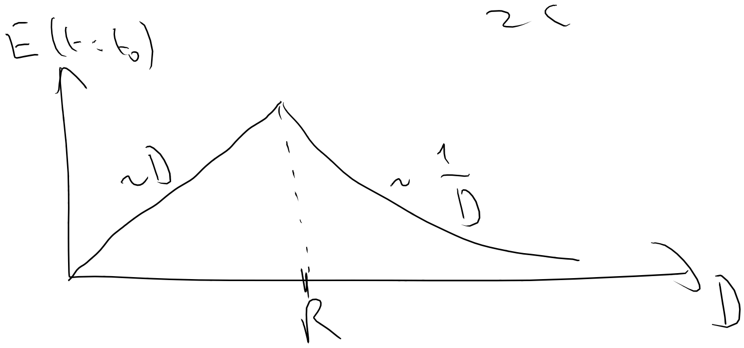
$$2\pi D\, E = -\frac{B_0 \pi R^2 \omega}{c}\cos(\omega t)$$

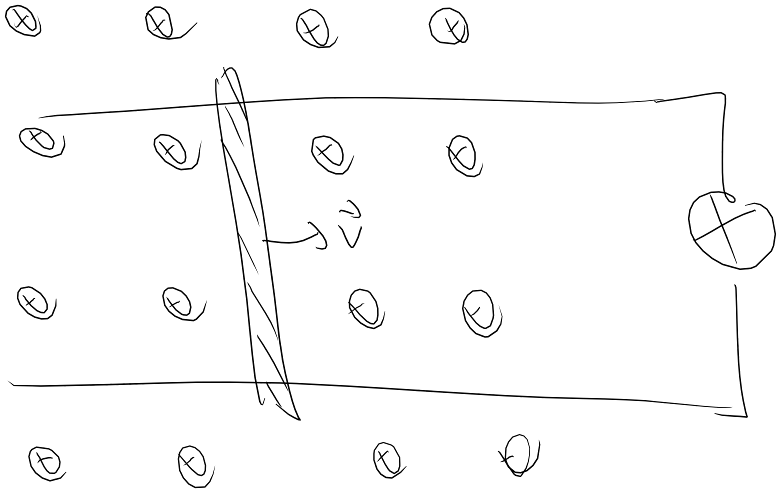
$$E = -\frac{B_0 R^2 \omega}{2c}\,\frac{\cos(\omega t)}{D}$$

E inside the solenoid $(D < R)$

$$2\pi D\, E = -B_0 \pi (D^2) \frac{\omega}{c} \cos(\omega t)$$

$$E = \frac{B_0 \omega}{2c} D \cos(\omega t)$$

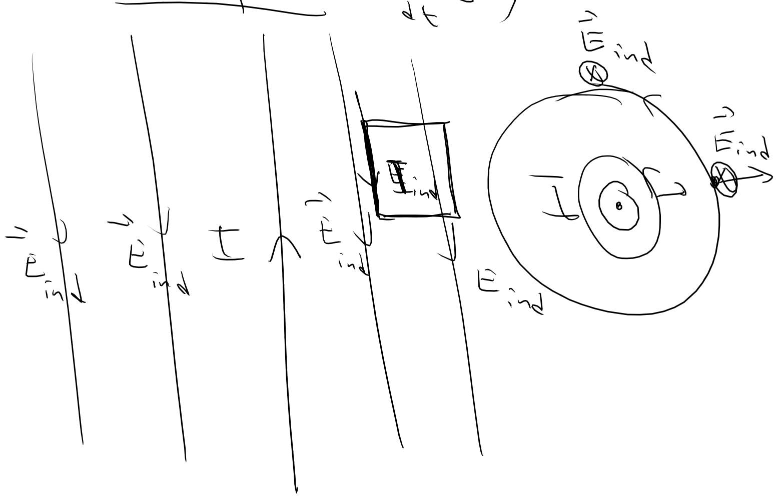




$$\vec{E} = -\vec{v} \times \vec{B}\,\frac{1}{c}$$
$$= \vec{B} \times \vec{v}\,\frac{1}{c}$$

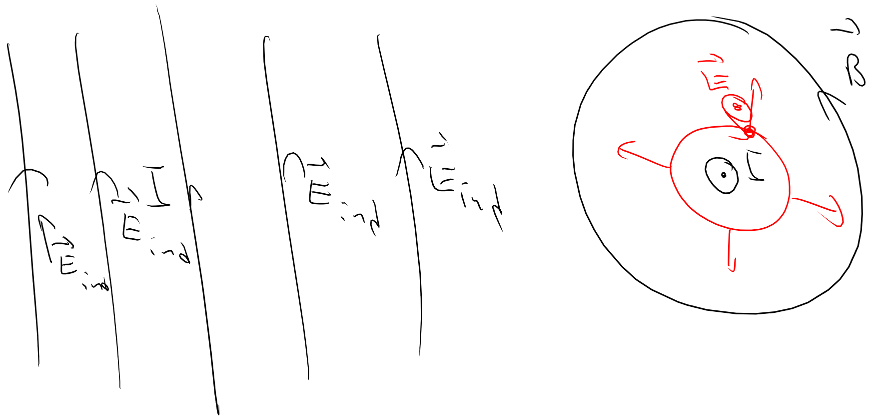
$\vec{v}$: velocity of $\vec{B}$ field lines

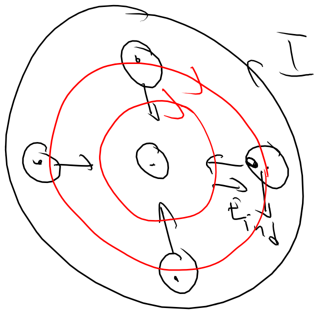
Example $\left(\frac{dI}{dt} > 0\right)$



Example $\left(\frac{dI}{dt} < 0\right)$

$\vec{E}_{ind} = \vec{B} \times \vec{v}$





$$\boxed{\vec{E}_{ind} = \vec{B} \times \frac{\vec{v}}{c}}$$

$$\boxed{\vec{B} = \frac{\vec{v}_0}{c} \times \vec{E}}$$

for a moving point charge.

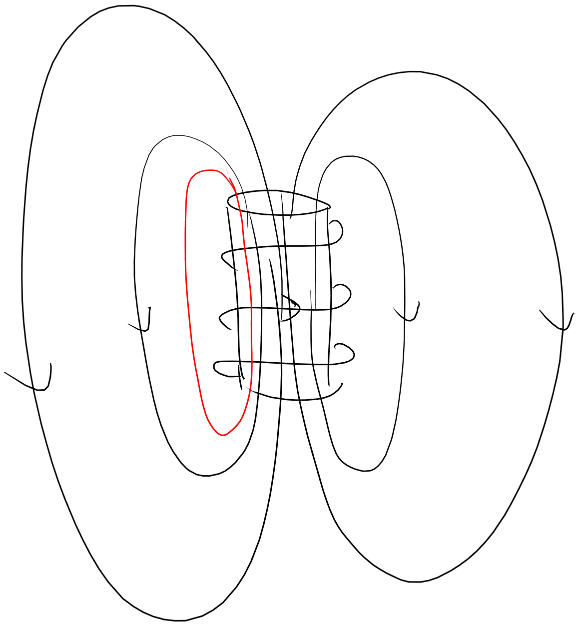
$$\vec{v} \times \vec{E}_{ind} = \vec{v} \times \left(\vec{B} \times \frac{\vec{v}}{c}\right)$$

$$= \vec{B}\left(\frac{v^2}{c}\right) - \vec{v}\left(\vec{B} \cdot \frac{\vec{v}}{c}\right)$$

assume $\vec{B} \perp \vec{v} \Rightarrow \vec{B} \cdot \vec{v} = 0$

$v = c$

$$\boxed{\vec{B} = \frac{\vec{v}}{c} \times \vec{E}_{ind}}$$



$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}}$$