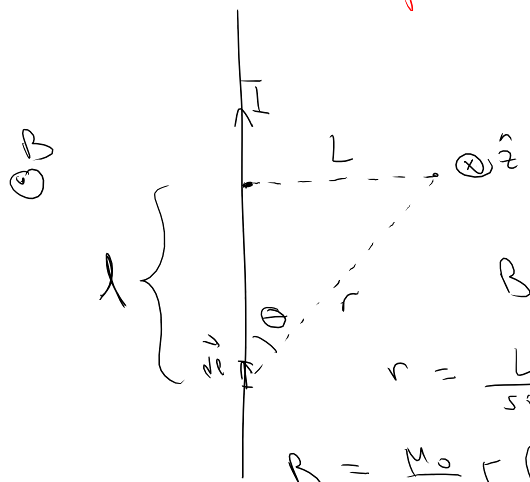
Hand in your HW.



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$d\vec{B} = dB\,\hat{z}$$

$$dB = \frac{\mu_0}{4\pi} I \frac{dl \sin\theta}{r^2}$$

$$B = \int dB = \frac{\mu_0}{4\pi} I \int_{-\infty}^{\infty} \frac{dl \sin\theta}{r^2} \Leftarrow$$

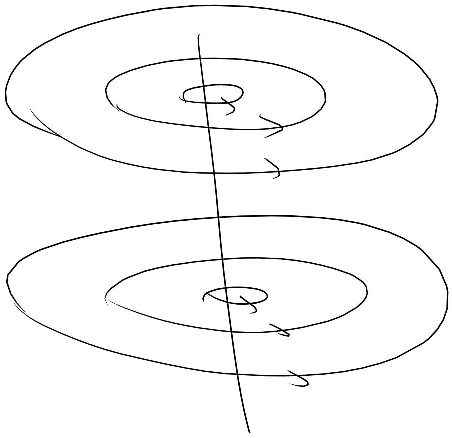
$$r = \frac{L}{\sin\theta} \; ; \; l = -\frac{L}{\tan\theta} \Rightarrow dl = -\frac{L}{\sin^2\theta} d\theta$$

$$B = \frac{\mu_0}{4\pi} I \int \left(-\frac{L}{\sin^2\theta} d\theta\right) \sin\theta \frac{\sin^2\theta}{L^2}$$

$$= -\frac{\mu_0 I}{4\pi L} \int_{\pi}^{0} \sin\theta \, d\theta$$

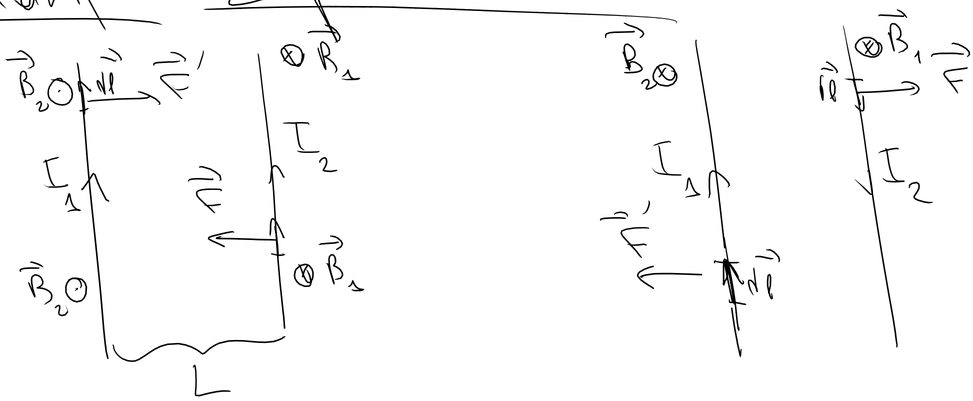
$$= -\frac{\mu_0 I}{4\pi L} \left(-\cos\theta\right)\Big|_{\theta=\pi}^{0} = -\frac{\mu_0 I}{4\pi} \left(-1 + (-1)\right)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi L}}$$



$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

Example 2 parallel wires



$$B_1 = \frac{\mu_0 I_1}{2\pi L}$$

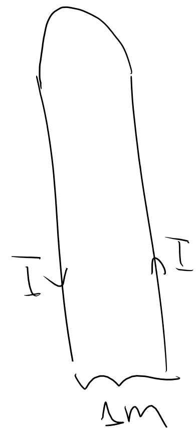
$$d\vec{F} = I_2\, d\vec{l} \times \vec{B}$$

$$F = I_2 h \frac{\mu_0 I_1}{2\pi L}$$

Force acting on a segment of length h.

$$F = \frac{\mu_0}{2\pi} h \frac{I_1 I_2}{L}$$

$$\mu_0 = 4\pi\, 10^{-7}\ \mathrm{Tm/A}$$



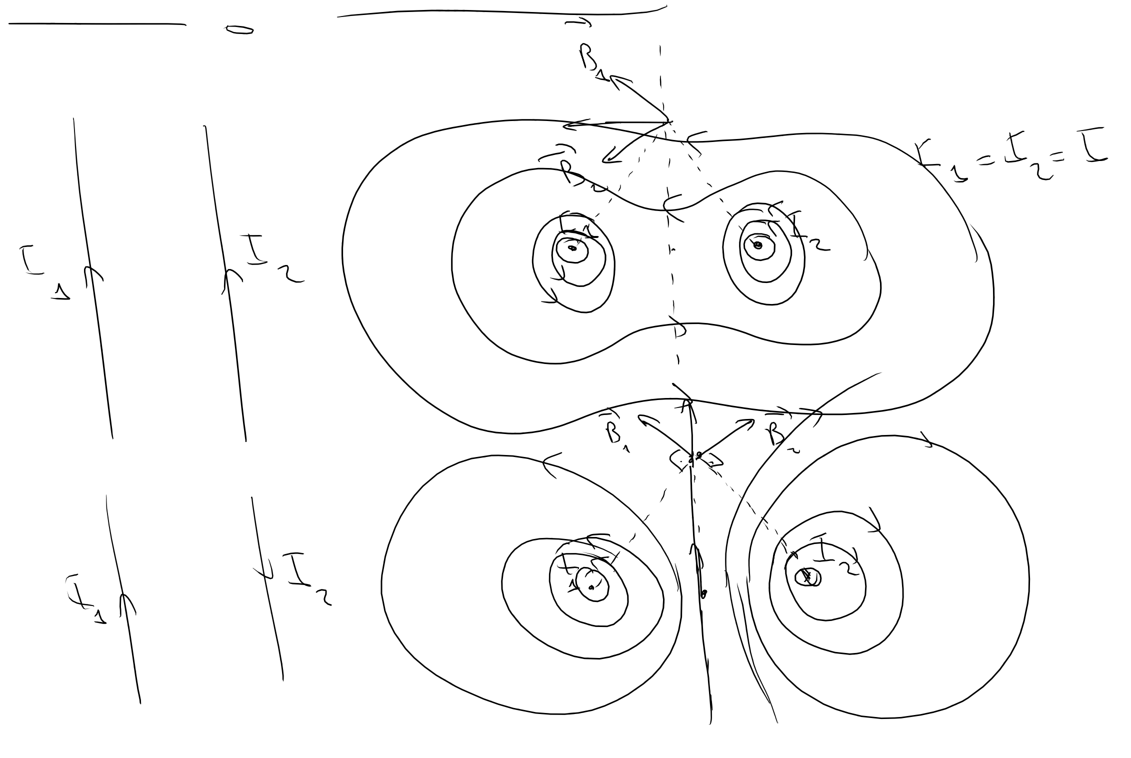
$$I = 1\,\mathrm{A}$$

$$\frac{F}{h} = \frac{4\pi\, 10^{-7}\ \mathrm{Tm/A}}{2\pi} \frac{(1\mathrm{A})^2}{1\mathrm{m}}$$

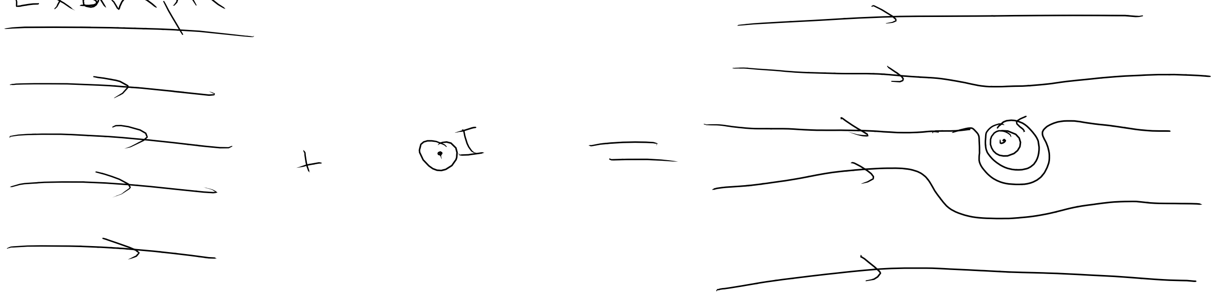
$$\frac{F}{h} = 2 \times 10^{-7}\ \mathrm{T}(1\mathrm{A})$$

$$\frac{F}{h} = 2 \times 10^{-7}\ \frac{\mathrm{N}}{\mathrm{m}}$$

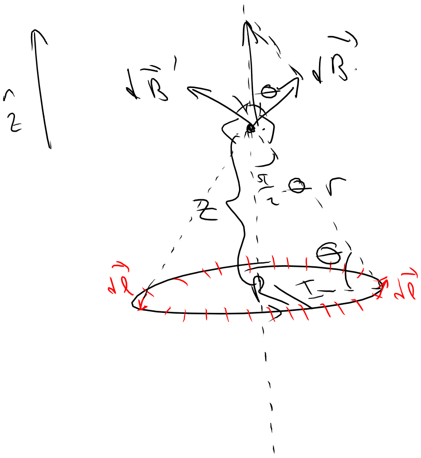
$$[T] = \frac{\mathrm{N}}{\mathrm{m\,A}}$$



Example



Example



$$\mu = I \pi R^2$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} \, d\vec{\ell} \times \hat{r}$$

$$d\vec{B} = dB \cos\theta \, \hat{z} + \dots$$

$$= \frac{\mu_0 I \cos\theta \, d\ell}{4\pi r^2} \hat{z} + \dots$$

$$\vec{B} = \sum d\vec{B} = \frac{\mu_0 I}{4\pi r^2} \left(\sum d\ell \right) \hat{z} \cos\theta$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi r^2} (2\pi R) \hat{z} \cos\theta}$$

$$\vec{B} = \frac{\mu_0 I R}{2 r^2} \frac{R}{r} \hat{z}$$

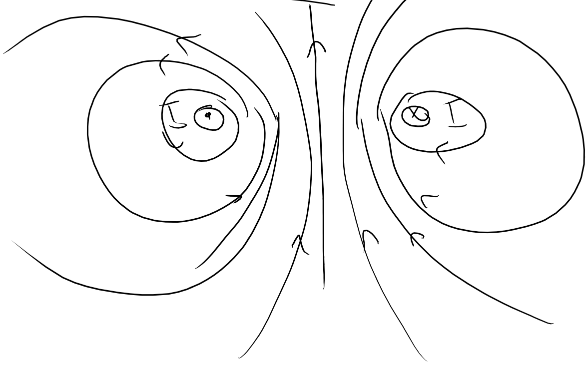
$$\boxed{\vec{B} = \frac{\mu_0}{2\pi r^3} \underbrace{(I \pi R^2) \hat{z}}_{\vec{\mu}}}$$

$$\boxed{\vec{B} = \frac{\mu_0}{2\pi r^3} \vec{\mu}}$$

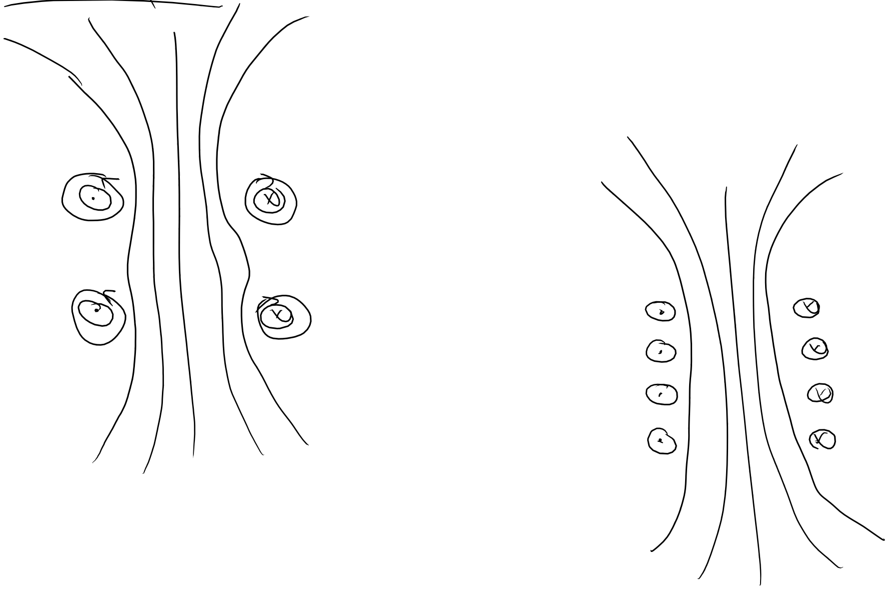
$$r^2 = (R^2 + z^2)$$

$$\boxed{\vec{B} = \frac{\mu_0}{2\pi (R^2 + z^2)^{3/2}} \vec{\mu}}$$

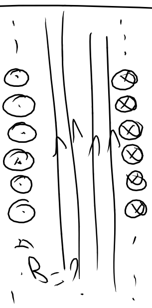
Examples



Example



Ideal Solenoid

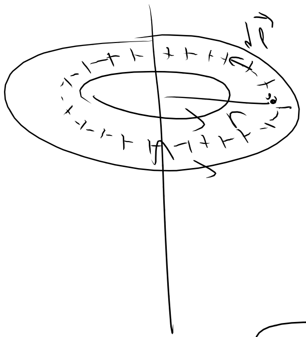


$\vec{B} = 0$

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2}\, d\vec{l} \times \hat{r}$$

Faraday's Law

(Analogous to Gauss' Law in electrostatics, in its usage)



$$B = \frac{\mu_0 I}{2\pi r}$$

$$\boxed{B(2\pi r) = \mu_0 I}$$

$$\sum B\, dl = \sum \vec{B} \cdot d\vec{l} = \oint \vec{B} \cdot d\vec{l}$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}}$$

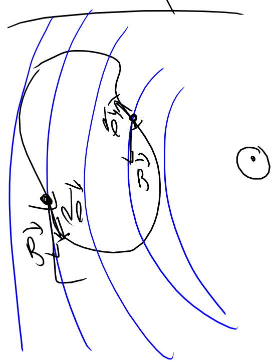
$$\oint_{L_1} \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 I_1$$

$$\oint_{L_2} \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 (I_1 - I_2)$$

$$\oint_{L_3} \vec{B}\cdot d\vec{l} = \mu_0 (I_2 - I_1)$$

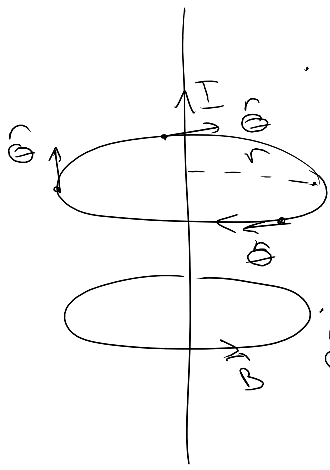
$$\oint_{L_4} \vec{B}\cdot d\vec{l} = 0$$

Example



$$\oint \vec{B}\cdot d\vec{l} = 0$$

Example



$$|\vec{B}| = B(r)$$

$$d\vec{l} = dl\,\hat{\theta}$$

$$\vec{B} = B(r)\,\hat{\theta}$$

$$\vec{B}\cdot d\vec{l} = B(r)\,dl$$

$$\oint \vec{B}\cdot d\vec{l} = \oint B(r)\,dl = B(r)\oint dl = 2\pi r\, B(r)$$

$$\parallel$$

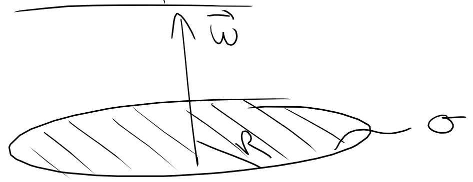
$$\mu_0 I_{enc} = \mu_0(-I)$$

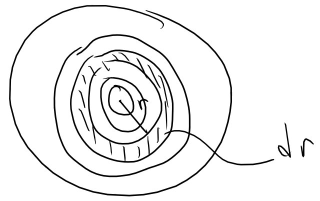
$$\Rightarrow \boxed{B(r) = -\frac{\mu_0}{2\pi r} I}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

Example

$\vec{\mu} = ?$





$$dA = 2\pi r\, dr$$

$$dq = 2\pi r\, dr \frac{\sigma}{\pi R^2}$$

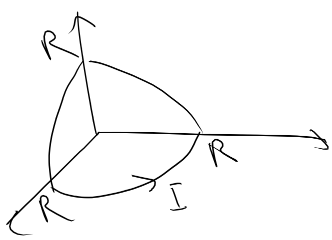
$$dI = \frac{dq}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi} dq$$

$$d\mu = (dI)\, \pi r^2 = \frac{\omega}{2\pi}(2\pi r\, dr)\frac{\sigma}{\pi R^2}\pi r^2$$

$$d\mu = \frac{\omega \sigma}{R^2} r^3\, dr$$

$$\mu = \int d\mu = \frac{\omega\sigma}{R^2}\int_0^R r^3\, dr = \frac{\omega\sigma}{R^2}\frac{R^4}{4} = \boxed{\frac{\omega\sigma R^2}{4} = \mu}$$

Example



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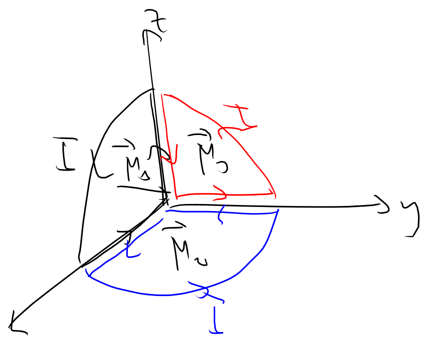
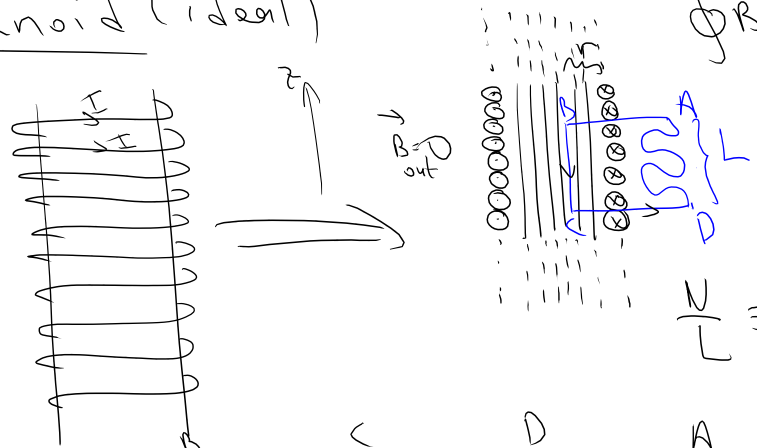


$$\vec{\mu}_1 = \frac{\pi R^2}{4}\hat{y}\ ;\ \vec{\mu}_2 = \frac{\pi R^2}{4}\hat{z}\ ;\ \vec{\mu}_3 = \frac{\pi R^2}{4}\hat{x}$$

$$\boxed{\vec{\mu} = \frac{\pi R^2}{4}(\hat{x}+\hat{y}+\hat{z})}$$

Solenoid (ideal)



$$\oint \vec B \cdot d\vec l = \mu_0 I_{enc}$$

Ampère's Law

$$\frac{N}{L} \equiv n = \text{\# of turns per unit length}$$

$$\oint \vec B \cdot d\vec l = \underbrace{\int_A^B \vec B \cdot d\vec l}_{=0} + \int_B^C \vec B \cdot d\vec l + \underbrace{\int_C^D \vec B \cdot d\vec l}_{=0} + \underbrace{\int_D^A \vec B \cdot d\vec l}_{=0}$$

$$\vec B = B(r)\,\hat z$$
$$d\vec l = dl\,(-\hat z)$$

$$\int_B^C \vec B \cdot d\vec l = -\int B(r)\,dl = -B(r)\int dl = -B(r)L$$

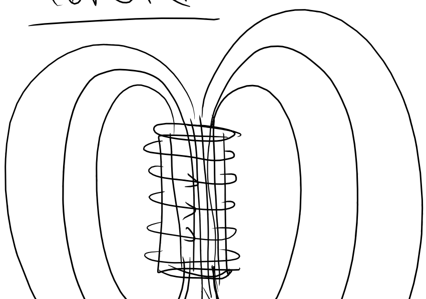
$$\oint \vec B \cdot d\vec l = -B(r)L = \mu_0 I_{enc} = \mu_0(-NI)$$

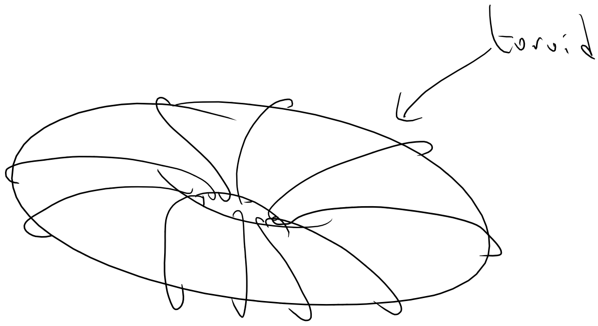
$$B(r) = \mu_0 \left(\frac{N}{L}\right) I = \mu_0 n I$$

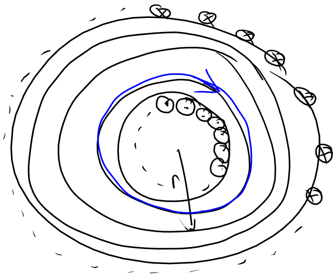
$$\boxed{B(r) = \mu_0 n I}$$ (independent of r)

$$\boxed{\vec B = \mu_0 n I\,\hat z}$$

Toroid







$$|\vec B| = B(r)$$
$$d\vec l = dl\,\hat\theta$$
$$\vec B = B(r)\,\hat\theta$$

$$\oint \vec B \cdot d\vec l = \oint B(r)\,dl = B(r)\oint dl = 2\pi r\,B(r)$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 (-N I)$$

$$B(r) = -\frac{\mu_0 N I}{2\pi r}$$

$$\vec{B} = -\frac{\mu_0 N I}{2\pi r} \hat{\theta}$$

Question Rail Gun

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$