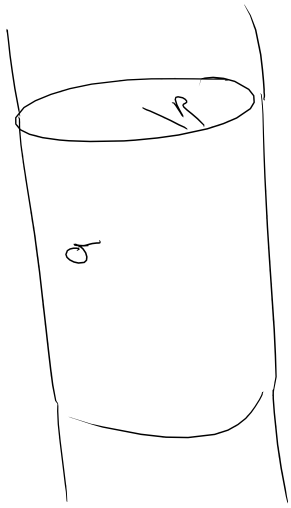


$\uparrow \vec{\omega}$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = 0$$ inside & outside the cylinder

$$\Rightarrow \vec{B} = -\vec{\nabla}\phi$$

$$|\vec{B}|(s)$$

$$\boxed{\vec{B} = B(s)\hat{z}}$$

$$B_z = -\frac{\partial \phi}{\partial z}$$

$$\Phi_M = \Phi_1(s)\, z + \Phi_2(s) \quad \Longleftarrow$$

$$\nabla^2 \Phi_M = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial \Phi_M}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 \Phi_M}{\partial \phi^2} + \frac{\partial^2 \Phi_M}{\partial z^2} = 0$$

$$\frac{1}{s}\frac{\partial}{\partial s}\, s\frac{\partial \Phi_i}{\partial s} \;\; S(s) - \frac{m^2}{s^2} = 0 \quad \Rightarrow \quad S(s) = A s^m + \frac{A'}{s^m}$$

$$\Rightarrow s\frac{d\Phi_i}{ds} = B \Rightarrow \frac{d\Phi_i}{ds} = \frac{B}{s}$$

$$\Phi_i = B_i \ln\left(\frac{s}{s_0}\right) + C_i$$

$$\Phi_m = \left[B_1 \ln\left(\frac{s}{s_0}\right) + C_1\right] z + B_2 \ln\left(\frac{s}{s_0}\right) + C_2$$

$$\Phi_m^{in} = C_1 z + C_2$$

$$\Phi_m^{outside} = 0$$

$B_\perp$ is continuous $\Rightarrow$ $\frac{\partial \Phi_m}{\partial s}$ is continuous

$$\boxed{\left(\frac{B_1}{R} + B_2\right) z = 0}$$

$B_\parallel$ is discontinuous $K = \omega R \sigma$

Electromagnetic Waves in Medium (Dielectrics)

$$\vec{\nabla}\cdot\vec{D} = 4\pi\rho$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

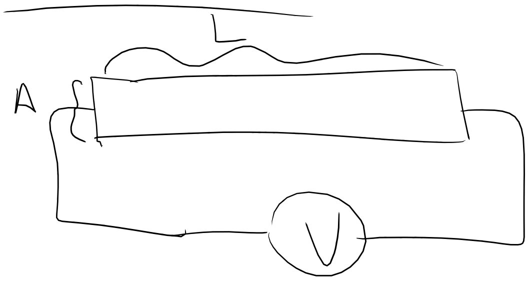
$$\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial D}{\partial t}$$

$$\begin{cases} \vec{D} = \varepsilon \vec{E} \\ \vec{B} = \mu \vec{H} \end{cases}$$

$$\vec{j} = \sigma(\omega)\vec{E}$$

Ohm's Law

Ohm's Law



$$V = LE$$

$$I = jA$$

$$\frac{V}{I} = \frac{LE}{jA} = \frac{L\cancel{E}}{\sigma \cancel{E} A}$$

$$\frac{1}{\sigma} = \rho \text{ : resistivity}$$

$$V = IR$$

$$R = \left(\frac{1}{\sigma}\right)\frac{L}{A}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad (\rho = 0 \; ; \; \vec{j} = 0)$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\frac{1}{\mu} \vec{\nabla} \times \vec{B} = + \frac{1}{c} \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \frac{\mu \varepsilon}{c} \frac{\partial \vec{E}}{\partial t}}$$

$$\vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t} \right) = \frac{\mu \varepsilon}{c} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla} \times \left(-c \, \vec{\nabla} \times \vec{E} \right) = \frac{\mu \varepsilon}{c} \frac{\partial^2 \vec{E}}{\partial t^2}$$

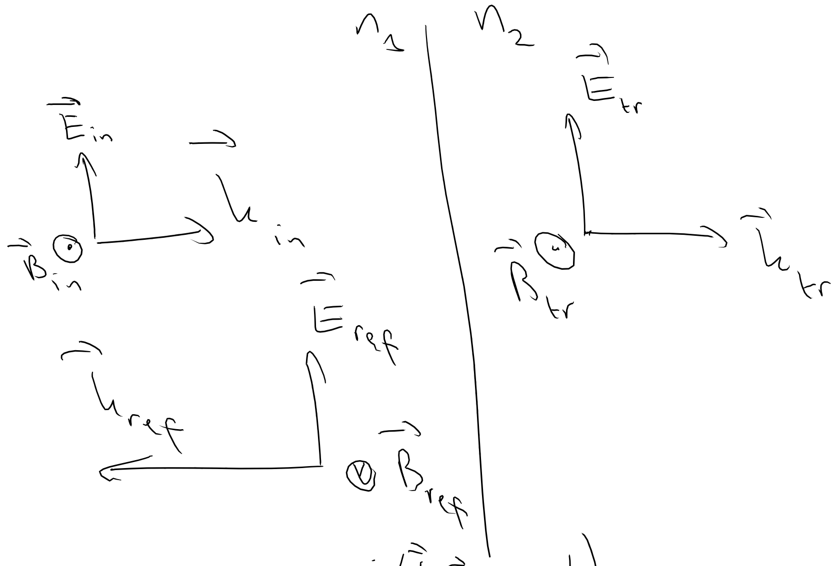
$$-c \left[\vec{\nabla} (\underbrace{\vec{\nabla} \cdot \vec{E}}_{=0}) - \nabla^2 \vec{E} \right] = \frac{\mu \varepsilon}{c} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{E} = 0$$

$$v = \frac{c}{\sqrt{\mu \varepsilon}} = \frac{c}{n} \qquad \boxed{v = \frac{\omega}{k}}$$

$$\boxed{n = \sqrt{\mu \varepsilon}} \simeq \sqrt{\varepsilon} \qquad \vec{B} = \hat{k} \times \vec{E}$$

Boundary value problem for EM waves



$$\vec{k}_{ref} = -\vec{k}_{in}$$

$$\vec{E}_{in} = \vec{E}^0_{in} e^{i(\vec{k}_{in}\cdot\vec{r} - wt)}$$

$$\vec{E}_{tr} = \vec{E}^0_{tr} e^{i(\vec{k}_{tr}\cdot\vec{r} - wt)}$$

$$\vec{E}_{ref} = \vec{E}^0_{tr} e^{i(\vec{k}_{ref}\cdot\vec{r} - wt)}$$

$$\vec{E}^0_{tr} = (?)\,\vec{E}^0_{in}$$

$$\vec{E}_{ref} = (?)\,\vec{E}^0_{in}$$

$$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow D_\perp \text{ is cont} \Rightarrow \underline{\varepsilon E_\perp \text{ is cont}}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \Rightarrow \vec{E}_\parallel \text{ is cont}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \underline{B_\perp \text{ is cont}}$$

$$\vec{\nabla}\times\vec{H} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} \Rightarrow \vec{H}_\parallel \text{ is cont} \Rightarrow \frac{\vec{B}_\parallel}{\mu} \text{ is cont}$$

$\vec{k}$ is $\perp$ to the interface

$$B_\perp = 0 \;;\; E_\perp = 0$$

$$\vec{E}^{\,0}_{in} e^{i(\vec{k}_{in}\cdot\vec{r} - \omega t)} + \vec{E}^{\,0}_{ref} e^{i(\vec{k}_{ref}\cdot\vec{r} - \omega t)}$$

$$= \vec{E}^{\,0}_{tr} e^{i(\vec{k}_{tr}\cdot\vec{r} - \omega t)}$$

on the interface (choose the xy plane on the interface)

$$\vec{k}_{in}\cdot\vec{r} = \vec{k}_{ref}\cdot\vec{r} = \vec{k}_{tr}\cdot\vec{r} = 0$$

$$\vec{k}\cdot\vec{r} = kz = 0$$

$$\boxed{\vec{E}^{\,0}_{in} + \vec{E}^{\,0}_{ref} = \vec{E}^{\,0}_{tr}} \Longleftarrow$$

$$\frac{\vec{B}_\parallel}{\mu} = \frac{\hat{k}\times\vec{E}}{\mu} \quad \text{is continuous}$$

~~$\Rightarrow \frac{\vec{E}}{\mu}$ is continuous~~

$$\boxed{\frac{1}{\mu_1}\left(\vec{E}^{\,0}_{in} - \vec{E}^{\,0}_{ref}\right) = \frac{\vec{E}^{\,0}_{tr}}{\mu_2}}$$

$$\frac{1}{\mu_1}\vec{E}^{\,0}_{in} - \frac{1}{\mu_1}\vec{E}^{\,0}_{ref} = \frac{1}{\mu_2}\vec{E}_{in} + \frac{1}{\mu_2}\vec{E}_{ref}$$

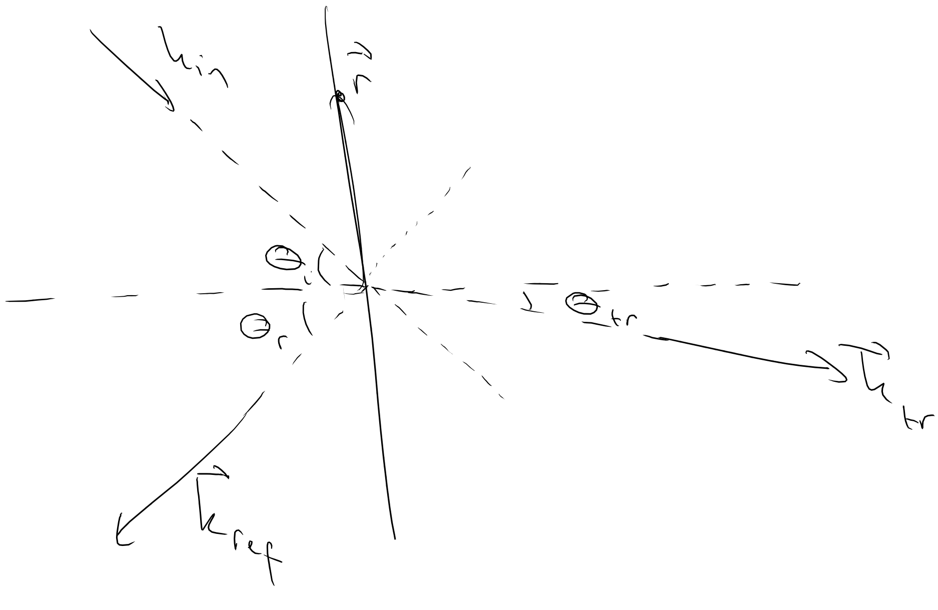
$$\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)\vec{E}^{\,0}_{in} = \left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\vec{E}^{\,0}_{ref}$$

$$\boxed{\vec{E}^{\,0}_{ref} = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\,\vec{E}^{\,0}_{in}}$$

$$\vec{E}^{\,0}_{tr} = \left(\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} + 1\right)\vec{E}^{\,0}_{in}$$

$$\boxed{\vec{E}^{\,0}_{tr} = \frac{2\mu_2}{\mu_2 + \mu_1}\,\vec{E}^{\,0}_{in}}$$

Plane wave incident with an angle



$$\vec{E}^{\,0}_{in\parallel} e^{i(\vec{k}_{in}\cdot\vec{r} - \omega t)} + \vec{E}^{\,0}_{ref\parallel} e^{i(\vec{k}_{ref}\cdot\vec{r} - \omega t)}$$

$$= \vec{E}^{\,0}_{tr\parallel} e^{i(\vec{k}_{tr}\cdot\vec{r} - \omega t)}$$

$$\checkmark\ \vec{k}_{in}\cdot\vec{r} = \vec{k}_{ref}\cdot\vec{r} = \vec{k}_{tr}\cdot\vec{r} \quad \Leftarrow$$

$$\begin{cases} k_{in}\, r \cos\left(\frac{\pi}{2}+\theta_i\right) = k_{ref}\, r \cos\left(\frac{\pi}{2}+\theta_r\right) \\ \quad = k_{tr}\, r \cos\left(\frac{\pi}{2}+\theta_{tr}\right) \end{cases}$$

$$k_{in} = \frac{\omega}{v} = \frac{\omega}{c} n$$

$$n_1(+\sin\theta_i) = n_1(+\sin\theta_r) = n_2(+\sin\theta_{tr})$$

$$n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_{tr}$$

$$\boxed{\theta_i = \theta_r}$$

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_{tr}}$$ Snell's Law

case i: $E_\perp = 0$

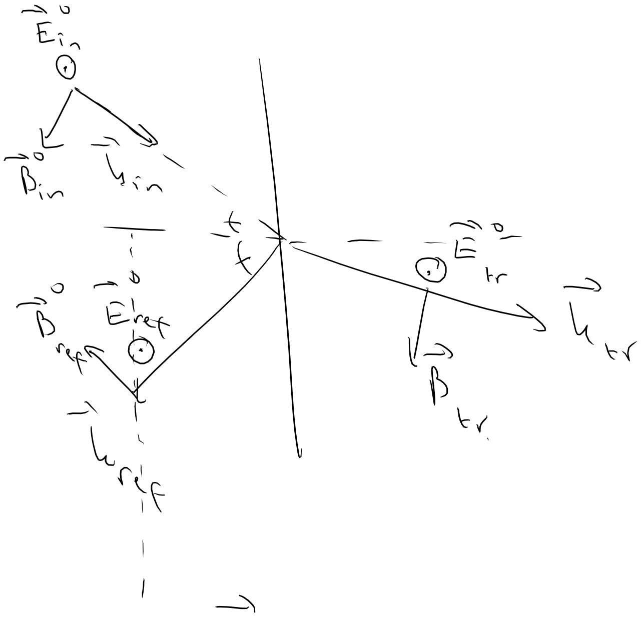
$$\boxed{\vec{E}^0_{in} + \vec{E}^0_{ref} = \vec{E}^0_{tr}}$$

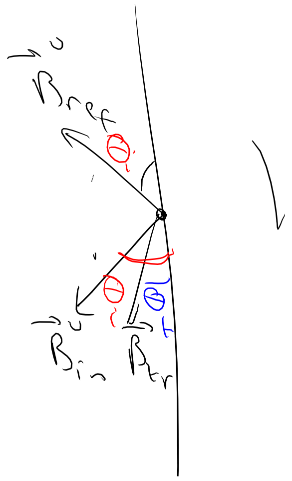
$B_\perp$ is cont $\quad \vec{B} = \hat{k} \times \vec{E}$

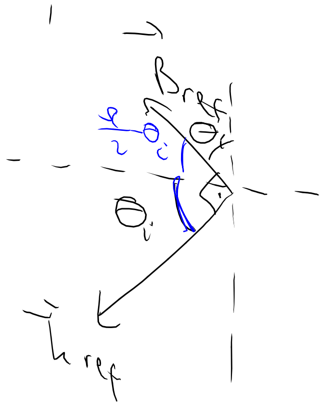
$\frac{\vec{B}_\parallel}{\mu}$ is cont.

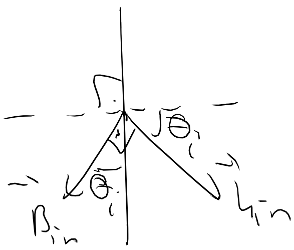
$$\left(\hat{k}_{in} \times \vec{E}_{in} + \hat{k}_{ref} \times \vec{E}_{ref}\right)_\perp = \left(\hat{k}_{tr} \times \vec{E}_{tr}\right)_\perp$$

$$\left(\frac{1}{\mu_1} \hat{k}_{in} \times \vec{E}_{in} + \frac{1}{\mu_1} \hat{k}_{ref} \times \vec{E}_{tr}\right)_\parallel = \left(\frac{1}{\mu_2} \hat{k}_{tr} \times \vec{E}_{tr}\right)_\parallel$$

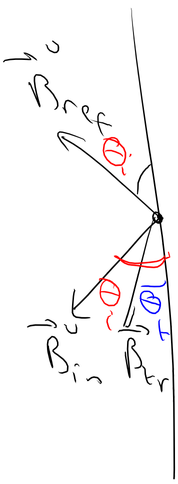








$$\vec{B} = \hat{k} \times \vec{E}$$
$$|\vec{B}| = |\vec{E}|$$



$B_\perp$ is continuous:

$$E^0_{in} \sin\theta_i + E^0_{ref} \sin\theta_i = E^0_{tr} \sin\theta_t$$

$\frac{B_\parallel}{\mu}$ is cont

$$\begin{cases} \dfrac{E^0_{in} \cos\theta_i}{\mu_1} - \dfrac{E^0_{ref} \cos\theta_i}{\mu_1} = \dfrac{E^0_{tr}}{\mu_2} \cos\theta_t \\ E^0_{in} + E^0_{r} = E^0_{tr} \end{cases}$$

$E^0_{tr} = ?$
$E^0_{ref} = ?$

$$\frac{1}{\mu_1} E_{in}^0 \cos\Theta_i - \frac{1}{\mu_1} E_{ref}^0 \cos\Theta_i = \frac{1}{\mu_2}\left(E_{in}^0 + E_r^0\right)\cos\Theta_t$$

$$\left[\frac{1}{\mu_1}\cos\Theta_i - \frac{1}{\mu_2}\cos\Theta_t\right] E_{in}^0 = E_r^0\left[\frac{1}{\mu_1}\cos\Theta_i + \frac{1}{\mu_2}\cos\Theta_t\right]$$

$$\boxed{E_r^0 = \frac{\frac{1}{\mu_1}\cos\Theta_i - \frac{1}{\mu_2}\cos\Theta_t}{\frac{1}{\mu_1}\cos\Theta_i + \frac{1}{\mu_2}\cos\Theta_t} E_{in}^0}$$

$$E_{tr}^0 = E_r^0 + E_{in}^0 = \boxed{\frac{\frac{2}{\mu_1}\cos\Theta_i}{\frac{1}{\mu_1}\cos\Theta_i + \frac{1}{\mu_2}\cos\Theta_t} E_{in}^0 = E_{tr}^0}$$

$$E_{in}^0 \sin\Theta_i + E_{ref}^0 \sin\Theta_i = E_{tr}^0 \sin\Theta_t$$

$$\cancel{E_{in}^0}\sin\Theta_i + \frac{\frac{1}{\mu_1}\cos\Theta_i - \frac{1}{\mu_2}\cos\Theta_t}{\frac{1}{\mu_1}\cos\Theta_i + \frac{1}{\mu_2}\cos\Theta_t}\cancel{E_{in}^0}\sin\Theta_i$$

$$\stackrel{?}{=} \frac{\frac{2}{\mu_1}\cos\Theta_i}{\frac{1}{\mu_1}\cos\Theta_i + \frac{1}{\mu_2}\cos\Theta_t}\cancel{E_{in}^0}\sin\Theta_t$$

$$\sin\theta_i \left(\frac{1}{\mu_1}\cos\theta_i + \cancel{\frac{1}{\mu_2}\cos\theta_t} \right)$$

$$+ \sin\theta_i \left(\frac{1}{\mu_1}\cos\theta_i - \cancel{\frac{1}{\mu_2}\cos\theta_t} \right) \stackrel{?}{=} \frac{2}{\mu_1}\sin\theta_t \cos\theta_i$$

$$\cancel{\frac{2}{\mu_1}} \sin\theta_i \cancel{\cos\theta_i} \stackrel{?}{=} \cancel{\frac{2}{\mu_1}} \sin\theta_t \cancel{\cos\theta_i}$$

$$\boxed{\sin\theta_i = \sin\theta_t} \, ?$$