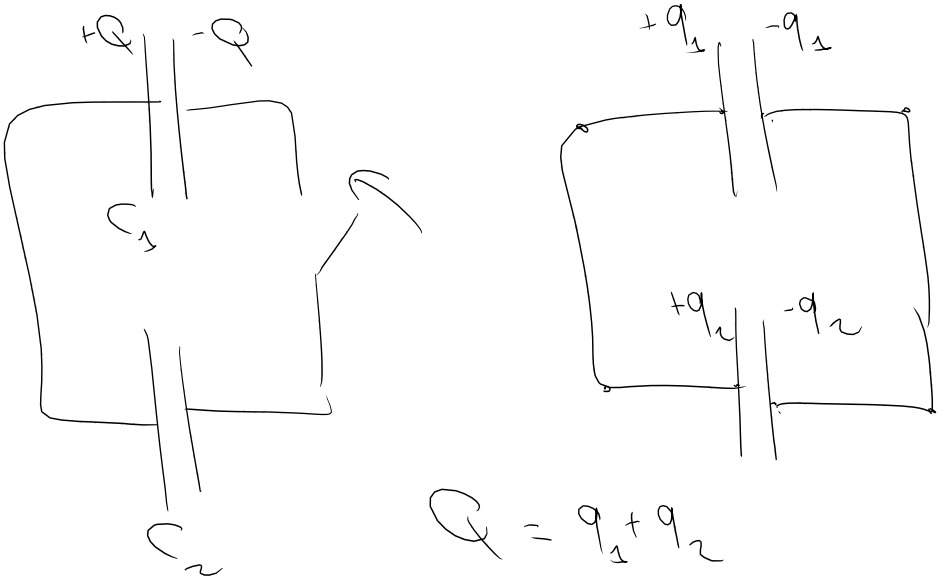
Hand in your HW!



$$Q = q_1 + q_2$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

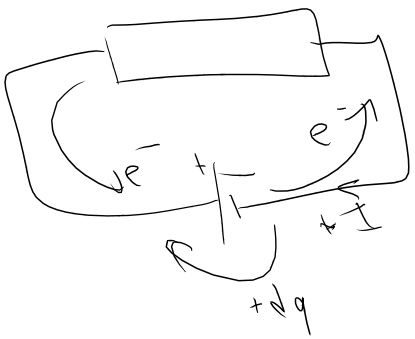
$$\frac{Q^2}{2C_1} \neq \frac{q_1^2}{2C_1} + \frac{q_2^2}{2C_2}$$

Circuits

$\mathcal{E}$: epsilon

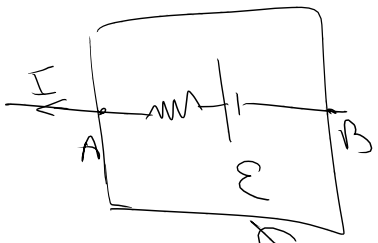
$\mathcal{E}$: electromotive force (emf)

$\mathcal{E}$ is NOT a FORCE!



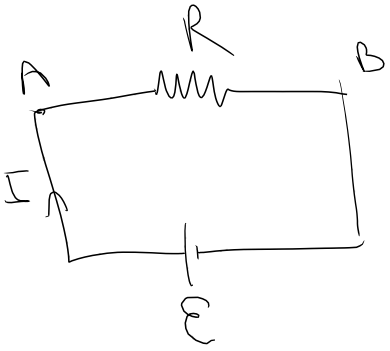
emf versus Voltage

a real battery



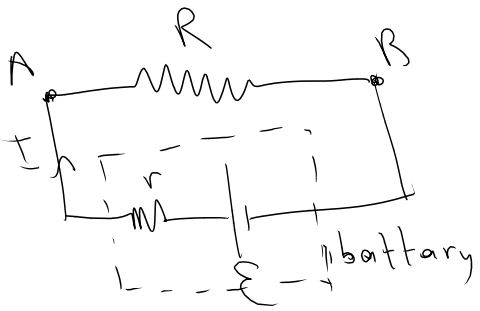
$V_{AB} \neq \mathcal{E}$

real battery



$V_{AB} = IR = \mathcal{E}$

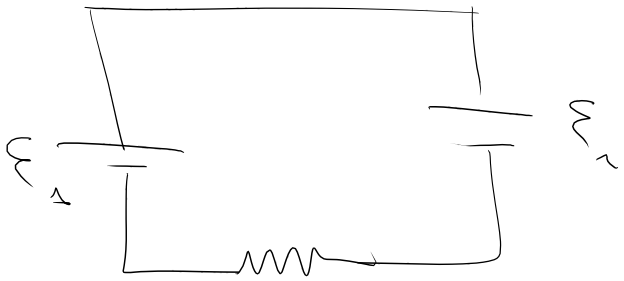
$\Rightarrow I = \frac{\mathcal{E}}{R}$

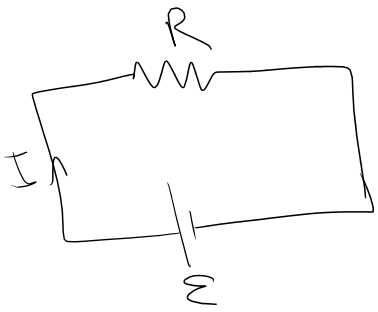


$R_{eq} = R + r$

$I = \frac{\mathcal{E}}{R+r}$

$V_{AB} = IR = \mathcal{E} - Ir < \mathcal{E}$



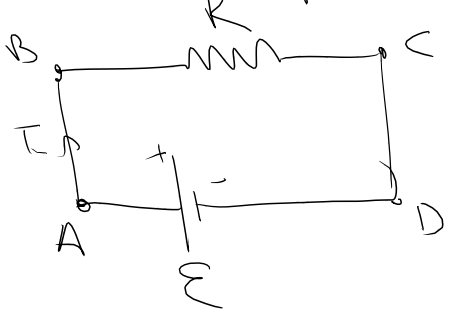


within a dt the work done by the battery

$$(I\,dt)\,\mathcal{E} = (IR)(I\,dt)$$

$$\boxed{\mathcal{E} = IR}$$

Change in the potential:

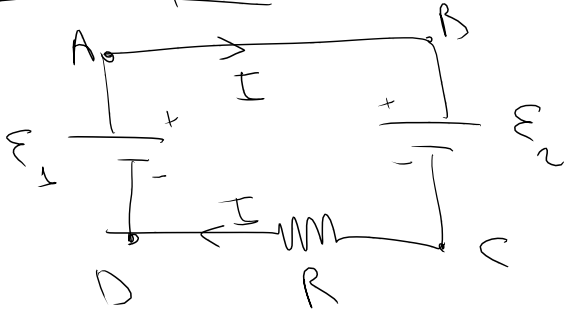


$$0 = V_{AA} = V_{AB} + V_{BC} + V_{CD} + V_{DA}$$

$$0 = 0 + IR + 0 + (-\mathcal{E})$$

$$\Rightarrow \boxed{\mathcal{E} = IR}$$

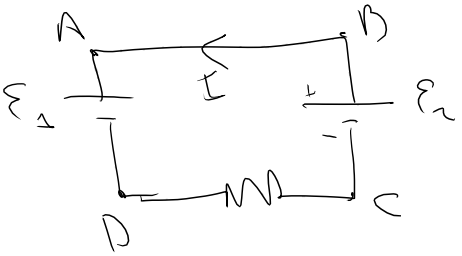
Example



$$0 = 0 + \mathcal{E}_2 + IR + (-\mathcal{E}_1)$$

$$\boxed{I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R}}$$

Alternative

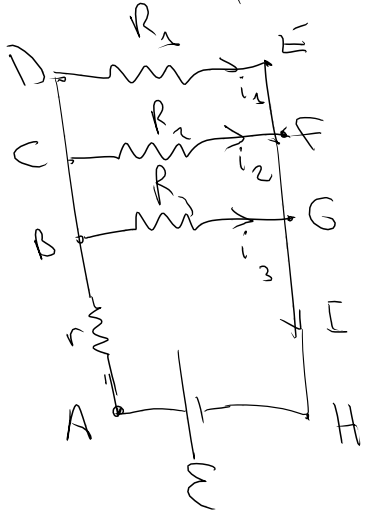


$$0 = V_{AB} + V_{BC} + V_{CD} + V_{DA}$$

$$= 0 + \mathcal{E}_2 + (-IR) + (-\mathcal{E}_1)$$

$$\boxed{I = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R}}$$

Multiloop Circuits



$$R_{eq} = r + \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$V_{AA} = 0 = V_{AB} + V_{BG} + V_{GH} + V_{HA}$$

$$0 = Ir + i_3 R_3 + 0 + (-\mathcal{E})$$

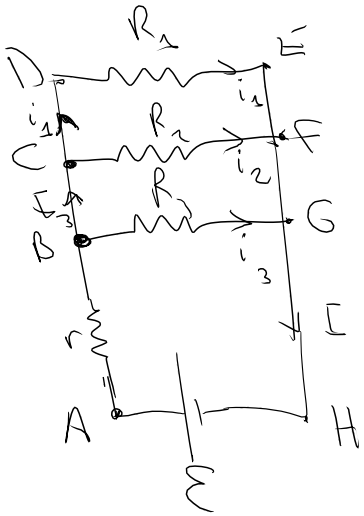
$$\boxed{Ir + i_3 R_3 = \mathcal{E}}$$

$$V_{AA} = V_{AB} + V_{BC} + V_{CF} + V_{FG} + V_{GH} + V_{HA}$$

$$= Ir + 0 + i_2 R_2 + 0 + 0 + (-\mathcal{E})$$

$$\boxed{Ir + i_2 R_2 = \mathcal{E}}$$

$$0 = V_{AA} = V_{AB} + V_{BC} + V_{CD} + V_{DE} + V_{EF} + V_{FG} + V_{GH} + V_{HA}$$



$$0 = Ir + 0 + 0 + i_1 R_1 + 0 + 0 + 0 + (-\mathcal{E})$$

$$\boxed{Ir + i_1 R_1 = \mathcal{E}}$$

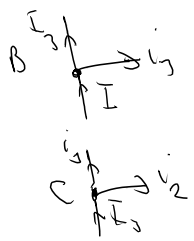
$$\boxed{Ir + i_2 R_2 = \mathcal{E}}$$

$$\boxed{Ir + i_3 R_3 = \mathcal{E}}$$

$$i_1 R_1 = i_2 R_2 = i_3 R_3$$

$$0 = V_{BB} = V_{BC} + V_{CF} + V_{FG} + V_{GB}$$

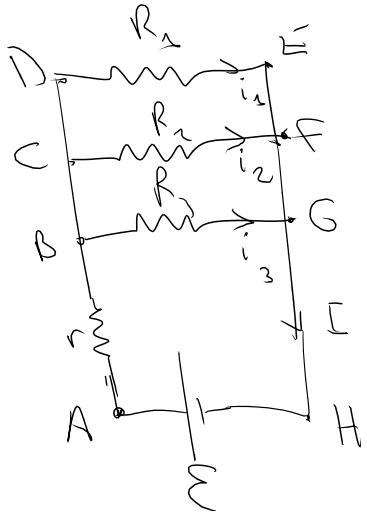
$$= 0 + (i_2 R_2) + 0 + (-i_3 R_3) \Rightarrow i_2 R_2 = i_3 R_3$$



$$I = i_3 + I_3 \Rightarrow I_3 = I - i_3$$

$$I_3 = i_1 + i_2$$

$$\boxed{I = i_1 + i_2 + i_3}$$

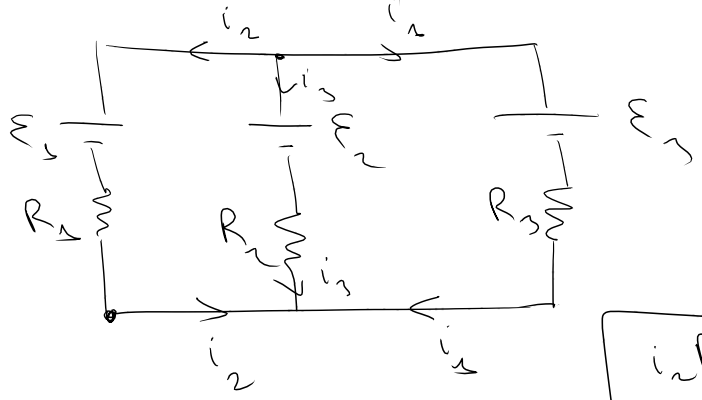


$$0 = V_{CD} + V_{DE} + V_{EF} + V_{FC}$$

$$0 = 0 + i_1 R_1 + 0 + (-i_2 R_2)$$

$$\boxed{i_1 R_1 = i_2 R_2}$$

Example



$$\boxed{0 = i_1 + i_2 + i_3}$$

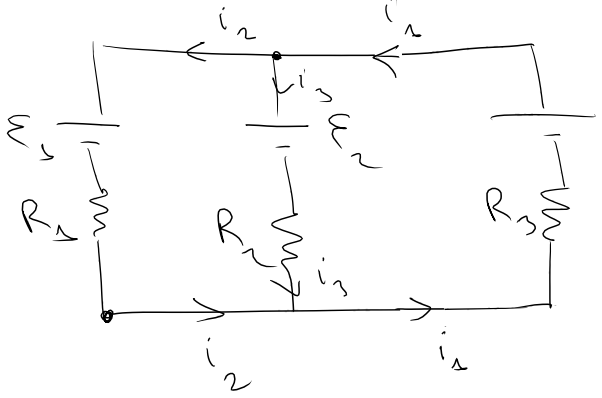
$$0 = (-i_3 R_2) + (-\varepsilon_2) + (\varepsilon_1) + (i_2 R_1)$$

$$\boxed{i_2 R_1 - i_3 R_2 = \varepsilon_2 - \varepsilon_1}$$

$$0 = (-i_1 R_3) + (-\varepsilon_3) + (\varepsilon_1) + (i_2 R_1)$$

$$\boxed{-i_3 R_3 + i_2 R_1 = \varepsilon_3 - \varepsilon_1}$$

Alternative



$$\boxed{i_1 = i_2 + i_3}$$

$$0 = i_2 + i_3 + (-i_1)$$

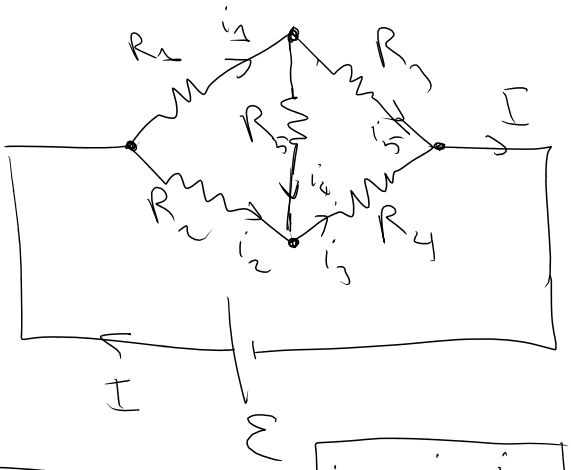
$$0 = 0 + (-i_3 R_2) + (-\varepsilon_2) + \varepsilon_1 + (i_2 R_1)$$

$$0 = 0 + (i_3 R_3) + (-\varepsilon_3) + \varepsilon_1 + (i_2 R_1)$$

Kirchoff's Rules:

i) Loop Rule

ii) Current Rule



$R_{eq} = ?$

$R_{eq} \equiv \frac{\varepsilon}{I}$

$0 = i_4 + i_4' \Rightarrow i_4' = -i_4$

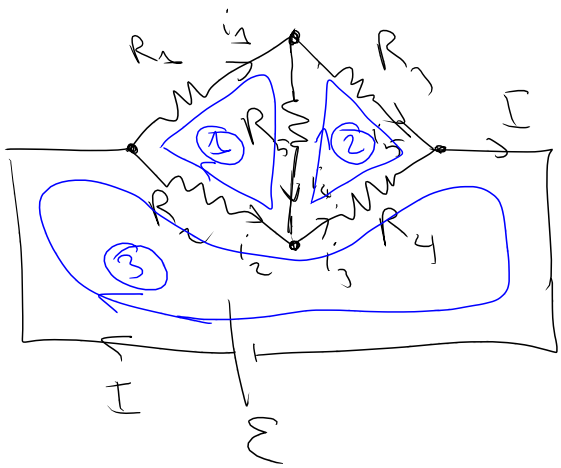
$I = i_1 + i_2$

$i_5 + i_3 = I$

$i_1 = i_4 + i_5$

$i_2 + i_4 = i_3$

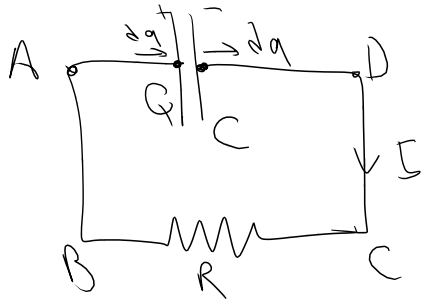
$i_1 + i_2 = i_3 + i_5 \Rightarrow (i_1 - i_5) + i_2 = i_3 \Rightarrow i_4 + i_2 = i_3$



① $\Rightarrow (-i_2 R_2) + (i_1 R_1) + (i_4 R_5) = 0$

② $\Rightarrow (-i_4 R_5) + (R_3 i_5) + (-i_3 R_4) = 0$

③ $\Rightarrow (i_2 R_2) + (i_3 R_4) + (-\varepsilon) = 0$



$$V_{AA} = V_{AB} + V_{BC} + V_{CD} + V_{DA}$$

$$0 = 0 + (-IR) + 0 + \left(-\frac{Q}{C}\right)$$

$$IR + \frac{Q}{C} = 0$$

$$Q = C\Delta V$$

$$I = \frac{dq}{dt}, \quad dq = dQ \quad \Rightarrow \quad I = \frac{dQ}{dt}$$

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0 \Rightarrow \frac{dQ}{dt} = -\frac{1}{RC}Q$$

$$\frac{d}{dt}(t^n) = nt^{n-1} \qquad \frac{d}{dt}(\sin \omega t) = \omega \cos \omega t$$

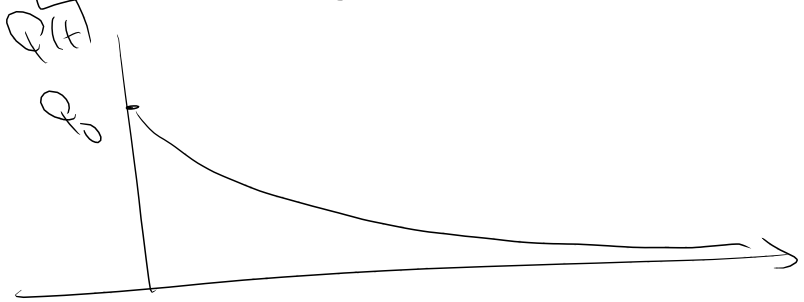
$$\frac{d}{dt}Ae^{\alpha t} = \alpha Ae^{\alpha t}$$

$$Q(t) = Ae^{-\frac{1}{RC}t}$$

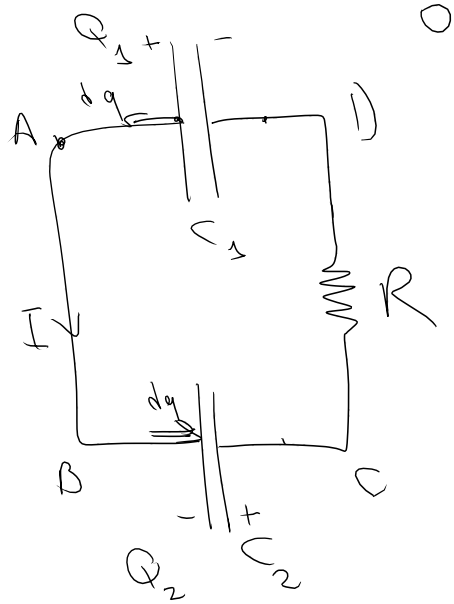
$$Q_0 = Q(t=0) = A$$

$$Q(t) = Q_0 e^{-t/\tau}$$

$\tau = RC$: time constant



Example



$$0 = V_{AA} = V_{AD} + V_{DC} + V_{CB} + V_{BA}$$

$$= \frac{Q_1}{C_1} + (-IR) + \left(\frac{Q_2}{C_2}\right)$$

$$\boxed{\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = IR}$$

$$I = \frac{dQ_1}{dt}$$

$$I = \frac{dQ_2}{dt}$$

$$\frac{1}{C_1}\underbrace{\frac{dQ_1}{dt}}_{-I} + \frac{1}{C_2}\underbrace{\frac{dQ_2}{dt}}_{-I} = \frac{dI}{dt}R$$

$$\frac{dI}{dt}R = -\left(\frac{1}{C_1} + \frac{1}{C_2}\right)I = -\frac{1}{C_{eq}}I$$

$$\boxed{\frac{dI}{dt} = -\frac{1}{RC_{eq}}I} \Rightarrow \boxed{I(t) = I_0 e^{-\frac{1}{RC_{eq}}t}}$$

$$\frac{dQ_1}{dt} = -I = -I_0 e^{-t/\tau} \qquad \tau \equiv RC_{eq}$$

$$\int_0^t \frac{dQ_1}{dt}\,dt = -I_0 \int_0^t e^{-t/\tau}\,dt$$

$$Q_1(t) - Q_1 = -I_0 \left(\frac{e^{-t/\tau}}{-1/\tau}\right)\Bigg|_{t=0}^{t} = I_0\tau\left(e^{-t/\tau} - 1\right)$$

$$\boxed{Q_1 = Q_{10} + I_0\tau\left(e^{-t/\tau} - 1\right)}$$

$$\boxed{Q_2 = Q_{20} + I_0\tau\left(e^{-t/\tau} - 1\right)}$$

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = IR \Rightarrow \frac{Q_{10}}{C_1} + \frac{Q_{20}}{C_2} + I_0 R\left(e^{-t/\tau} - 1\right)\underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}_{\frac{1}{C_{eq}}}$$

$$= R I_0 e^{-t/\tau}$$

$$\tau = R C_{eq} \Rightarrow \frac{\tau}{C_{eq}} = R$$

$$\frac{Q_{10}}{C_1} + \frac{Q_{20}}{C_2} - I_0 R = 0 \Rightarrow \boxed{\frac{Q_{10}}{C_1} + \frac{Q_{20}}{C_2} = I_0 R}$$