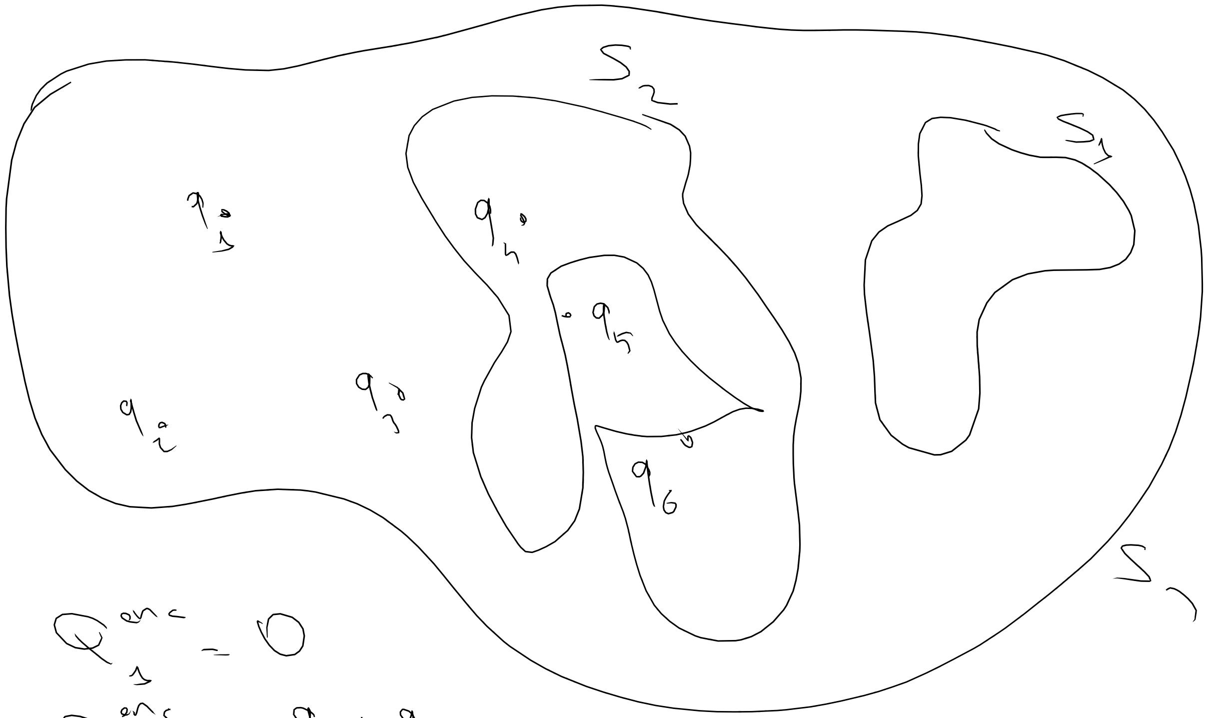
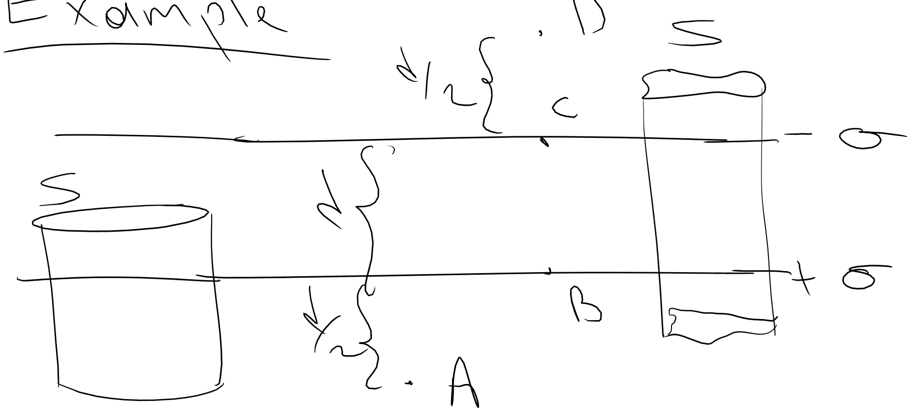


$Q_1^{enc} = 0$

$Q_2^{enc} = q_4 + q_6$

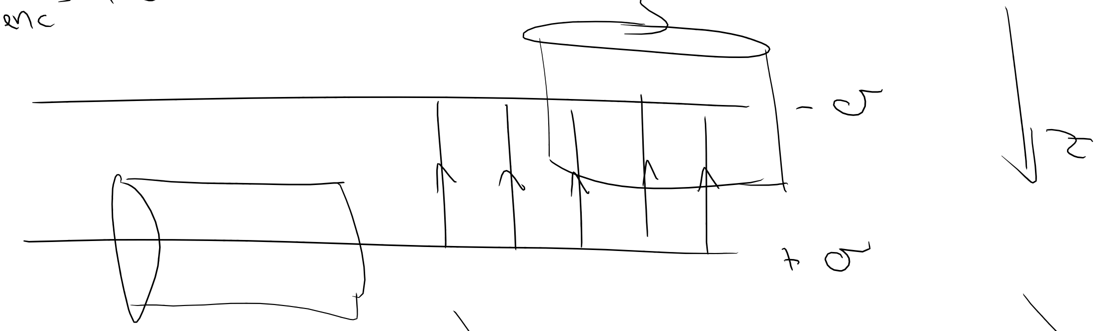
$Q_3^{enc} = q_1 + q_2 + q_3 + \dots + q_6$

Example

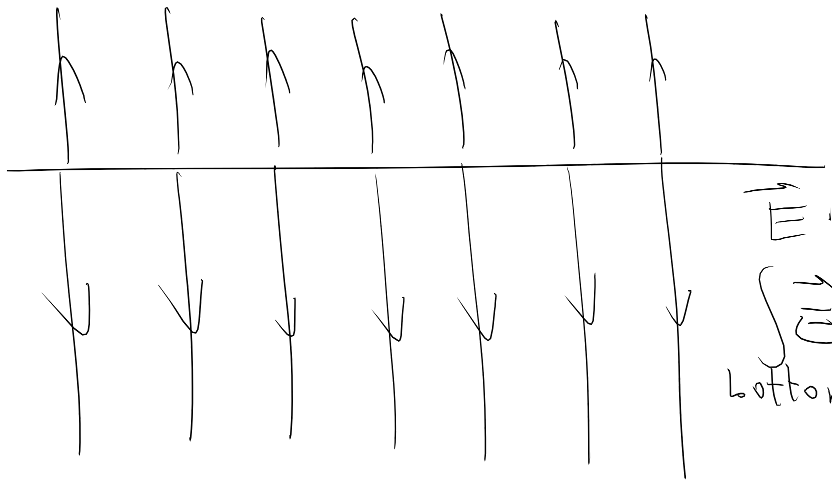


$$Q_{enc} = S(-\sigma) + S(+\sigma) = 0$$

$Q_{enc} = +\sigma S$



$$\oint \vec{E}\cdot d\vec{S} = \int_{top} \underbrace{\vec{E}}_{=0}\cdot d\vec{S} + \int_{bottom} \vec{E}\cdot d\vec{S} + \int_{side} \vec{E}\cdot d\vec{S}$$



$$\vec{E}(\text{botton}) = E_0 \hat{z}$$

$$d\vec{S} = dS\, \hat{z}$$

$$\vec{E}\cdot d\vec{S} = E_0\, dS$$

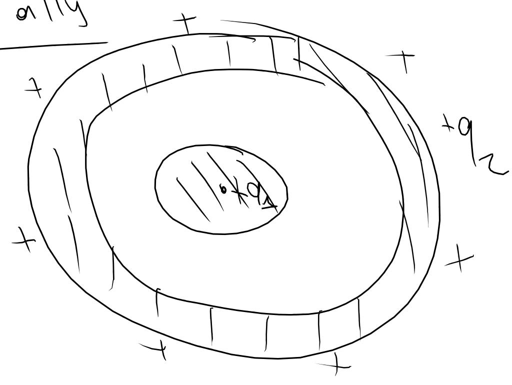
$$\int_{\text{botton}} \vec{E}\cdot d\vec{S} = E_0 S$$

$$\oint \vec{E}\cdot d\vec{S} = E_0 S = \frac{Q_{enc}}{\varepsilon_0} = \frac{S(-\sigma)}{\varepsilon_0}$$

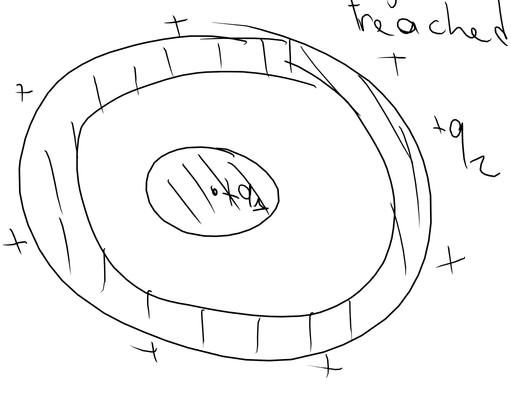
$$\Rightarrow \boxed{E_0 = -\frac{\sigma}{\varepsilon_0}}$$

Example (sphere)

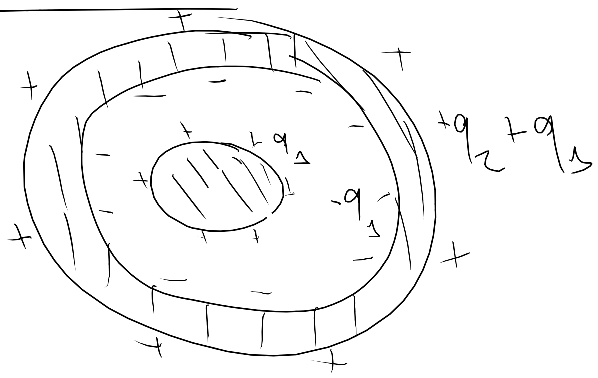
initially

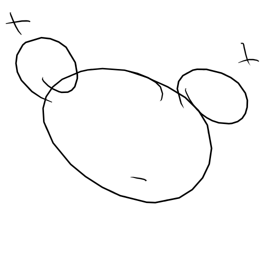


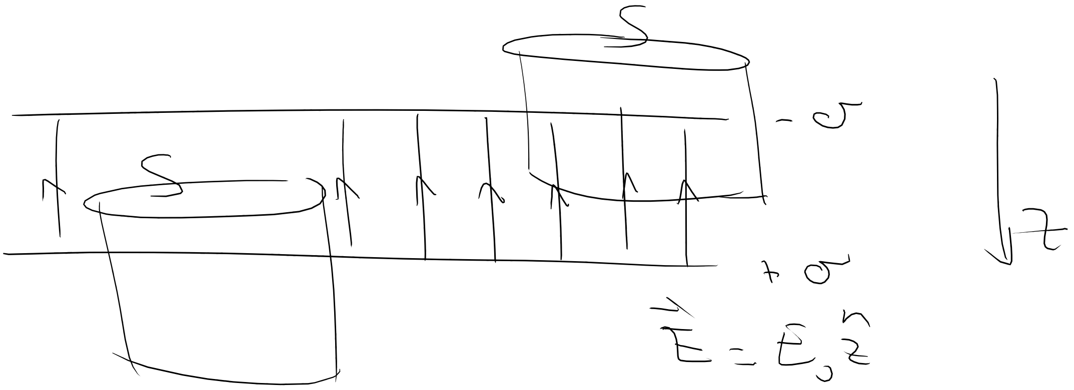
insulator after static configuration is reached



conductor







$$\vec{E} = E_0 \hat{z}$$

$$\oint \vec{E}\cdot d\vec{S} = \int_{top} \vec{E}\cdot d\vec{S} + \cancel{\int_{side} \vec{E}\cdot d\vec{S}}_{0} + \cancel{\int_{bottom} \vec{E}\cdot d\vec{S}}_{0}$$

$$\left.\begin{array}{l} d\vec{S} = -dS\hat{z} \\ \vec{E}\cdot d\vec{S} = -E_0\, dS \end{array}\right\} \int_{top} \vec{E}\cdot d\vec{S} = -E_0 S$$

$$-E_0 S = \frac{(+\sigma) S}{\varepsilon_0} \Longrightarrow \boxed{E_0 = -\frac{\sigma}{\varepsilon_0}}$$

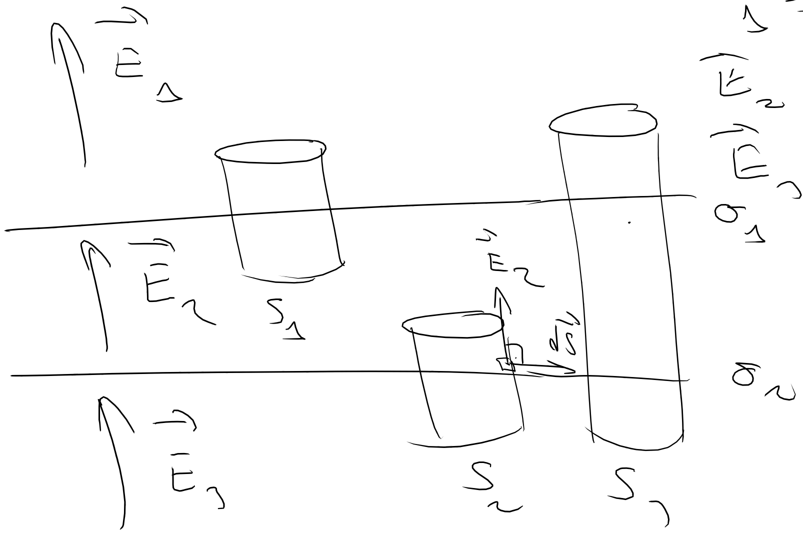
Example

$$\oint \vec{E}\cdot d\vec{S} = ?$$

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

Example

$$\vec{E}_1 = E_1 \hat{z}$$
$$\vec{E}_2 = E_2 \hat{z}$$
$$\vec{E}_3 = E_3 \hat{z}$$

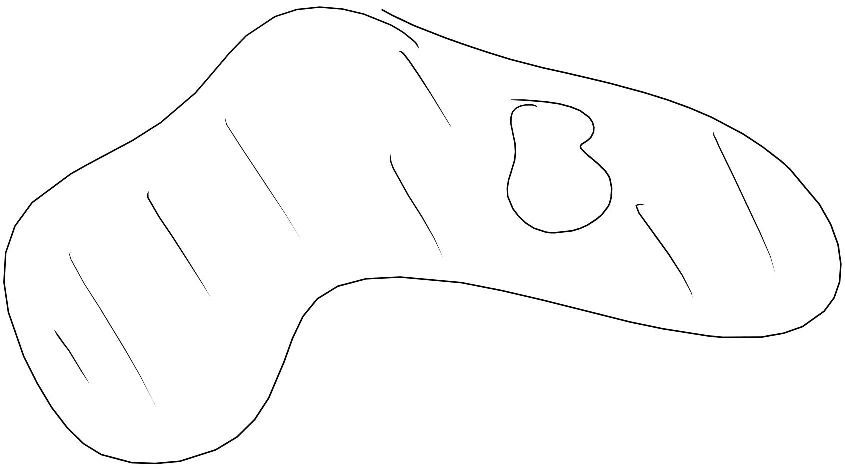


$$\oint_{S_1} \vec{E} \cdot d\vec{S} = (E_1 - E_2) S_1 = \frac{\sigma_1 S_1}{\varepsilon_0}$$

$$\boxed{E_1 - E_2 = \frac{\sigma_1}{\varepsilon_0}}$$

$$E_1 - E_3 = \frac{\sigma_1 + \sigma_2}{\varepsilon_0}$$

$$\boxed{E_2 - E_3 = \frac{\sigma_2}{\varepsilon_0}}$$



Potential & Pot. Energy

q_1 q_2 q_3 q_4

$$\vec{E} = \lim_{q' \to 0} \frac{\vec{F}}{q'}$$

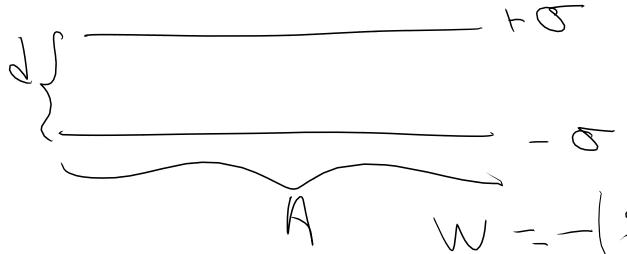
$$\vec{F} = q\vec{E}$$

q_1 q_2

$$\Delta U = -q_5 \int \vec{E} \cdot d\vec{l}$$

$$\Delta U = q_5 V(P)$$

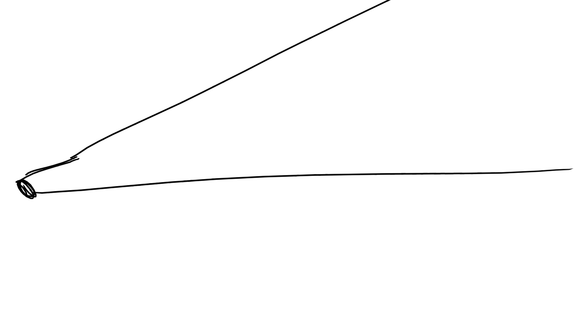
Example

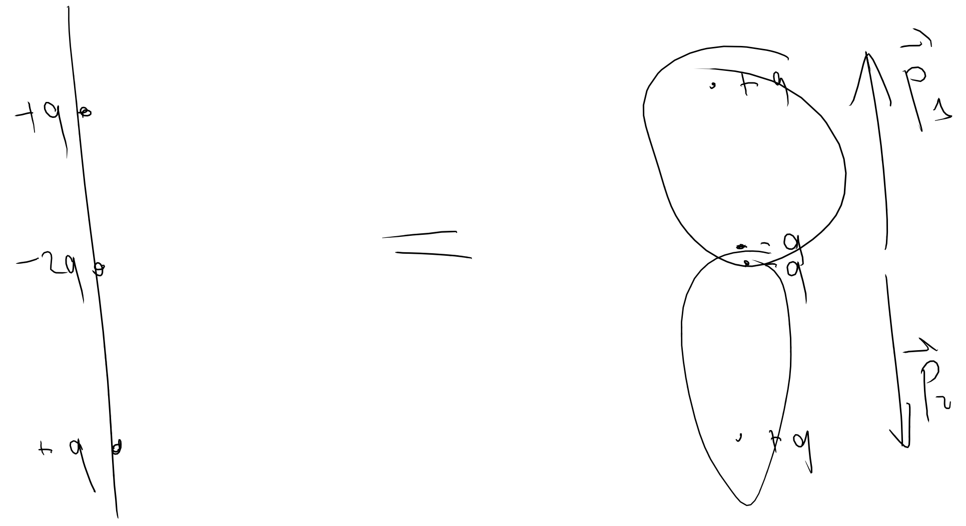


$$F = \underbrace{\left(\frac{\sigma}{2\varepsilon_0}\right)}_{E}\underbrace{(\sigma A)}_{q}$$

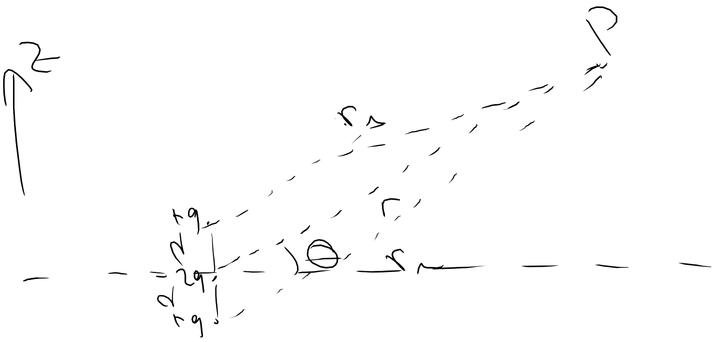
$$W_{45} = -\left(\frac{\sigma}{2\varepsilon_0}\right)(\sigma A)\, d$$

Example





$$\vec p_1 + \vec p_2 = 0$$



$$V(P) = \left(\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right)\frac{1}{4\pi\varepsilon_0}$$

$$\vec r_1 = -d\hat z + \vec r$$

$$r_1^2 = r^2 + d^2 - 2d\,\vec r\cdot\hat z = r^2 + d^2 - 2dr\cos\left(\frac{\pi}{2} - \theta\right)$$

$$\boxed{r_1^2 = r^2 + d^2 - 2rd\sin\theta}$$

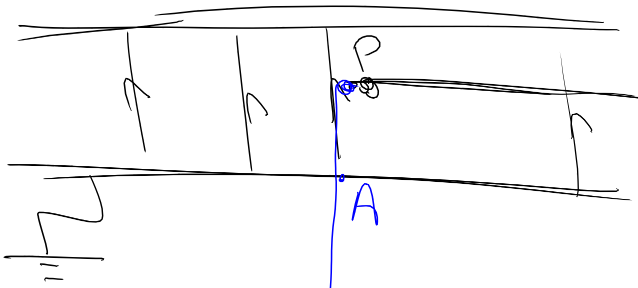
$$\vec r_2 = d\hat z + \vec r$$

$$r_2^2 = r^2 + d^2 + 2rd\sin\theta$$

$$\frac{1}{r_1} = \left(r^2 + d^2 - 2rd\sin\theta\right)^{-1/2} = \frac{1}{r}\left(1 + \left(\frac{d}{r}\right)^2 - 2\frac{d}{r}\sin\theta\right)^{-1/2}$$

$$= \frac{1}{r}\left(1 + x\right)^{-1/2} = \frac{1}{r}\left(1 - \frac{1}{2}x + \left(-\frac{1}{2}\right)\left(-\frac{1}{2} - 1\right)\frac{x^2}{2} + \dots\right)$$

$$x = \left(\frac{d}{r}\right)^2 - 2\frac{d}{r}\sin\theta$$



$$\Delta U = -\int \vec{F}\cdot d\vec{l} = -\int q\vec{E}\cdot d\vec{l} = 0$$

$$\Delta U = q\left(-\int_{\infty}^{P} \vec{E}\cdot d\vec{l}\right)$$

$$= q\left(V(P) - \overset{0}{V(\infty)}\right)$$

$$= q\left(V(P) - V(A)\right) + q\left(V(A) - V(\infty)\right)$$

isolated sphere

$$C = 4\pi\varepsilon_0 R$$

$$q = C\Delta V \Rightarrow \Delta V = \frac{q}{C}$$

Example