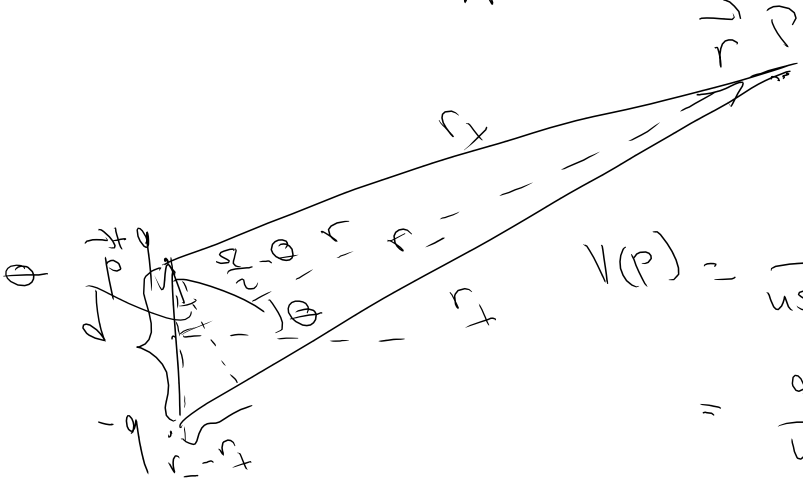
$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{l} = -U(B) + U(A)$$

$$U(B) = -\int_A^B \vec{F}\cdot d\vec{l} + U(A)$$



$$V(P) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r_+} + \frac{-q}{4\pi\varepsilon_0}\frac{1}{r_-}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$

$$r_+ \simeq r + O\left(\frac{d}{r}\right) \qquad \frac{1}{r_+ r_-} \simeq \frac{1}{r^2}$$

$$r_- \simeq r - O\left(\frac{d}{r}\right)$$

$$r_- - r_+ \simeq d\sin\theta$$

$$V(P) = \frac{q}{4\pi\varepsilon_0}\frac{d\sin\theta}{r^2}$$

$$= \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$$

$$r_+ = \frac{r_+ + r_-}{2} + \frac{r_+ - r_-}{2}$$

$$r_+ \simeq r - \frac{1}{2}d\sin\theta$$

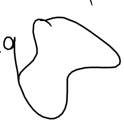
$$r_- \simeq r + \frac{1}{2}d\sin\theta$$

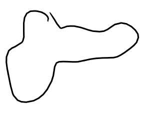
$$r_\pm = r \mp \frac{1}{2}d\sin\theta$$

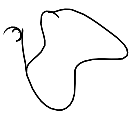
$$= r\left(1 \mp \frac{1}{2}\sin\theta\,\frac{d}{r}\right)$$

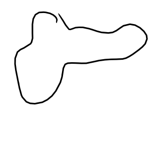
$$= r\left(1 + O\left(\frac{d}{r}\right)\right)$$

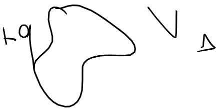
Example

$+q$  $\vec{E}_q(\vec{r})$

$-q$ 

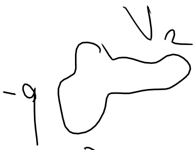
$+2q$  $\vec{E}_{2q}(\vec{r}) = ?$

$-2q$ 



$$\vec{E}(\vec{r}) = q\,\vec{f}(\vec{r})$$

$\vec{f}(\vec{r})$ is indep. of q.

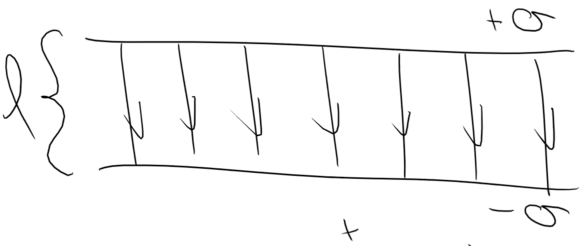


$$V_1 - V_2 = \Delta V = -\int_{\vec{r}_2}^{\vec{r}_1} \vec{E}\cdot d\vec{l} = q\underbrace{\left(-\int_{\vec{r}_2}^{\vec{r}_1} \vec{f}(\vec{r})\cdot d\vec{l}\right)}_{\frac{1}{C}}$$

$$\boxed{q = C\,\Delta V}$$

C: capacitance

<u>Example</u> Parallel Plate Capacitor

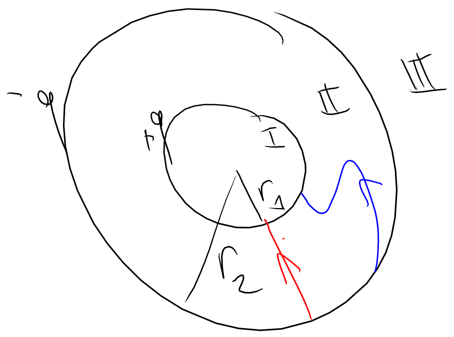


$$E = \frac{\sigma}{\varepsilon_0}$$

$$\Delta V = \int_{-}^{+} \vec{E}\cdot d\vec{l} = \vec{E}\cdot\Delta\vec{l} = El \quad \text{(ignoring signs)}$$

$$C = \frac{q}{\Delta V} = \frac{q}{El} = \frac{\cancel{\sigma} A}{\frac{\cancel{\sigma}}{\varepsilon_0} l} = \boxed{\varepsilon_0 \frac{A}{l} = C}$$

Example Spherical capacitor



$\vec{E}_I = 0$

$\vec{E}_{III} = 0$

$\vec{E}_{II} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

$\Delta V = \Delta V$

$$\Delta V = -\int_{r_2}^{r_1} \vec{E}\cdot\vec{dl}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \; ; \; \vec{dl} = dr\,\hat{r} \; ; \; dr < 0$$

$$\vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr$$

$$\Delta V = -\int_{r_2}^{r_1}\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

$$C = \frac{q}{\Delta V} = \frac{\cancel{q}}{\frac{\cancel{q}}{4\pi\varepsilon_0}\frac{r_2 - r_1}{r_2 r_1}}$$

$$\boxed{C = 4\pi\varepsilon_0\frac{r_2 r_1}{r_2 - r_1}}$$

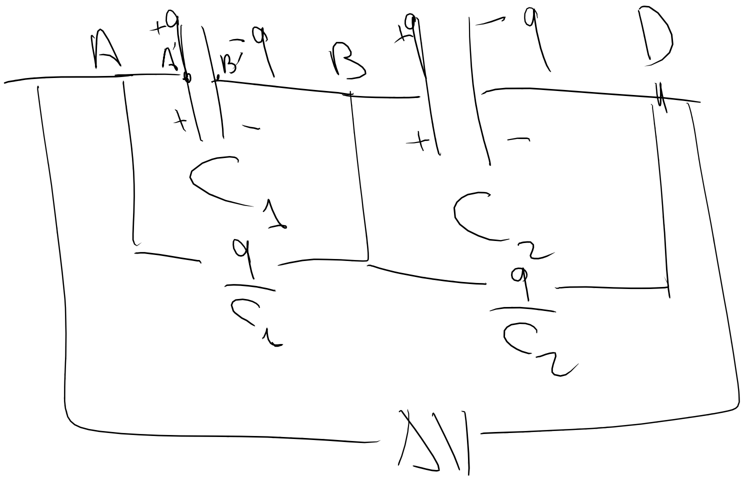
$$\boxed{q = C\Delta V}$$

$q > 0$
$C > 0$
$\Delta V > 0$

Connecting Capacitors

Series & Parallel

Series



$$q = C_{eq}\Delta V$$

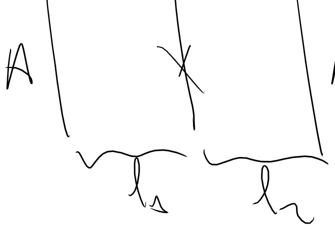
$$\Delta V = \frac{q}{C_1} + \frac{q}{C_2}$$

$$V_{AB} = V_{A'B'}$$

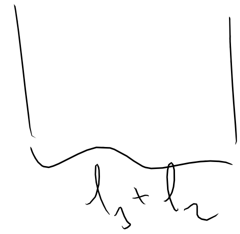
$$\frac{1}{C_{eq}} = \frac{\Delta V}{q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_1 = \varepsilon_0 \frac{A}{l_2}$$

$$C_2 = \varepsilon_0 \frac{A}{l_2}$$



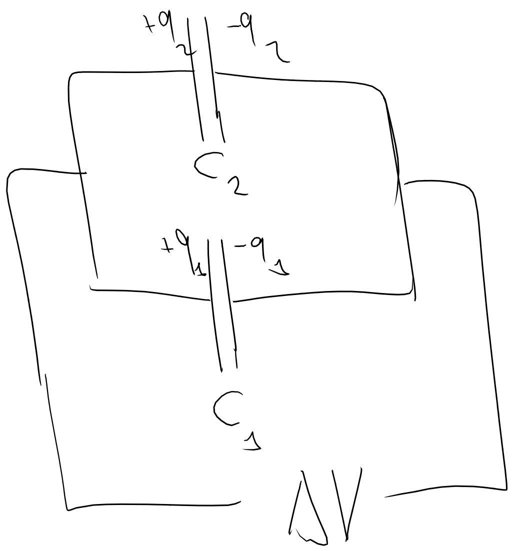
$\Rightarrow$



$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$q = C\Delta V \Leftrightarrow \Delta V = \frac{q}{C}$$

Parallel



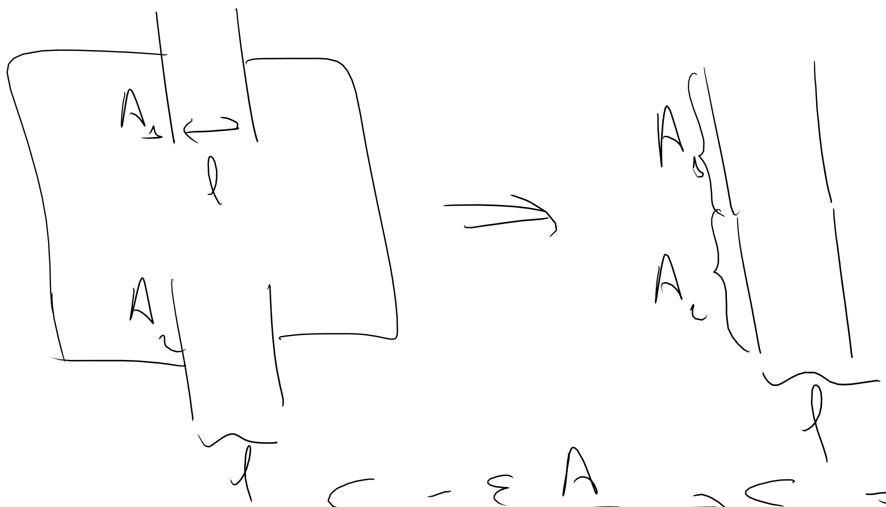
$$q_1 = C_1 \Delta V$$

$$q_2 = C_2 \Delta V$$

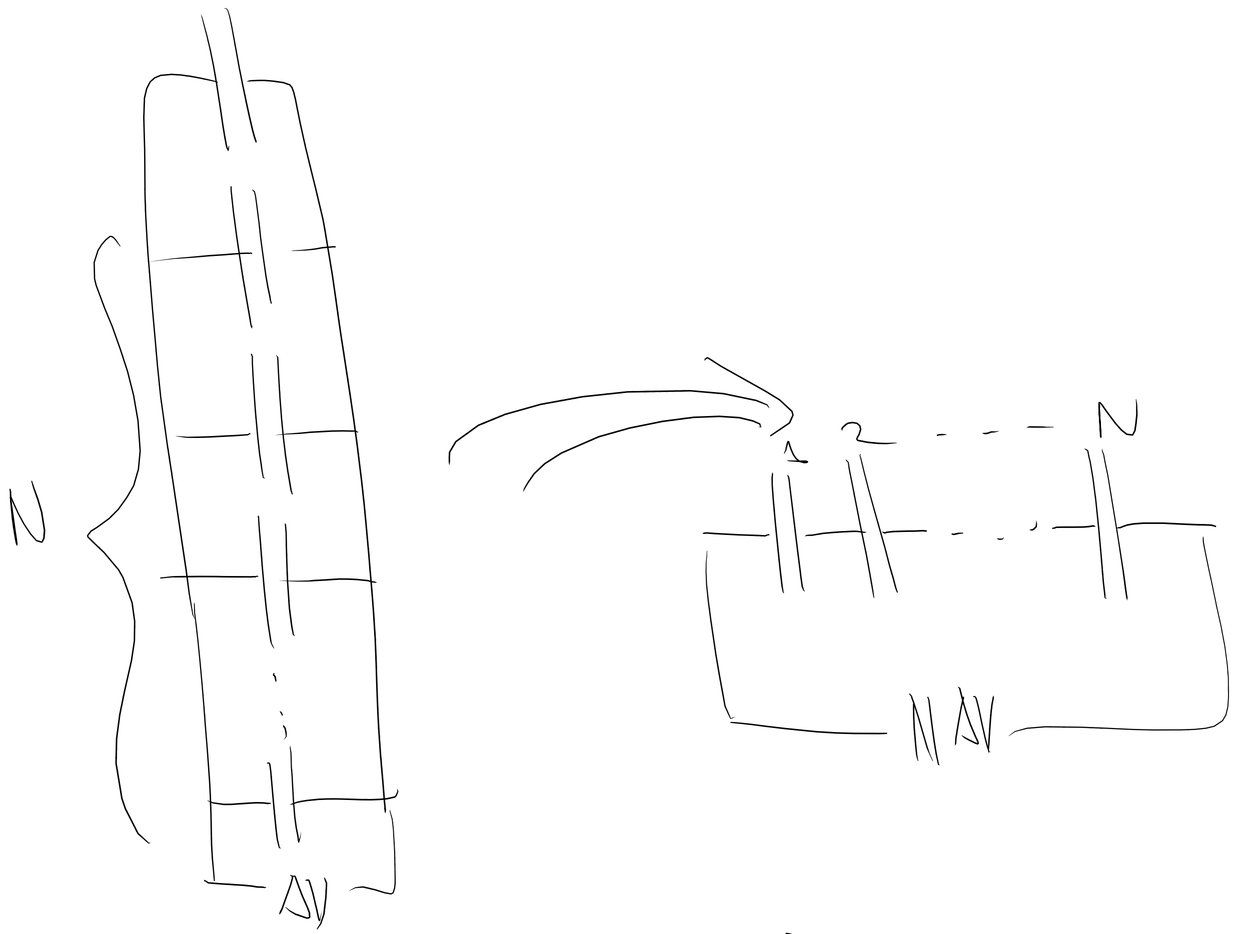
$$Q = q_1 + q_2 = C_{eq}\Delta V$$

$$C_1 \cancel{\Delta V} + C_2 \cancel{\Delta V} = C_{eq} \cancel{\Delta V}$$

$$C_1 + C_2 = C_{eq}$$

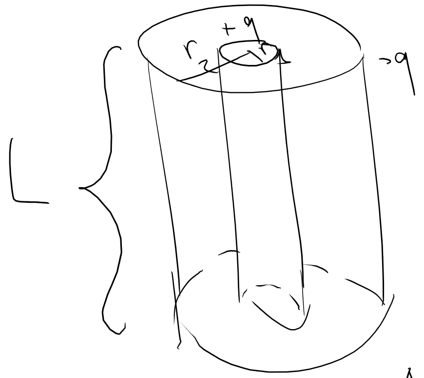


$$C = \varepsilon_0 \frac{A}{l} \Rightarrow C_{eq} = C_1 + C_2$$



$$C = \varepsilon_0 \frac{A}{l}$$

Example Cylindrical Capacitor

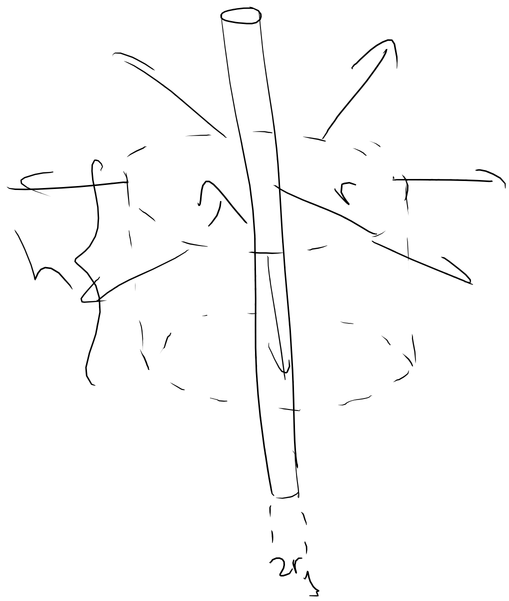


$$C = ?$$

$$E = \frac{q/L}{2\pi \varepsilon_0} \frac{1}{r}$$

$$\Delta V = \int \vec{E} \cdot d\vec{r} = \frac{q}{2\pi \varepsilon_0 L} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$\Delta V = \frac{q}{2\pi \varepsilon_0 L} \ln\left(\frac{r_2}{r_1}\right) \equiv \frac{q}{C}$$



λ: charge density

$$\vec{E}\cdot d\vec{A} = E\,dA\cos\theta = E\,dA$$

$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot d\vec{A} + \int_{botton} \vec{E}\cdot d\vec{A} + \int_{side} \vec{E}\cdot d\vec{A}$$

$$= E\int_{side} dA = E\,2\pi r h$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda h}{\varepsilon_0}$$

λ: charge per unit length

$$E\,2\pi r\cancel{h} = \frac{\lambda \cancel{h}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2\pi r\,\varepsilon_0}}$$