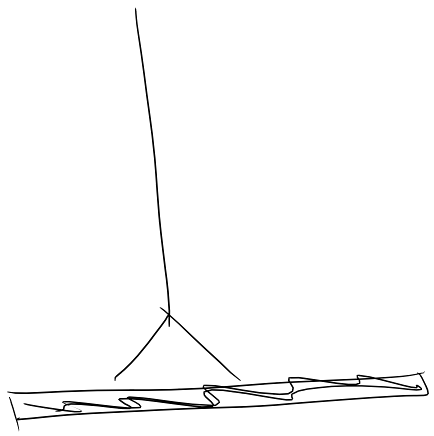
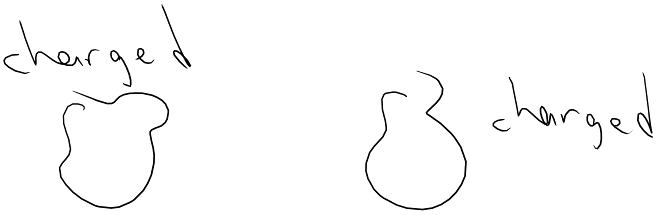


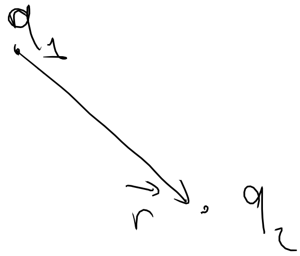
$$\vec{F}_{tot} = 0 \Leftarrow$$

$$\vec{\tau} = 0$$

$$F \propto \frac{1}{d^2}$$



$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$



$$\boxed{q_e = -q_p} = -1.6 \times 10^{-19}\ C$$

$$q = N_e q_e + N_p q_p = (N_p - N_e)\, q_p$$

$$\boxed{[\varepsilon_0] = \frac{C^2}{m^2 N}}$$

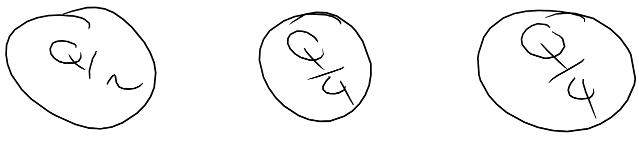
Charging objects: by friction

Conductors: contain charges that can move
insulators : does not contain " " " "

Conductors : metals: electrons are free to move
salty water: contains negatively and positively charged ions

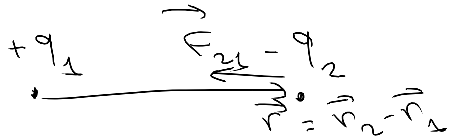
charging objects: induction

seperate two spheres

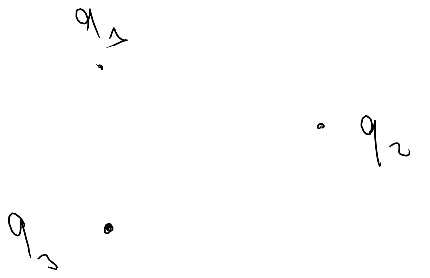


Coulomb's Law

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3}$$



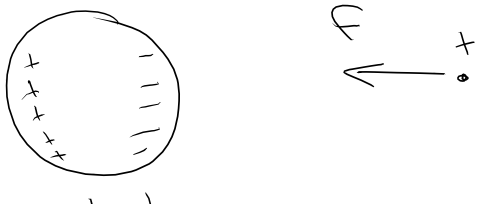
Ex



$$\boxed{\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23}}$$

superposition principle

Ex



neutral conductor

a) $F = 0$

b) $F \neq 0$ and is towards the sphere

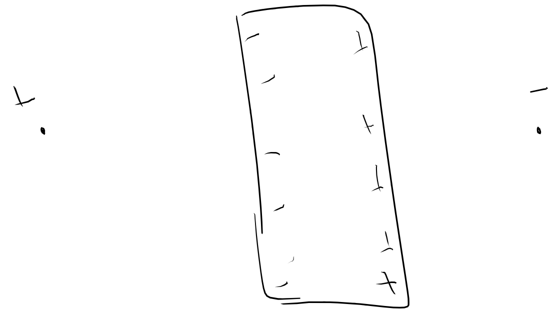
c) $F \neq 0$ and is away from the sphere

Ex

neutral conductor

$$\vec{F}_+ = \vec{F}' + \vec{F}_{\text{due to negative charge}}$$

$$\vec{F}' \overset{?}{=} \vec{F}$$

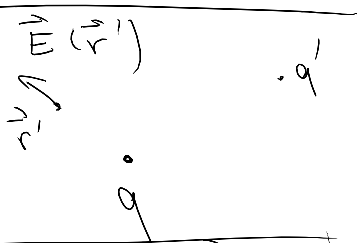


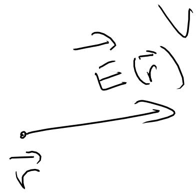
$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$ Force acting on q_2 due to q_1.

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

$$|\vec{F}_{12}| = \frac{1}{4\pi\varepsilon_0} |q_1 q_2| \frac{\cancel{|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|^{\cancel{3}\,2}} = \frac{1}{4\pi\varepsilon_0} |q_1 q_2| \frac{1}{|\vec{r}_1 - \vec{r}_2|^2}$$

Electric Field

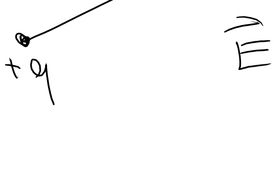




Vector Field: a function whose values are vectors.

$$\boxed{\vec{E} = \lim_{q' \to 0} \frac{\vec{F}_{q'}}{q'}} \qquad \boxed{\vec{F}_{q'} = q' \vec{E}}$$

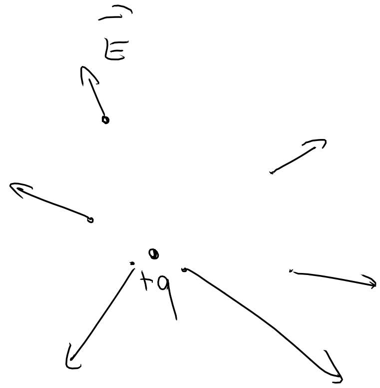
Example $\vec{E}$ of a point charge



$$\vec{F}_{q'} = \frac{1}{4\pi\varepsilon_0} \frac{q q'}{r^2} \hat{r}$$

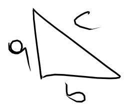
$$\vec{E}(\vec{r}) = \lim_{q' \to 0} \frac{\vec{F}_{q'}}{q'} = \boxed{\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} = \vec{E}(\vec{r})}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



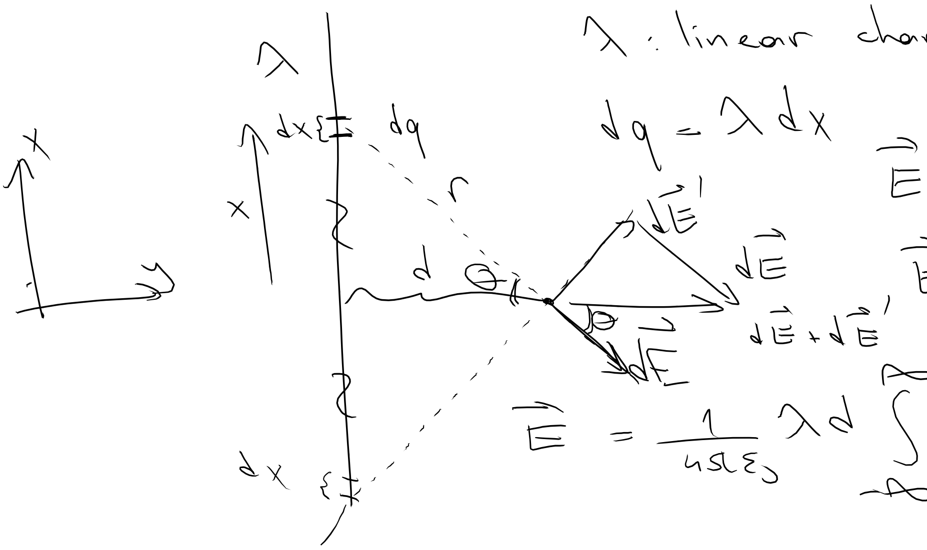
<u>Example</u> Electric field of a line of charge.

λ: linear charge density.



$a^2+b^2=c^2$

$dq = \lambda\, dx$



$$\vec{E} = \sum (dE_y)\hat{y}$$

$$\vec{E} = \sum \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2+d^2)} \overbrace{\frac{d}{(x^2+d^2)^{1/2}}}^{\cos\theta} \hat{y}$$

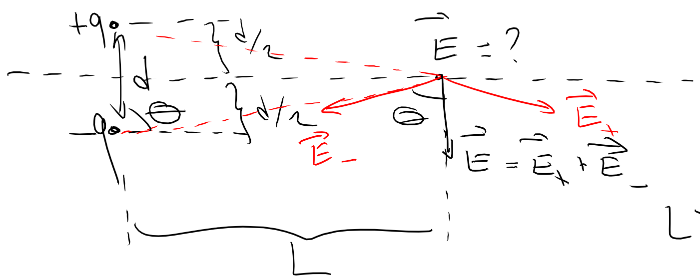
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\lambda d \int_{-\infty}^{\infty}\frac{dx}{(x^2+d^2)^{3/2}}\hat{y} = \frac{\lambda}{2\pi\varepsilon_0 d}\hat{y}$$

$$\boxed{\vec{F} = q\vec{E}}$$

if $q>0$, $\vec{F}$ and $\vec{E}$ are in the same direction

if $q<0$, $\vec{F}$ and $\vec{E}$ are in opposite directions

<u>Ex Electric Dipole</u>



$$|\vec{E}_+| = |\vec{E}_-|$$

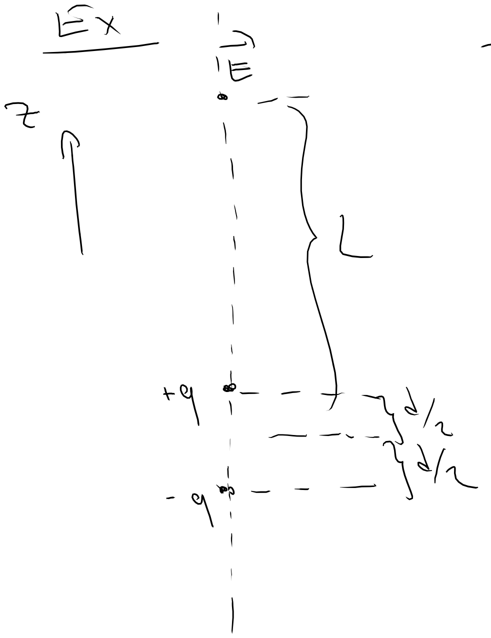
$$|\vec{E}| = 2|\vec{E}_-|\cos\theta$$

$$= \cancel{2}\,\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(L^2+\frac{d^2}{4}\right)}\overbrace{\frac{d/\cancel{2}}{\left(L^2+\frac{d^2}{4}\right)^{1/2}}}^{\cos\theta}$$

$L \gg d$

$$\cong \frac{1}{4\pi\varepsilon_0}\frac{(qd)}{L^3}$$

Ex



$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{\left(L-\frac{d}{2}\right)^2} - \frac{1}{\left(L+\frac{d}{2}\right)^2} \right) \hat{z}$$

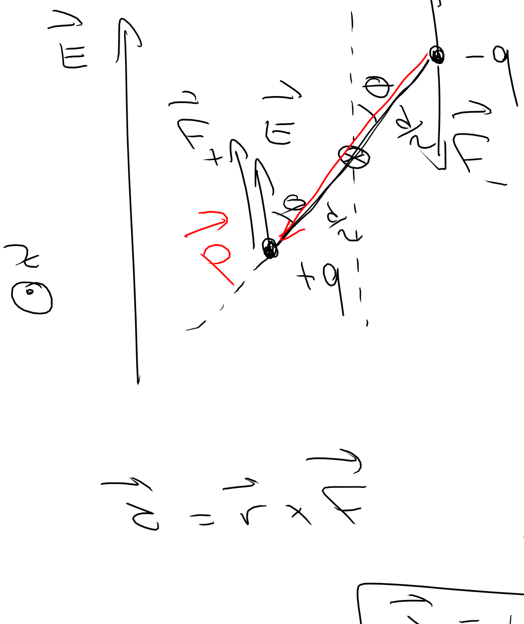
$$= \frac{q}{4\pi\varepsilon_0} \frac{\left(L+\frac{d}{2}\right)^2 - \left(L-\frac{d}{2}\right)^2}{\left(L^2 - \frac{d^2}{4}\right)^2} \hat{z}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{\left(L^2 + Ld + \frac{d^2}{4}\right) - \left(L^2 - Ld + \frac{d^2}{4}\right)}{\left(L^2 - \frac{d^2}{4}\right)^2} \hat{z}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{2Ld}{L^4} \hat{z} = \frac{1}{2\pi\varepsilon_0} \frac{(qd)}{L^3} \hat{z}$$

$$\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{\boxed{(qd)}}{L^3} \hat{z} \left[1 + O\left(\frac{d^2}{L^2}\right) \right]$$

Example



$\vec{E}$ is uniform : $\vec{E}$ is the same at every point.

$$\boxed{\vec{F} = q\vec{E}}$$

$$|\vec{F}_+| = |\vec{F}_-| \neq 0$$

$\vec{F} = \vec{F}_+ + \vec{F}_- = 0$: the force acting on a dipole in a uniform electric field is zero.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

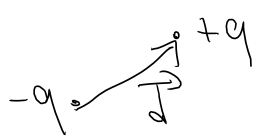
$$\vec{\tau}_+ = -\hat{z} \frac{d}{2} F_+ \sin(\pi - \theta) = -\hat{z} \frac{d}{2} qE \sin\theta$$

$$\vec{\tau}_- = -\hat{z} \frac{d}{2} F_- \sin(\pi - \theta) = -\hat{z} \frac{d}{2} qE \sin\theta$$

$$\boxed{\vec{\tau} = (-\hat{z})(qd) E \sin\theta} = \vec{p} \times \vec{E}$$

$qd \equiv p$: electric dipole moment

$$\boxed{\vec{p} = q\vec{d}}$$



March 2, 2016