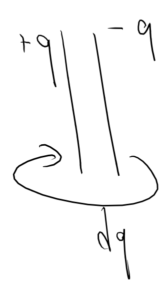
Hand in your HW!

$$F_G = G_N \frac{Mm}{r^2} + \frac{(\ )}{r^3}$$

How much energy is stored in a capacitor?



$$q = C\Delta V$$

$$\Delta V = \frac{q}{C}$$

$$\Delta U = dq\,\Delta V = \frac{q}{C}\,dq$$

$$U = \sum \Delta U = \sum \frac{q}{C}\,dq = \int_0^Q \frac{q}{C}\,dq$$

$$U = \frac{1}{2C} q^2 \Big|_{q=0}^{Q} = \boxed{\frac{1}{2C} Q^2 = U}$$

Example $U = \frac{Q^2}{2C}$; $C = \varepsilon_0 \frac{A}{l}$

$$E_+ = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$$

$$F_- = E_+ Q = \frac{Q^2}{2\varepsilon_0 A}$$

$$\boxed{W = F\Delta l = \frac{Q^2}{2\varepsilon_0 A}(l_f - l_i)}$$

$$\Delta U = \Delta \frac{Q^2}{2C} = \Delta \frac{Q^2}{2\left(\varepsilon_0 \frac{A}{l}\right)} = \Delta\left(\frac{Q^2 l}{2\varepsilon_0 A}\right)$$

$$\Delta U = \frac{Q^2}{2\varepsilon_0 A}\Delta l \quad \text{compare with } W = \frac{Q^2}{\varepsilon_0 A}\Delta l$$

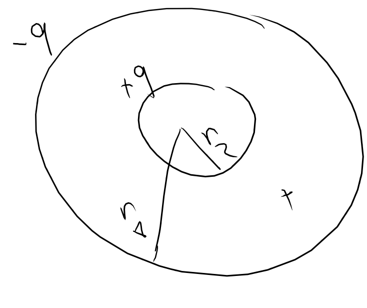
$$U = \frac{1}{2C}Q^2 = \frac{1}{2}\left(\frac{Q}{C}\right)Q = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

$$U = \frac{1}{2}\varepsilon_0 \frac{A}{l}(E\,l)^2 = \left(\frac{1}{2}\varepsilon_0 E^2\right)A l$$

$$\boxed{U = \left(\frac{1}{2}\varepsilon_0 E^2\right)\left(\begin{array}{c}\text{volume} \\ \text{where } \vec{E}\neq 0\end{array}\right)}$$

<u>Example</u> Spherical Capacitor

$$\boxed{C = 4\pi\varepsilon_0 \frac{r_1 r_2}{r_1 - r_2}}$$



$$C = \frac{q}{\Delta V}$$

$$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

$$\Delta V = \int_{r_1}^{r_2} \vec{E}\cdot\vec{dl} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\boxed{\Delta V = \frac{q}{4\pi\varepsilon_0}\frac{r_1 - r_2}{r_1 r_2}}$$

$$\boxed{C = 4\pi\varepsilon_0 \frac{r_1 r_2}{r_1 - r_2}}$$

C for an isolated sphere

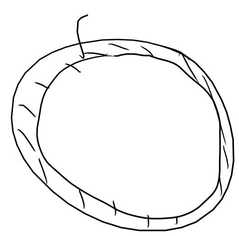
$$C = \lim_{r_1 \to \infty} 4\pi\varepsilon_0 \frac{r_1 r_2}{r_1 - r_2} = 4\pi\varepsilon_0 r_2$$

$$U = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$$

$$= \frac{Q^2}{2}\left[\frac{r_1 - r_2}{r_1 r_2} \frac{1}{4\pi\varepsilon_0}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{2} \frac{r_1 - r_2}{r_1 r_2}$$

Approximation $r_1 - r_2 = dr \ll$ small



$$r_1 \simeq r_2 \equiv r$$

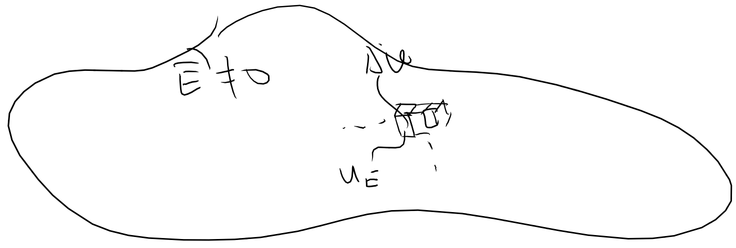
$$U = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0} \frac{Q^2}{r^2}\right) dr$$

$$= \frac{1}{2}\varepsilon_0 \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}\right)^2 4\pi r^2 dr$$

$$U = \left(\frac{1}{2}\varepsilon_0 E^2\right) \cdot \left(\text{volume where } \vec{E} \neq 0\right)$$

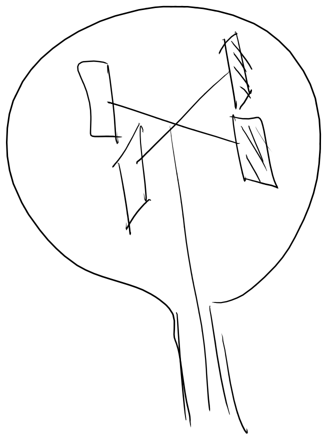
$$U = \sum_{\text{volume}} \left(\frac{1}{2}\varepsilon_0 E^2\right) \underbrace{\Delta v}_{\text{very small volume element}}$$

$$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$$: energy density stored in the electric field.



$$\Delta U = u_E \Delta v$$

$$U = \sum \Delta U = \sum u_E \Delta v$$



vacuum

$$U = \frac{1}{2} \sum_{i=} q_i V(q_i)$$

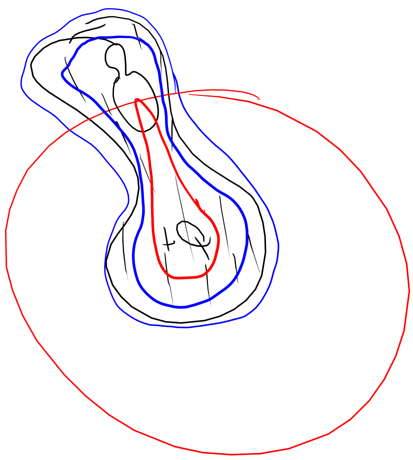
q_1 q_2 $$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

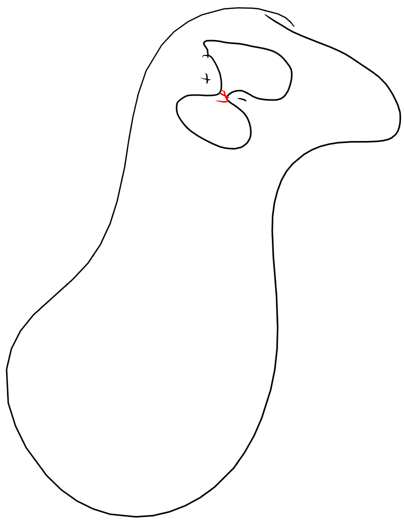
$$U = \frac{1}{2} q_1 \left(\frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_{12}} \right) + \frac{1}{2} q_2 \left(\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{12}} \right)$$

$$= \frac{1}{2} q_1 V(q_1) + \frac{1}{2} q_2 V(q_2)$$

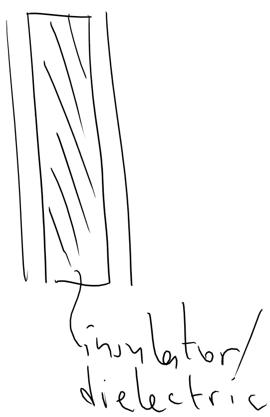
$$U = \sum_{\substack{ij \\ i<j}} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} = \sum_{\substack{ij \\ j<i}} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

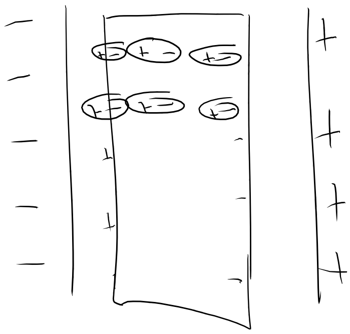
$$U = \frac{1}{2} \left(\sum_{\substack{ij \\ i<j}} + \sum_{\substack{ij \\ j<i}} \right) \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$

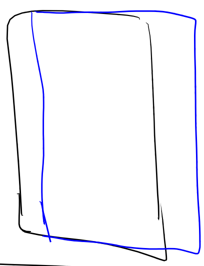


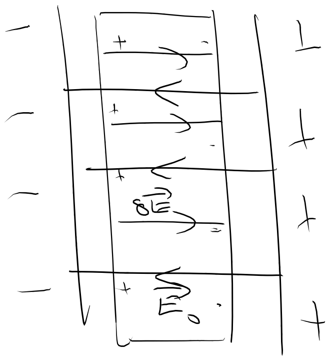


Dielectrics inside a Capacitor









$$\boxed{\vec{E} = \vec{E}_0 + \delta\vec{E}}$$

$$|\vec{E}| < |\vec{E}_0|$$

V_0 : potential difference in the absence of dielectric

V : pot. dif. in the presence of dielectric

$$V < V_0$$

$$\frac{q}{V} > \frac{q}{V_0} \Rightarrow C > C_0$$

$$\vec{E} = ?$$

$$\vec{E}(\vec{E}_0) \simeq \vec{E}(\vec{E}_0 = 0) + \underbrace{\left(\frac{d\vec{E}}{d()}\right)}_{= \frac{1}{K}} \cdot \vec{E}_0 + \dots$$

K: dielectric constant

$$\boxed{\vec{E} = \frac{\vec{E}_0}{K}}$$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \dots$$

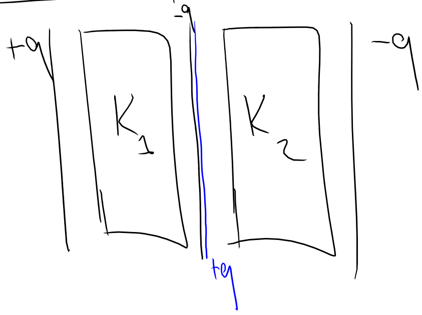
$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

$$E = \frac{E_0}{K} = \frac{q}{AK\varepsilon_0}$$

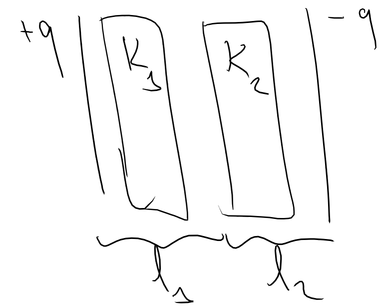
$$\Delta V = El = \frac{q}{AK\varepsilon_0} l$$

$$C = \frac{q}{\Delta V} = (K\varepsilon_0)\frac{A}{l} = K C_0$$

Example



second alternative



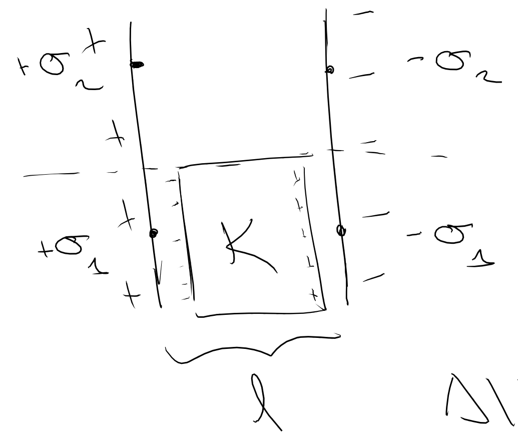
$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

$$E_1 = \frac{E_0}{K_1} \; ; \; E_2 = \frac{E_0}{K_2}$$

$$\Delta V = E_1 l_1 + E_2 l_2$$

$$\Delta V = \frac{q}{A\varepsilon_0} l_1 + \frac{q}{A\varepsilon_0} l_2 \equiv \frac{q}{C}$$

Example



$$E_0^{\uparrow} = \frac{\sigma_2}{\varepsilon_0} \quad \Delta V^{\uparrow} = \frac{\sigma_2}{\varepsilon_0} l$$

$$E_{0\downarrow} = \frac{\sigma_1}{\varepsilon_0} \quad \Delta V^{\downarrow} = \frac{\sigma_1}{K\varepsilon_0} l$$

$$\Delta V^{\uparrow} = \Delta V^{\downarrow} \Rightarrow \boxed{\sigma_2 = \frac{\sigma_1}{K}}$$

$$\sigma_1 \frac{A}{2} + \sigma_2 \frac{A}{2} = q$$

$$\sigma_1 \frac{A}{2} + \frac{\sigma_1}{K}\frac{A}{2} = q \Longrightarrow \frac{A}{2}\sigma_1\left(1 + \frac{1}{K}\right) = q$$

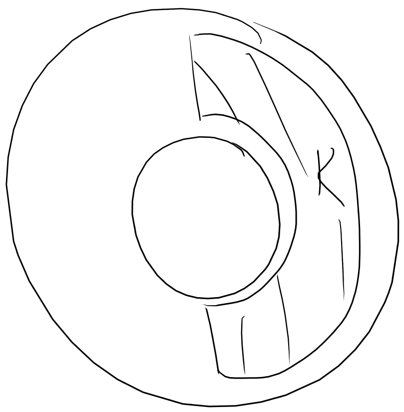
$$\sigma_1 = \frac{2}{A} q \frac{K}{K+1}$$

$$\Delta V = \Delta V^{\downarrow} = \frac{\sigma_1}{K\varepsilon_0} l = \frac{2}{A} q \frac{\cancel{K}}{K+1} \frac{1}{\cancel{K}\varepsilon_0} l$$

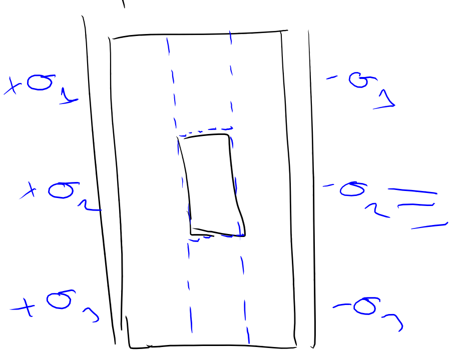
$$= \frac{2}{K+1} \frac{l}{A\varepsilon_0} q \equiv \frac{q}{C}$$

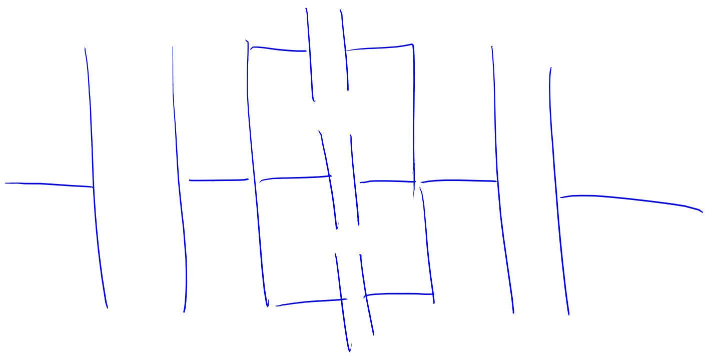
$$\boxed{C = \frac{K+1}{2} \frac{A\varepsilon_0}{l}}$$

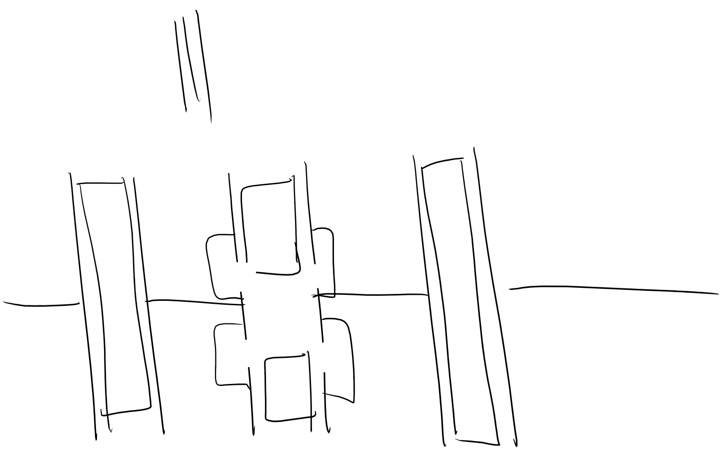
$$C = \left(K \varepsilon_0 \frac{A}{2} \frac{1}{l}\right) + \left(\varepsilon_0 \frac{A}{2} \frac{1}{l}\right)$$



Example

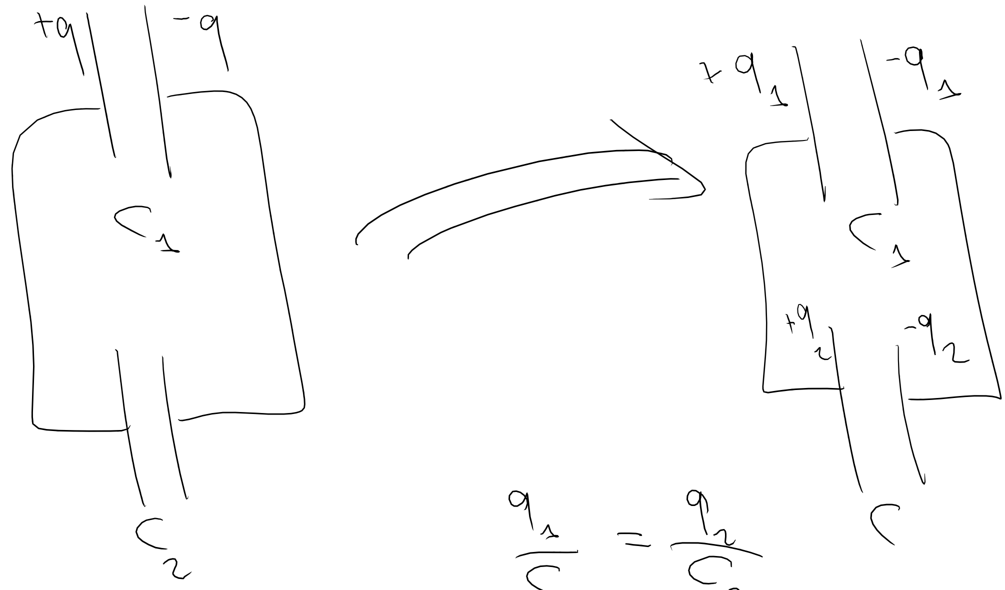






$K\varepsilon_0 \equiv \varepsilon$: electric permitivity of the dielectric

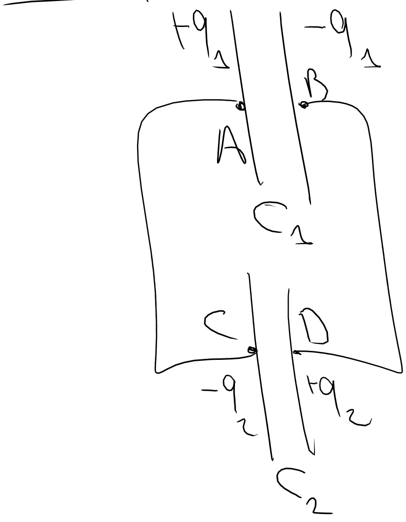
Connecting Charged Capacitors



$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

$$q_1 + q_2 = q$$

Example



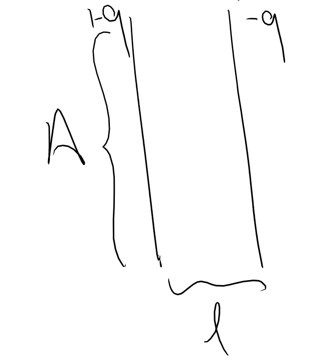
$$V_{AB} = V_A - V_B = \frac{q_1}{C_1}$$

$$V_{CD} = V_C - V_D = -\frac{q_2}{C_2}$$

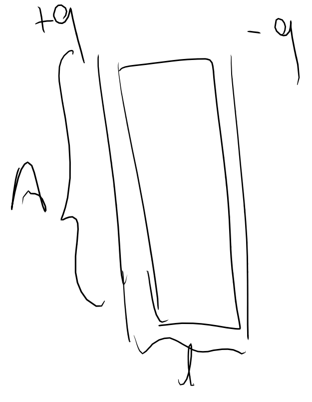
$$\left.\begin{array}{l} V_A = V_C \\ V_B = V_D \end{array}\right\} V_{AB} = V_{CD}$$

$$\frac{q_1}{C_1} = \frac{-q_2}{C_2}$$

Example



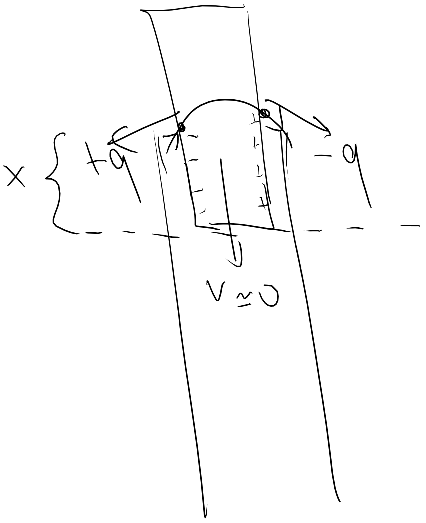
$$C_0 = \varepsilon_0 \frac{A}{l} \qquad U_0 = \frac{q^2}{2C_0}$$



$$C = K \varepsilon_0 \frac{A}{l} \qquad U = \frac{q^2}{2C}$$

$$U = \frac{q^2}{2C_0 K}$$

$$U = \frac{U_0}{K} < U_0$$



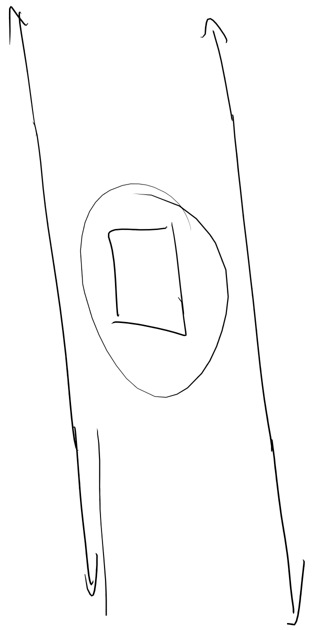
$$F \cdot dx = dU$$

HW

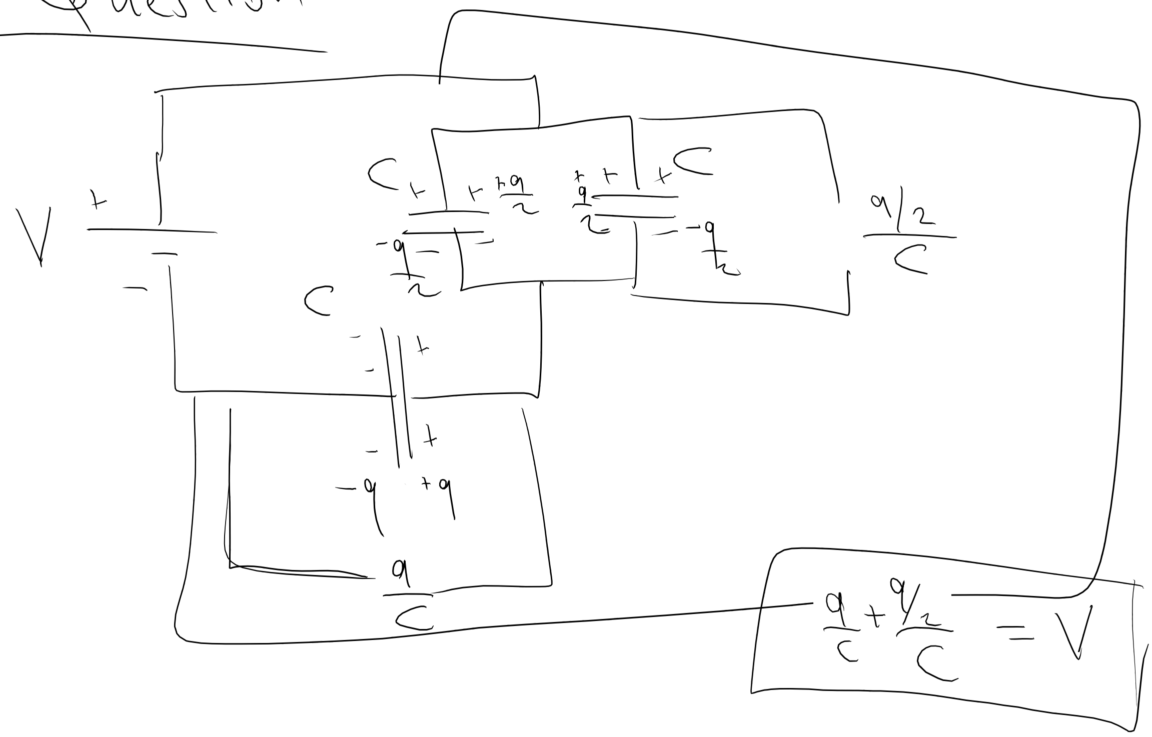
$$\left(-F_{us} \cdot dx\right) = dU$$

$$F_{us} = -F_{cap}$$

$$F = \frac{dU}{dx}$$



Question



V
C
C
C
$-\frac{q}{2}$
$+\frac{q}{2}$
$\frac{q}{2}$
$-\frac{q}{2}$
$-q$
$+q$
$\frac{q/2}{C}$
$\frac{q}{C}$
$\frac{q}{C}+\frac{q/2}{C}=V$