March 2, 2016

Coulomb Law:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$\vec{E}(\vec{r}) = \lim_{q' \to 0} \frac{\vec{F}_{q'}(\vec{r})}{q'}$$

point charge: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$

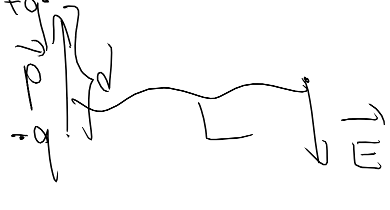
infinite wire

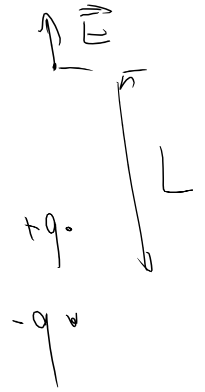
$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$$

dipole

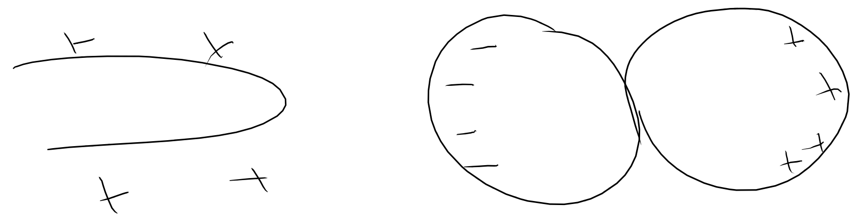
$$|\vec{p}| = qd$$

$$L \gg d \qquad E = \frac{1}{4\pi\varepsilon_0} \frac{p}{L^3}$$



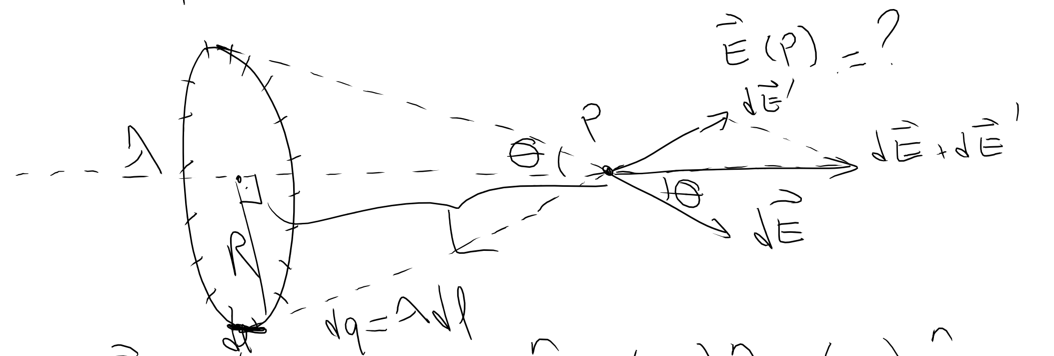


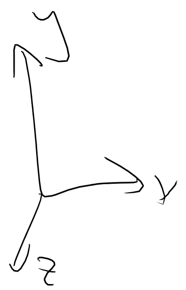
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{L^3}$$



$$n \to p + e^- + \bar{\nu}_e$$
$$0 \quad +1 \quad -1 \quad 0 = q$$

<u>Example</u> Electric Field of a ring charge





$$d\vec E = (dE)\cos\theta\,\hat x + (\ )\,\hat y + (\ )\,\hat z$$

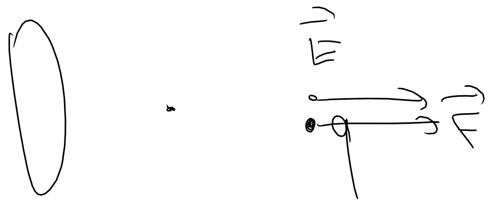
$$d\vec E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dl}{\left(\sqrt{R^2+L^2}\right)^2}\frac{L}{\sqrt{R^2+L^2}}\,\hat x + \dots$$

$$\vec E = \sum d\vec E = \frac{1}{2\cancel{4\pi}\varepsilon_0}\frac{\lambda L}{(R^2+L^2)^{3/2}}(2\cancel{\pi}R)\,\hat x + 0$$

$$\boxed{\vec E = \frac{1}{2\varepsilon_0}\frac{\lambda L R}{(R^2+L^2)^{3/2}}\hat x} = \frac{1}{4\pi\varepsilon_0}\frac{QL}{(R^2+L^2)^{3/2}}\hat x$$

$Q = (2\pi R)\lambda$

Q: total charge of the ring.



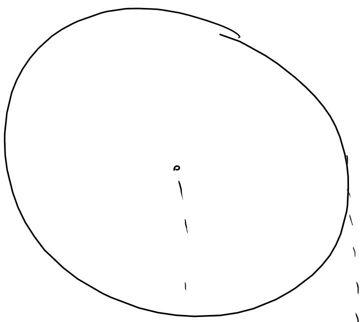
$$E = \frac{1}{4\pi\varepsilon_0} \frac{qL}{(R^2+L^2)^{3/2}}$$

$$\vec{F} = q\vec{E}$$

↳ $\vec{E}$ created by all the <u>other</u> charges

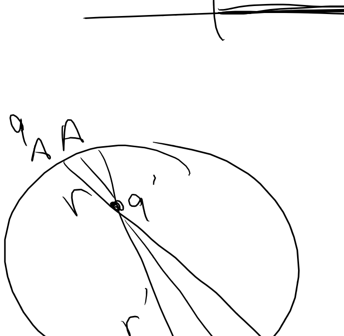
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}\hat{r}}$$

<u>Shell Theorem</u>



a spherical shell with uniformly distributed surface charge
Q: total charge of the shell.

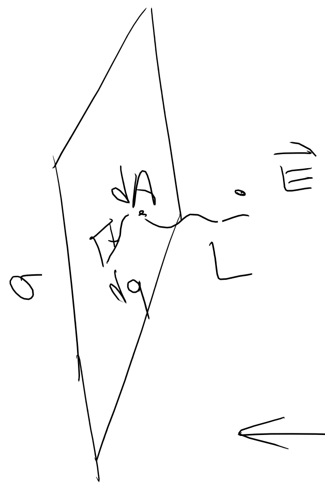
$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$



$$\frac{q_A}{q_{A'}} = \frac{A}{A'} = \frac{r^2}{r'^2}$$

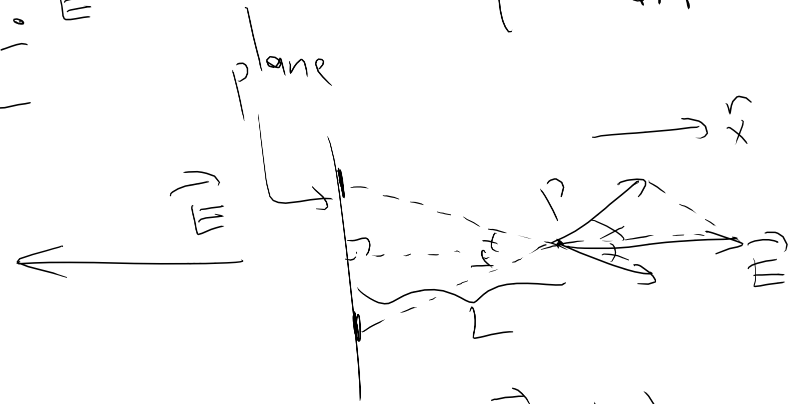
$$\Rightarrow \frac{q_A}{r^2} = \frac{q_{A'}}{r'^2}$$

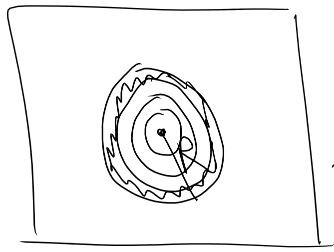
Example Infinite Plane Charge



σ: sigma

$dq = \sigma\, dA$





$$\vec{E}(P) = \sum_{\text{rings}} \vec{E}_{\text{rings}}(P)$$

$$\vec{E} = \sum_{\text{rings}} \frac{1}{4\pi\varepsilon_0} \frac{Q_{\text{ring}}\, L}{(L^2+R^2)^{3/2}}\, \hat{x}$$

$$Q_{\text{ring}} = \sigma A_{\text{ring}}$$
$$= \sigma (2\pi R)\, dR$$

$$\vec{E} = \sum_{\text{rings}} \frac{1}{4\pi\varepsilon_0} \frac{\sigma(2\pi R)\, dR\, L}{(L^2+R^2)^{3/2}}\, \hat{x}$$

$$= \frac{1}{4\pi\varepsilon_0}\, \sigma L \int_0^\infty \frac{2\pi R\, dR}{(L^2+R^2)^{3/2}}\, \hat{x}$$

$$= \frac{1}{4\pi\varepsilon_0}\, \sigma L \int_0^\infty (-2\pi)\left(\frac{d}{dR} \frac{1}{(L^2+R^2)^{1/2}}\right) dR\, \hat{x}$$

$$= -\frac{1}{2\varepsilon_0}\, \sigma L \left.\frac{1}{(L^2+R^2)^{1/2}}\right|_{R=0}^{\infty} \hat{x} = \boxed{\frac{\sigma}{2\varepsilon_0}\, \hat{x} = \vec{E}}$$