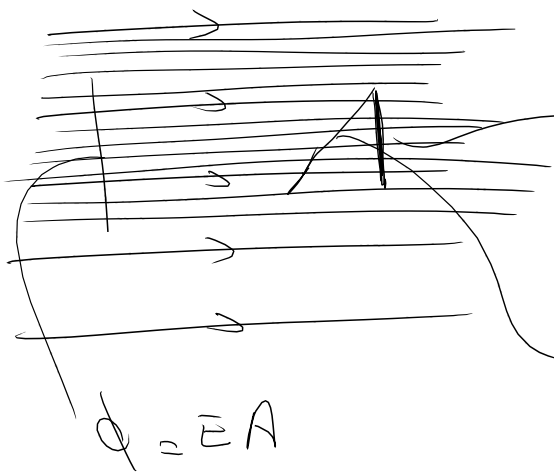


$$\Phi = \vec{E} \cdot \vec{A}$$

A



$$\Phi_E = \vec{E} \cdot \vec{A}$$

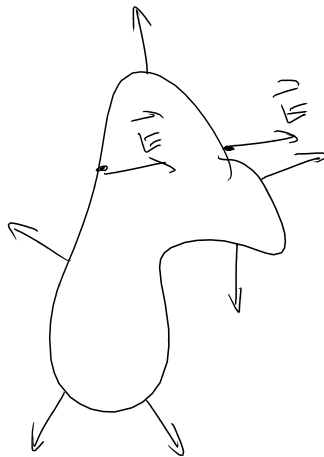
$$A \cos \theta$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi = \vec{E} A \cos \theta$$

arbitrary surface

to

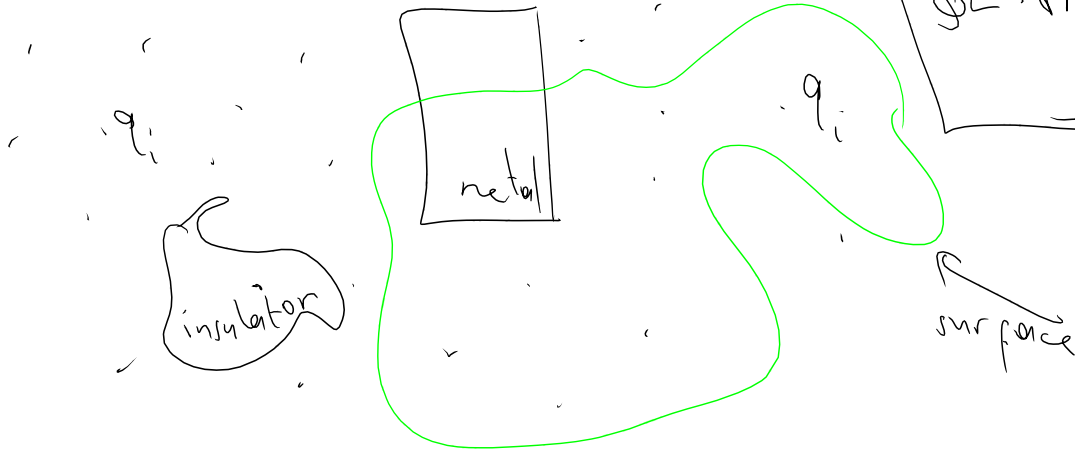


$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

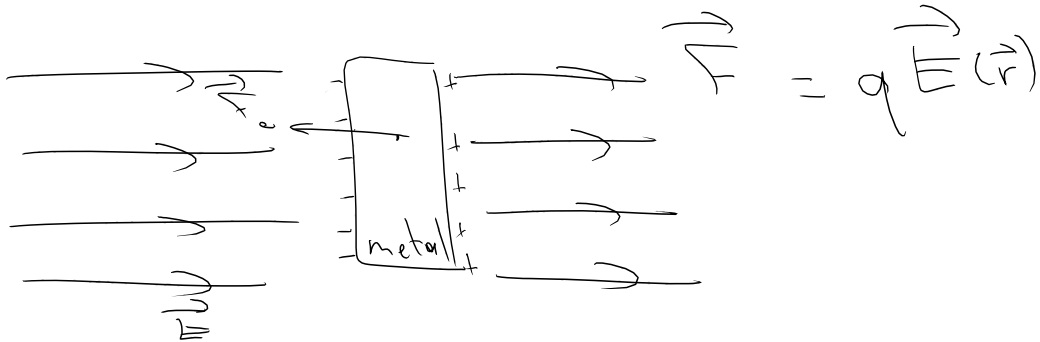
Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$



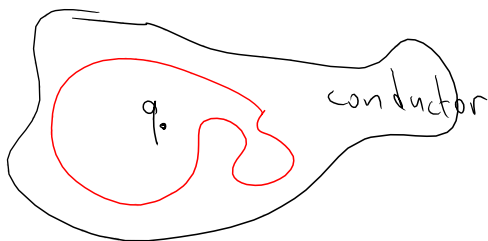
\oint over a closed surface

Electric Field in a Conductor = 0
in electrostatics



Example

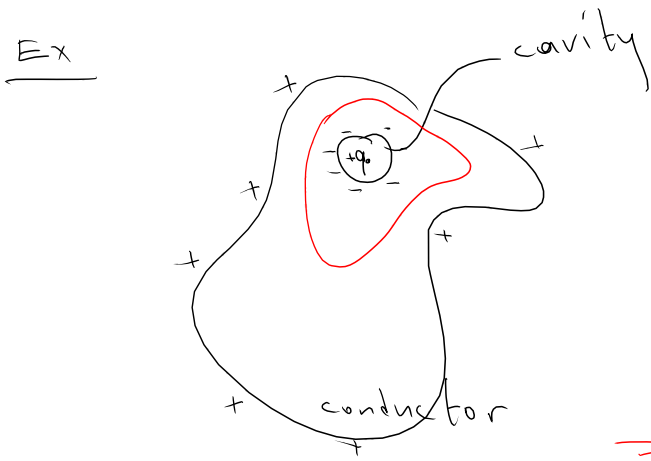
$q = ?$



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$= 0$$

$$\Rightarrow Q_{enc} = q = 0$$

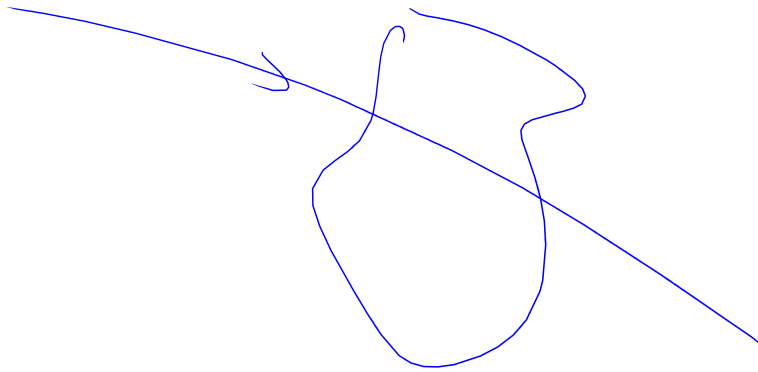


- what is the total charge in the inner surface?

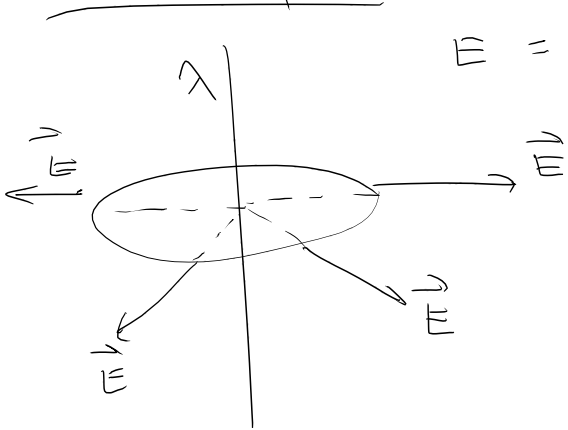
$$\oint \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = (+q) + Q_{inner\ surface}$$

$$Q_{inner\ surface} = -q$$

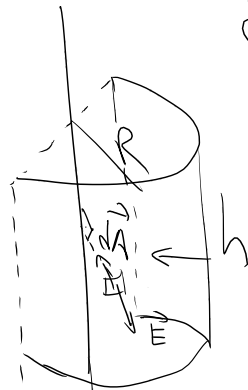


Example



$$E = \frac{\lambda}{2\pi\epsilon_0 R}$$

$\Phi_E = ?$



$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

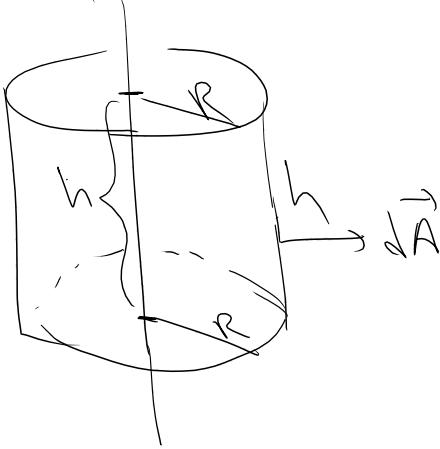
$$\vec{E} \cdot d\vec{A} = E dA \cos(\alpha)$$

$$\vec{E} \cdot d\vec{A} = - \frac{\lambda}{2\pi\epsilon_0 R} dA$$

$$\begin{aligned} \Phi_E &= \int \vec{E} \cdot d\vec{A} = \sum \left(- \frac{\lambda}{2\pi\epsilon_0 R} dA \right) \\ &= - \frac{\lambda}{2\pi\epsilon_0 R} \sum dA = - \frac{\lambda}{2\pi\epsilon_0 R} \cdot R \cdot h \end{aligned}$$

$$\Phi_E = - \left(\frac{\lambda h}{\epsilon_0} \right) \left(\frac{1}{2} \right)$$

Example



$$\Phi_E = \left(\frac{\lambda h}{\epsilon_0} \right) = \frac{\lambda h}{\epsilon_0}$$

$$\int \vec{E} \cdot d\vec{A} = \frac{\rho_{enc}}{\epsilon_0}$$

Example



$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{top}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = 0 + 0$$

$$+ \int_{\text{side}} \vec{E} \cdot d\vec{A}$$

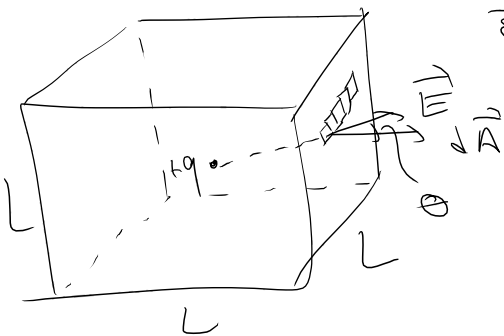
$$= \frac{\lambda h}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{\rho_{enc}}{\epsilon_0}$$

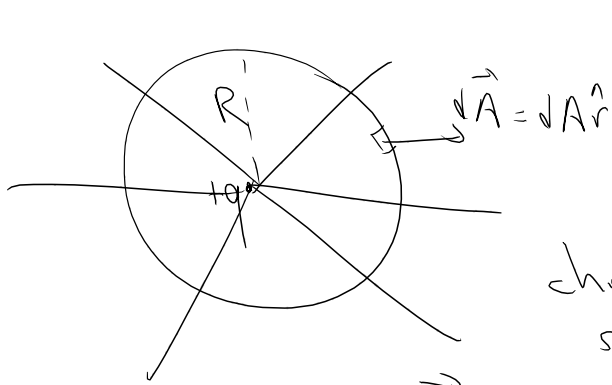
Example

$$\Phi_{\text{side}} = ?$$

$$\frac{\rho_{enc}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{A} = \sum \Phi_{\text{side}} = 6 \Phi_{\text{side}}$$



Example Electric field of a point charge using Gauss' Law.



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\vec{E} = E(r) \hat{r}$$

choose a spherical Gauss surface.

$$d\vec{A} = dA \hat{r}$$

$$\vec{E} \cdot d\vec{A} = E(R) dA \cos(\theta) = E(R) dA$$

$$\oint \vec{E} \cdot d\vec{A} = \oint E(R) dA = E(R) 4\pi R^2 = \frac{q}{\epsilon_0}$$

$$E(R) = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$