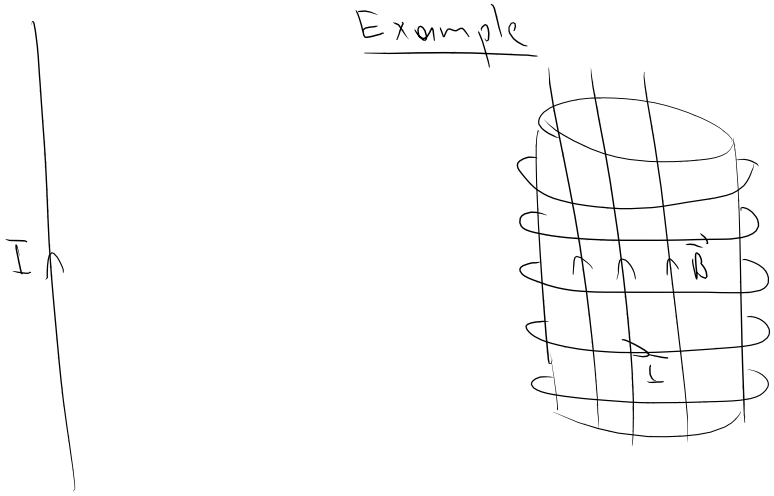
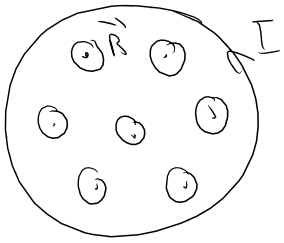
Example

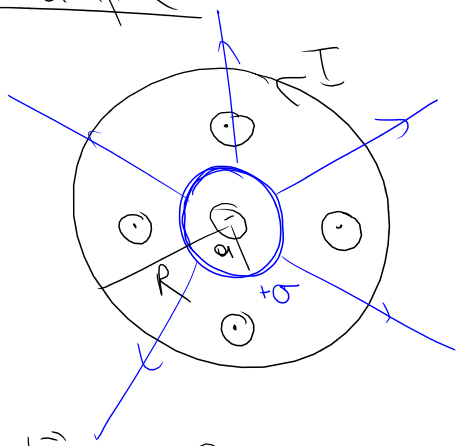




case i) $\frac{dI}{dt} > 0$ induced $\mathcal{E}$ will resist change

case ii) $\frac{dI}{dt} < 0$ " " will resist change.

Example



$\oint \vec{E} \cdot d\vec{S} = 4\pi Q_{enc}$

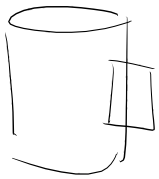
$B = \frac{4\pi I}{c} n$

$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$

$E = \frac{\left(\frac{4\pi}{c}\right)(2\pi a \sigma)}{2\pi s}$

$\vec{E} = 4\pi a \frac{\sigma}{s} \hat{n}$

$\vec{B} = \frac{4\pi}{c} I n \hat{z}$



$\oint \vec{B} \cdot d\vec{R} = \frac{4\pi}{c} I_{enc}$

$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I_{enc}$

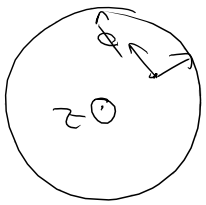
$\vec{S} = 4\pi \frac{c}{4\pi} \left(a \frac{\sigma}{s} \hat{s}\right) \times \left(\frac{4\pi}{c} I n \hat{z}\right)$

$$\vec{S}(r) = 4\pi a \frac{\sigma I n}{s} \left(-\hat{\phi}\right)$$

$$\frac{\vec{L}}{l} = \frac{1}{l}\int\left(\vec{r}\times\frac{\vec{S}}{c^2}\right)dV$$

$$= \frac{4\pi}{c^2 l}\int_0^l dz\int_0^{2\pi} d\phi\int_a^R s\,ds\,(s\hat{s}+z\hat{z})\times\left(\frac{a\sigma I_n}{s}\right)(-\hat{\phi})$$

$$= \frac{4\pi a\sigma I_n}{c^2 l}\int_0^l dz\int_0^{2\pi} d\phi\int_a^R ds\left(s(-\hat{z})+z\,\hat{s}\right)$$



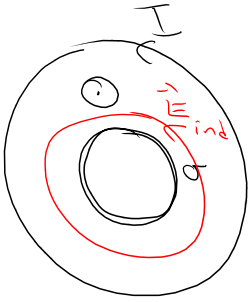
$$\int_0^{2\pi} d\phi\;\hat{s} = 0$$

$$\int_0^{2\pi} d\phi\,(\cos\phi\,\hat{x}+\sin\phi\,\hat{y}) = 0$$

$$\frac{\vec{L}}{l} = \frac{4\pi a\sigma I_n}{c^2}(-\hat{z})\int_0^{2\pi} d\phi\int_a^R s\,ds$$

$$= -\frac{4\pi a\sigma I_n}{c^2}(\hat{z})\,2\pi\left(\frac{R^2-a^2}{2}\right)$$

$$\boxed{\frac{\vec{L}}{l}^{em} = \frac{4\pi a\sigma I_n \pi}{c^2}\left(R^2-a^2\right)(-\hat{z})}$$



$$\oint\vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{d\Phi_B}{dt}$$

$$E\,2\pi s = -\frac{d}{dt}\left(\frac{4\pi^2}{c}I_n s^2\right)$$

$$E = -\frac{2\pi s}{c^2}n\frac{dI}{dt}$$

$$dF = E\,dq$$

$$dq = \sigma\,dS$$

$$\vec{r}_2 = \vec{r}_1 + \vec{r}_{12}$$

$$dS = b\,d\phi\,dz$$

$$|\vec{r}_{12}| = a$$

$$\vec{r}_{12} = a\,\hat{r}(z)$$

$$F = z\left(-\frac{2\pi a n}{c^2}\frac{dI}{dt}\right)\sigma\,b\,d\phi\,dz$$

$$\frac{\vec{L}}{l} = \hat{z}\left(-\frac{2\pi a n}{c} a \sigma \frac{dI}{dt}\right) 2\pi a$$

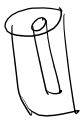
$$\int_{t=0}^{t=\infty} \frac{d}{dt}\left(\frac{\vec{L}_a}{l}\right) = \frac{\vec{L}}{l} = \hat{z}\left(-\frac{4\pi^2 a^2 n \sigma}{c^2} \frac{dI}{dt}\right)$$

$$\frac{(\vec{L}_a)^f}{l} = -\frac{4\pi^2 a^2 n \sigma \hat{z}}{c^2}\left(I_f - I_0\right)$$

$$\boxed{\left(\frac{\vec{L}_a}{l}\right)^f = \frac{4\pi^2 I a^2 n \sigma \hat{z}}{c^2}}$$

$$\frac{\vec{L}^{em}}{l} = \frac{4\pi a \sigma I n \pi}{c^2}\left(R^2 - a^2\right)(-\hat{z})$$

$$= \frac{4\pi a \sigma I n \pi}{c^2} R^2(-\hat{z}) + \frac{a^2 \sigma I n \pi}{c^2}\hat{z}$$



$E \sim \frac{1}{r}$

$B \sim \frac{1}{r^3}$

$S \sim \frac{1}{r^4}$

$L \sim \int \frac{dr}{r}$

$\vec{B} \sim \frac{1}{r^3}$

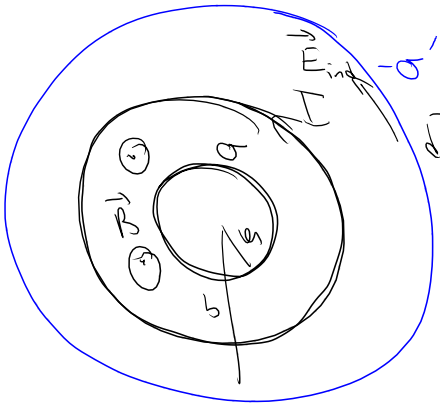
$\vec{E} \sim \frac{1}{r^2}$

$S \sim \frac{1}{r^5}$

$r \times S \sim \frac{1}{r^4}$

$L \sim \int r \times S \, dV \sim \int \frac{dr}{r^2}$

$dV \sim r^2 dr$



$d\vec{S}$ dq

$\sigma a = \sigma' b$

$\vec{E} = 0$

outside

$$E_{ind} \cdot 2\pi s = -\frac{1}{c}\frac{d}{dt}\left(\pi R^2 \frac{4\pi}{c} I n\right)$$

$$\boxed{E_{in} = -\frac{2\pi R^2 \cdot n}{c^2 s}\frac{dI}{dt}}$$

$$dq = (-\sigma')\, dS$$

$$dF = \left(\frac{-2\pi R^2 n}{c^2 s}\frac{dI}{dt}\right)(-\sigma')\, dS$$

$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{r}| = b$

$$d\tau = \frac{2\pi R^2 n}{c^2 b}\frac{dI}{dt}\sigma' b\, dS$$

$$d\tau = \frac{2\pi R^2 n}{c^2}\left(\frac{dI}{dt}\right) dS\, \sigma'$$

$$\frac{\tau}{l} = \frac{2\pi R^2 n}{c^2}\left(\frac{dI}{dt}\right) 2\pi b\, \sigma'$$

$$\frac{\vec{L}_b^{\,f}}{l} = \frac{2\pi R^2 n}{c^2}(2\pi b)\sigma'(-I)\hat{z}$$

$$= 4\pi^2 \frac{R^2 n (b\sigma')}{c^2} I(-\hat{z}) \Leftarrow$$

$$\frac{\vec{L}_b^{\,f}}{l} = \frac{4\pi^2 R^2 n\, a\sigma}{c^2} I(-\hat{z})$$

$$\left(\frac{\vec{L}_a}{l}\right)^f = \frac{4\pi I a^2 n \sigma \hat{z}}{c^2}$$

$$\left(\frac{\vec{L}_a + \vec{L}_b}{l}\right)^f = \frac{4\pi^2 a\sigma I n (R^2 - a^2)(-\hat{z})}{c^2}$$

$$\frac{\vec{L}^{em}}{t} = \frac{4\pi\sigma a \sigma I n s_1}{c^2}\left(R^2 - a^2\right)(-\hat{z})$$

$$\sigma a = a' b \leftarrow$$

Vector and Scalar Potentials in Electrodynamics

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{A}\right)$$

$$= -\vec{\nabla}\times\left(\frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right)$$

$$\vec{\nabla}\times\left(\vec{E} + \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right) = 0$$

$$\Rightarrow \vec{E} + \frac{1}{c}\frac{\partial\vec{A}}{\partial t} = -\vec{\nabla}\phi$$

$$\boxed{\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{A}_0 \longrightarrow \vec{A} = \vec{A}_0 + \vec{\nabla}\Lambda$$

$$\vec{E} = -\vec{\nabla}\phi_0 - \frac{1}{c}\frac{\partial\vec{A}_0}{\partial t} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$-\vec{\nabla}\phi_0 - \cancel{\frac{1}{c}\frac{\partial\vec{A}_0}{\partial t}} = -\vec{\nabla}\phi - \cancel{\frac{1}{c}\frac{\partial\vec{A}_0}{\partial t}} - \frac{1}{c}\frac{\partial\vec{\nabla}\Lambda}{\partial t}$$

$$\phi = \phi_0 - \frac{1}{c}\frac{\partial \Lambda}{\partial t}$$

$$\left.\begin{aligned} \vec{A} &\to \vec{A} + \vec{\nabla}\Lambda \\ \phi &\to \phi - \frac{1}{c}\frac{\partial \Lambda}{\partial t} \end{aligned}\right\} \text{Gauge Transformations}$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\boxed{-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}\right) = 4\pi\rho(\vec{r},t)}$$

in Coulomb Gauge $\vec{\nabla}\cdot\vec{A} = 0$

$$\nabla^2\phi = -4\pi\rho(\vec{r},t)$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial}{\partial t}\left(-\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \nabla^2\vec{A} = \frac{4\pi}{c}\vec{J} - \vec{\nabla}\left(\frac{1}{c}\frac{\partial}{\partial t}\phi\right) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} - \nabla^2\vec{A}\right) = \frac{4\pi}{c}\vec{J} - \vec{\nabla}\left(\frac{1}{c}\frac{\partial}{\partial t}\phi + \vec{\nabla}\cdot\vec{A}\right)$$

$$\frac{1}{c}\frac{\partial}{\partial t}\phi + \vec{\nabla}\cdot\vec{A} = 0 \quad : \text{Lorentz Gauge}$$

$$\vec{\nabla}\cdot\vec{A} = -\frac{1}{c}\frac{\partial}{\partial t}\phi$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{A} = \frac{4\pi}{c}\vec{J}$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}\right) = 4\pi\rho(\vec{r},t)$$

$$-\nabla^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial}{\partial t}\phi\right) = 4\pi\rho(\vec{r},t)$$

$$\boxed{\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = 4\pi\rho(\vec{r},t)}$$

in free space

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\begin{pmatrix} A_x \\ A_y \\ A_z \\ \phi \end{pmatrix} = 0$$

wave eqn.: $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{d^2}{dx^2}\right)\varphi = 0 \Rightarrow \varphi = f(x-ct) + g(x+ct)$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}\right)$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{B} = 0$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial\vec{E}}{\partial t}\right)$$

$\vec{\nabla}\cdot\vec{E} = 0$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{E} = 0 \quad \Longleftarrow$$

$$\vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$-\frac{\omega^2}{c^2} + k^2 = 0 \Rightarrow \boxed{\omega = kc} \Longleftarrow \text{Dispersion relation}$$

Direction of $\vec{E}_0$ is call the polarization of the em wave.

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \qquad \vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

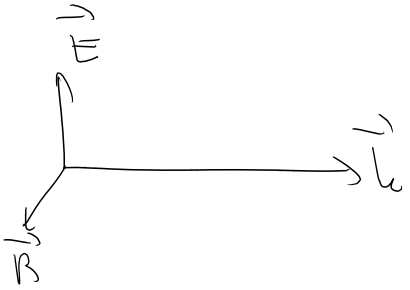
$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$i\vec{k} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad \not\Leftarrow$$

$$i\vec{k} \times \vec{E} = +\frac{1}{c}(+i\omega)\vec{B}$$

$$\vec{k} \times \vec{E} = \underbrace{\frac{\omega}{c}}_{k} \vec{B}$$

$$\boxed{\vec{B} = \hat{k} \times \vec{E}}$$



$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$$

$$\vec{E} = \vec{E}_0 e^{i(kx - \omega t)} = a\hat{\varepsilon}_1 e^{i(kx-\omega t)} + b\hat{\varepsilon}_2 e^{i(kx-\omega t)}$$

$$a = |a| e^{i\phi_1}$$

$$b = |b| e^{i\phi_2}$$

$$\vec{E} = |a|\hat{\varepsilon}_1 e^{i(kx - \omega t + \phi_1)} + |b|\hat{\varepsilon}_2 e^{i(kx - \omega t + \phi_2)}$$

$$\mathrm{Re}(\vec{E}) = |a|\hat{\varepsilon}_1 \cos(kx - \omega t) + |b|\hat{\varepsilon}_2 \cos(kx - \omega t + \phi_2)$$

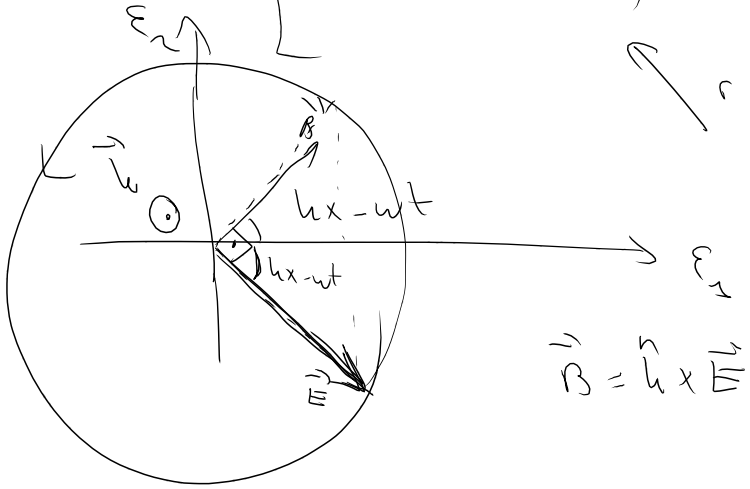
assume $\phi_1 = 0$

Simple case $\boxed{|a| = |b|}$ $\phi_2 = \frac{\pi}{2}$

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$$

$$\mathrm{Re}(\vec{E}) = |a| \left[\hat{\varepsilon}_1 \cos(kx-\omega t) - \hat{\varepsilon}_2 \sin(kx-\omega t) \right]$$

right circular polarized wave.



$$\vec{B} = \hat{k} \times \vec{E}$$