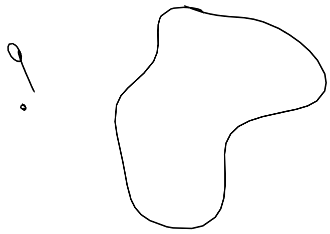
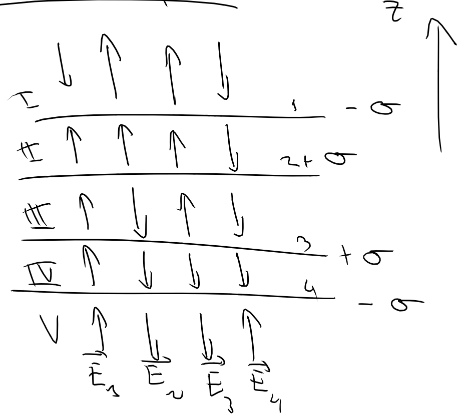


$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

if $Q_{enc} = 0$

$$\Rightarrow \oint \vec{E} \cdot d\vec{A} = 0 \not\Rightarrow \vec{E} = 0$$

Example



$\vec{E}$ everywhere?

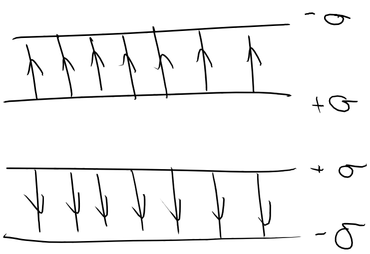
$\vec{E}_i$: electric field created by the plate i

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

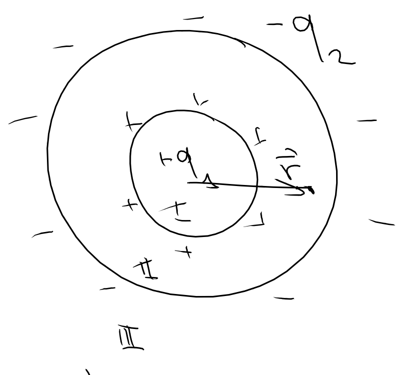
$$E_i = \frac{\sigma}{2\varepsilon_0} \quad \forall i$$

$\vec{E}_I = 0$ $\quad \vec{E}_V = 0$

$\vec{E}_{III} = 0$ $\quad \vec{E}_{II} = \frac{\sigma}{\varepsilon_0}\hat{z}$ $\quad \vec{E}_{IV} = \frac{\sigma}{\varepsilon_0}(-\hat{z})$



Example

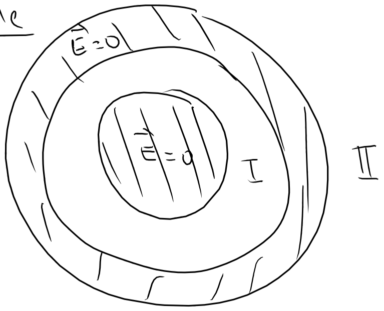


$$\vec{E}_I(\vec{r}) = 0$$

$$\vec{E}_{II}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r} + 0$$

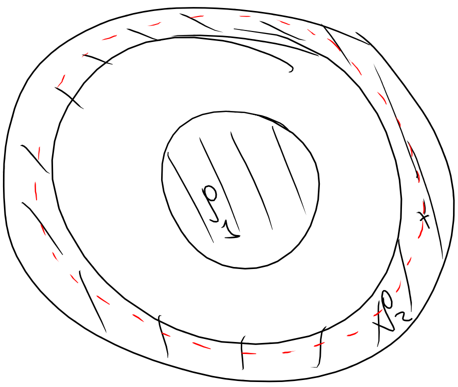
$$\vec{E}_{III}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r} + \frac{1}{4\pi\varepsilon_0}\frac{(-q_2)}{r^2}\hat{r}$$

Example



everything conducting

Example

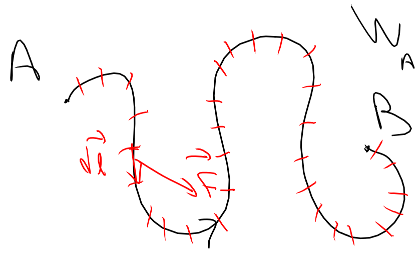


everything insulator, charge uniformly distributed over the volume.

Work and Potential Energy

$\vec{F}$: any force

$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{\ell}$$



$$W_{A\to B} = \sum \vec{F}\cdot d\vec{\ell}$$

Conservative force: a force for which the work done is indep. of path.

Constant force is conservative

$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{\ell} = \vec{F}\cdot\int_A^B d\vec{\ell} = \vec{F}\cdot(\vec{r}_B - \vec{r}_A)$$

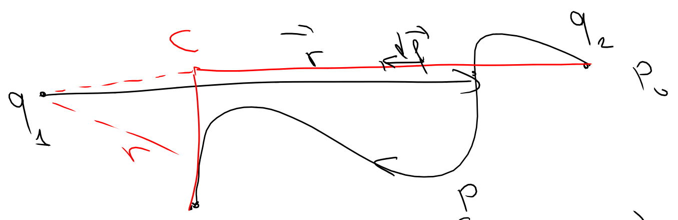
$$\left.\begin{aligned} \vec{F}_G &= G_N \frac{m_1 m_2}{r^2}\hat{r} \\ \vec{F}_e &= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \end{aligned}\right\} \text{conservative force}$$

For a conservative force

$$\int_A^B \vec{F}\cdot d\vec{\ell} = U(A) - U(B) \underset{\text{constant force}}{=} -\vec{F}\cdot\vec{r}_A + \vec{F}\cdot\vec{r}_B$$

For a constant force $\vec{F}$, $U(\vec{r}) = -\vec{F}\cdot\vec{r} + U_0$

Electric Pot. Energy



$$\vec F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat r \qquad U(P) = \int_{P_0}^{P} (-\vec F)\cdot d\vec l + U(P_0)$$

$$U(P) = -\int_{P_0}^{C} \vec F\cdot d\vec l + \underbrace{\int_{C}^{P} (-\vec F)\cdot d\vec l}_{=0} + U(P_0)$$

$$\vec F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat r \qquad d\vec l = dl\,\hat r \qquad (dl < 0)$$

$$\vec F\cdot d\vec l = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,dl$$

$$\int_{r_0}^{r_f} \vec F\cdot d\vec l = \int_{r_0}^{r_f} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,dl \qquad dl = dr$$

$$= \frac{1}{4\pi\varepsilon_0}\, q_1 q_2 \left. \frac{-1}{r}\right|_{r=r_0}^{r=r_f}$$

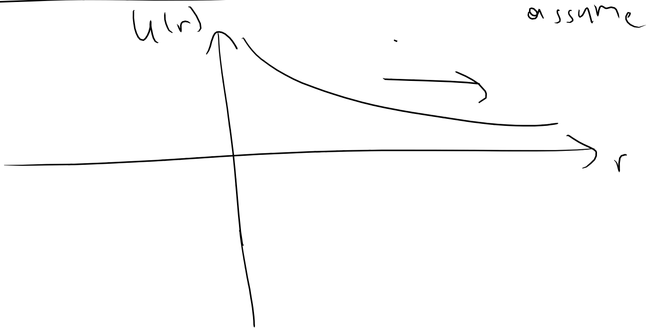
$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_f} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_0}$$

$$U(P) = +\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_f} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_0} + U(P_0)$$

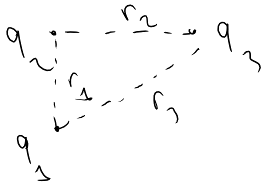
choose $U(P_0) = +\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_0}$

$$\boxed{U(\vec r) = +\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

assume $q_1 q_2 > 0$



Example



$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_3} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_2}$$

$$-\int_A^B \vec{F}\cdot d\vec{l} = -\int_A^B \vec{F}_1\cdot d\vec{l} - \int_A^B \vec{F}_2\cdot d\vec{l}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_3} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_2}$$

$$= q_3 \underbrace{\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_3} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}\right)}_{\text{electric potential at the position of } q_3}$$

$$U(P) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{l} + U(P_0)$$

$$\vec{F} = q'\vec{E}$$

$$U(P) = U(P_0) - q'\int_{P_0}^{P} \vec{E}\cdot d\vec{l}$$

$V \equiv \frac{U}{q'}$: electric potential $\left(\vec{E} = \frac{\vec{F}}{q'}\right)$

$$\boxed{V(P) = V(P_0) - \int_{P_0}^{P} \vec{E}\cdot d\vec{l}}$$ electric potential

$$\vec{F} = q'\vec{E}(P)$$

$\Delta U = q' V(P)$ ← work done to bring it from infinity

$$U = qV$$

$$\Delta U = q' \Delta V$$