Hand in your HW!

$$V(P) = -\int_{P_0}^{P} \vec{E}\cdot d\vec{l} + V(P_0)$$

$dl \sim \Delta l$

$$\Delta V = -\vec{E}\cdot d\vec{l} = -E\,dl\cos\theta$$

$$\boxed{\Delta V = -E\,\Delta l\cos\theta}$$

for fixed Δl, ΔV is maximum if $\cos\theta = -1$

$\Rightarrow \theta = \pi$

$$(\Delta V)_{max} = +E\,\Delta l$$

$$\Rightarrow \boxed{E = +\frac{(\Delta V)_{max}}{\Delta l}}$$

$$\Delta V = -\vec{E}\cdot d\vec{l} = V(x,y,z) - V(x_0,y_0,z_0)$$

$(x,y,z) = (x_0,y_0,z_0) + d\vec{l}$

$x = x_0 + (dl)_x$

$y = y_0 + (dl)_y$

$z = z_0 + (dl)_z$

$d\vec{l} = dl\,\hat{x}$ $\quad$ $x = x_0 + dl$ $\quad$ $y = y_0$ $\quad$ $z = z_0$

$$\frac{V(x_0+dl, y_0, z_0) - V(x_0,y_0,z_0)}{dl} = \frac{-E_x dl}{dl}$$

$\vec{E}\cdot d\vec{l} = E_x\,dl$

$$E_x = -\lim_{dl\to 0} \frac{V(x_0+dl, y_0, z_0) - V(x_0,y_0,z_0)}{dl}$$

$$E_x(x_0,y_0,z_0) = -\left.\frac{\partial V}{\partial x_0}\right|_{(x_0,y_0,z_0)}$$

$$\boxed{E_x(\vec{r}) = -\frac{\partial V}{\partial x}} \; ; \; E_y = -\frac{\partial V}{\partial y} \; ; \; E_z = -\frac{\partial V}{\partial z}$$

<u>Example</u> El. field of a point charge

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad r = \sqrt{x^2+y^2+z^2}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{1}{4\pi\varepsilon_0} q \frac{\partial}{\partial x}\left(\frac{1}{r}\right)$$

$$= -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\frac{\partial r}{\partial x}$$

$$= +\frac{q}{4\pi\varepsilon_0}\left(+\frac{1}{r^2}\right)\frac{1}{2\sqrt{x^2+y^2+z^2}}\,2x$$

$$\boxed{E_x = \frac{q}{4\pi\varepsilon_0}\frac{x}{r^3}}$$

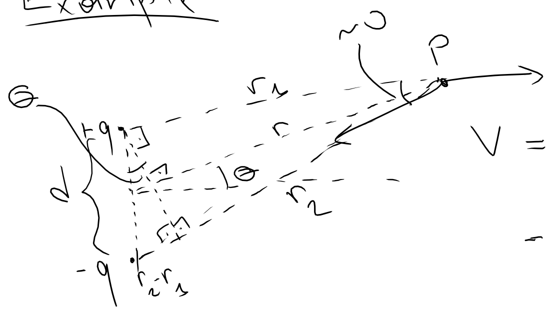
$$\boxed{E_y = \frac{q}{4\pi\varepsilon_0}\frac{y}{r^3}}$$

$$\boxed{E_z = \frac{q}{4\pi\varepsilon_0}\frac{z}{r^3}}$$

$$\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^3}\underbrace{\left(x\hat{x}+y\hat{y}+z\hat{z}\right)}_{\vec{r}}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} \qquad \hat{r} = \frac{\vec{r}}{r}$$

Example



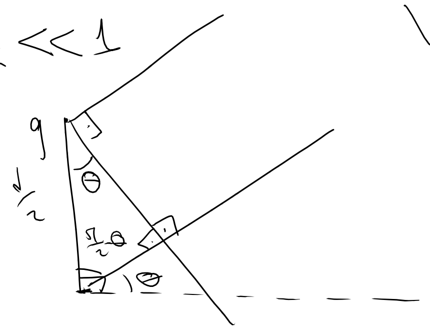
$V(P) = ?$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{r_2}$$

$$= \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$\frac{d}{r} \ll 1$

$$V = \frac{1}{4\pi\varepsilon_0} q \frac{r_2 - r_1}{r_1 r_2}$$



$$r_2 - r_1 \simeq d\sin\theta$$

$$r_1 \simeq r_2 \simeq r \mp \frac{d}{2}\sin\theta = r\left(1 \mp \frac{\sin\theta}{2}\frac{d}{r}\right)$$

$$r_1 \sim r \mp \frac{d}{2}\sin\theta \sim r$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{(qd)\sin\theta}{r^2}\left(1 + ()\frac{d}{r}\right)$$

$p = qd$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{(qd)\, r\cos\left(\frac{\pi}{2}-\theta\right)}{r^3}$$

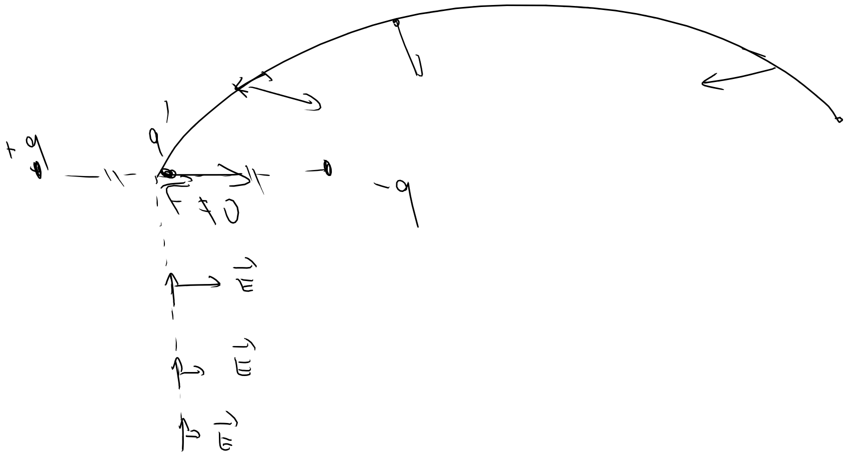
$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}}$$

HW

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x}$$

$$\sin\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} z}$$



Equipotential Surfaces

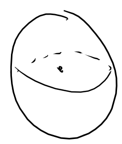
$$\boxed{\Delta V = -E\,\Delta l\cos\theta}$$

$\Delta V = 0$ if $\cos\theta = 0 \Rightarrow \theta = \mp\frac{\pi}{2}$

A surface for which every point on the surface is at the same electrostatic potential.

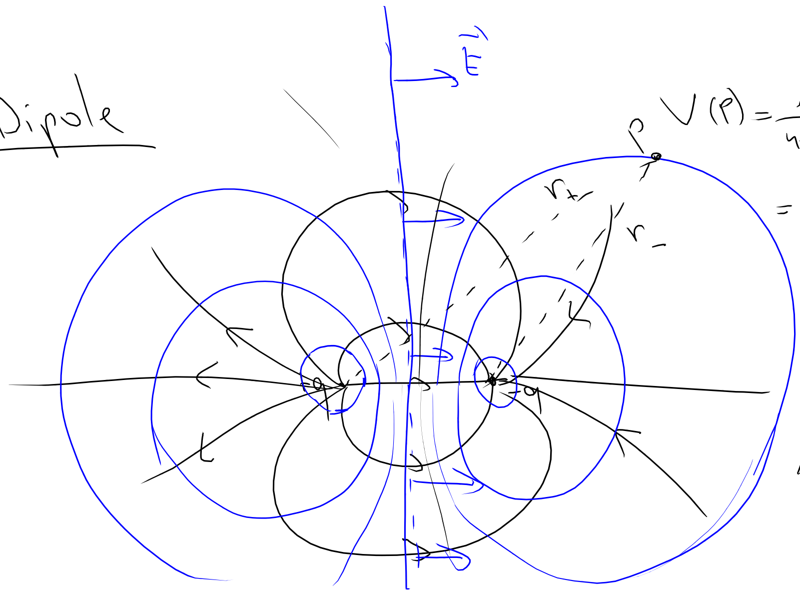
Examples

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



$r = \text{const}$ defines equipotential surfaces.

Dipole



$$V(P) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} + \frac{-q}{r_-}\right)$$

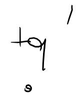
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$\frac{1}{r_+} - \frac{1}{r_-} = \text{const}$$

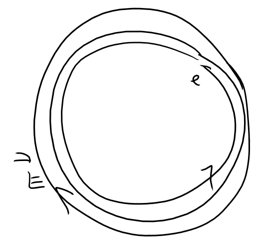
$$\Delta V = -\vec{E}\cdot d\vec{l}$$

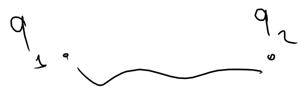
The surface of a conductor is an equipotential surface.

$$\Delta V = -\int_{P_0}^{P_1} \vec{E}\cdot d\vec{l} = 0$$

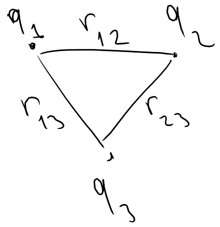


electric field on the surface of a conductor is perpendicular to the surface.
(in electrostatics!)





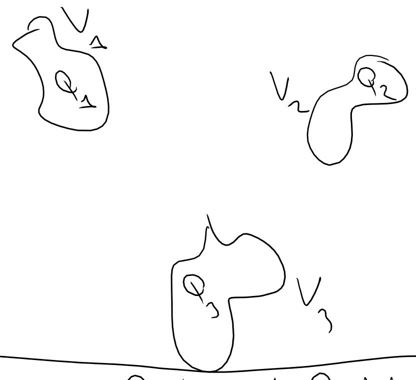
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$
$$= \frac{1}{2}q_1\left(\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r}\right) + \frac{1}{2}q_2\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}\right)$$
$$= \frac{1}{2}q_1 V(\vec{r}_1) + \frac{1}{2}q_2 V(\vec{r}_2)$$



$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$
$$= \frac{1}{2}q_1\left[\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{13}}\right]$$
$$+ \frac{1}{2}q_2\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{23}}\right]$$
$$+ \frac{1}{2}q_3\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}\right]$$

$$U = \frac{1}{2}q_1 V(\vec{r}_1) + \frac{1}{2}q_2 V(\vec{r}_2) + \frac{1}{2}q_3 V(\vec{r}_3)$$

$$U = \frac{1}{2}\sum q_i V(\vec{r}_i)$$



$$\boxed{U = \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 + \frac{1}{2} Q_3 V_3}$$

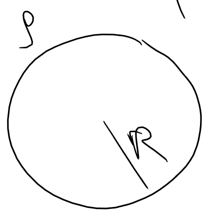
$$U = \frac{1}{2} \sum q_i V(\vec{r}_i)$$

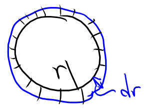
$$= \frac{1}{2} \sum_{\text{cond. 1}} q_i V(\vec{r}_i) + \frac{1}{2} \sum_{\text{cond 2.}} q_i V(\vec{r}_i) + \frac{1}{2} \sum_{\text{cond. 3}} q_i V(\vec{r}_i)$$

$$= \frac{1}{2} \sum_{\text{cond. 1}} q_i V_1 + \frac{1}{2} \sum_{\text{cond 2}} q_i V_2 + \frac{1}{2} \sum_{\text{cond 3}} q_i V_3$$

$$U = \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 + \frac{1}{2} Q_3 V_3$$

Example: Uniformly charged sphere.





$$dU = \frac{1}{4\pi\varepsilon_0} \left(\rho \frac{4}{3} \pi r^3\right) \frac{1}{r}\, dq$$

$$dq = (4\pi r^2 dr)\, \rho$$

$$\int_0^U dU = \int_0^R \frac{1}{4\pi\varepsilon_0} \rho^2 \frac{(4\pi)^2}{3} r^4 dr$$

$$U = \frac{4\pi}{3\varepsilon_0} \rho^2 \frac{r^5}{5} \Big|_{r=0}^{R}$$

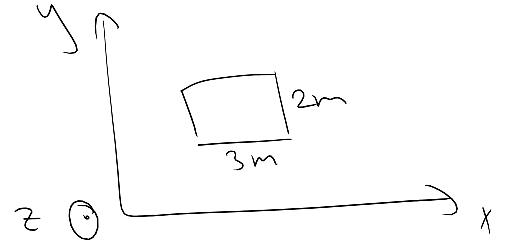
$$\boxed{U = \frac{4\pi}{15\varepsilon_0} \rho^2 R^5}$$

$$U = \frac{4\pi}{15\,\varepsilon_0}\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)^2 R^5$$

$$= \frac{\cancel{4\pi}}{\cancel{15}_{5}\,\varepsilon_0}\,\frac{Q^2}{R}\,\frac{1}{(4\pi)^{\cancel{2}}}\cdot \cancel{9}^{3}$$

$$\boxed{U = \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\,\frac{Q^2}{R}\right)}$$

Quiz 2



$\vec{E} = E_0\,\hat{z}$

$E_0 = 5\ \mathrm{V/m}$

$\phi_E = ?$