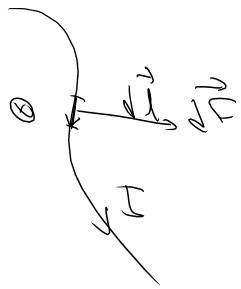
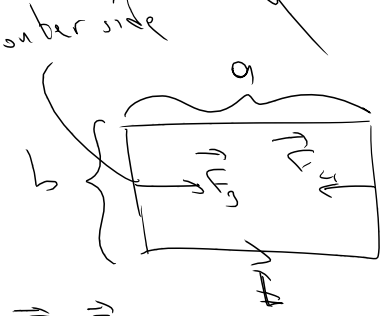
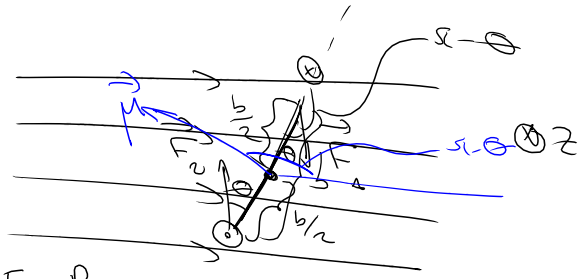
$\vec{F} = q\vec{v} \times \vec{B}$

$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$







$\vec{F}_3 + \vec{F}_4 = 0$

$\vec{\tau} = \vec{r}_3 \times \vec{F}_3$

$F_1 = IaB$

$F_2 = IaB$

$\vec{\tau} = \hat{z}\left[\left(\frac{b}{2}\right) IaB \sin(\pi - \theta) + \left(\frac{b}{2}\right) IaB \sin(\pi - \theta)\right]$

$\vec{\tau} = \hat{z}\left[(Iab)B \sin(\pi - \theta)\right]$

$|\vec{\mu}| = I(ab) = IA$

$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$

$\vec{\mu} = I\vec{A}$

$\vec{F}_{EM} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Example

$I = \frac{(2\pi R\lambda)}{\omega} = \omega R\lambda$

$\boxed{I = \omega R\lambda}$

$\mu = (\omega R\lambda)\pi R^2 \Rightarrow \vec{\omega}\left(\pi\lambda R^3\right) = \vec{\omega}\,\pi R^2 \frac{Q}{2\pi R}$

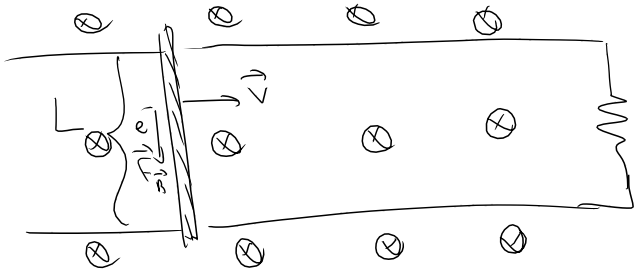
$\boxed{\vec{\mu} = \frac{Q}{2} R^2 \vec{\omega}}$

$\vec{L} = I\vec{\omega} = mR^2\vec{\omega}$ (moi)

$\Rightarrow \vec{\omega} = \frac{\vec{L}}{mR^2}$

$$\vec{\mu} = \frac{Q}{2} R^2 \frac{\vec{L}}{mR^2} = \boxed{\frac{Q}{2m}\vec{L} = \vec{\mu}}$$

Example

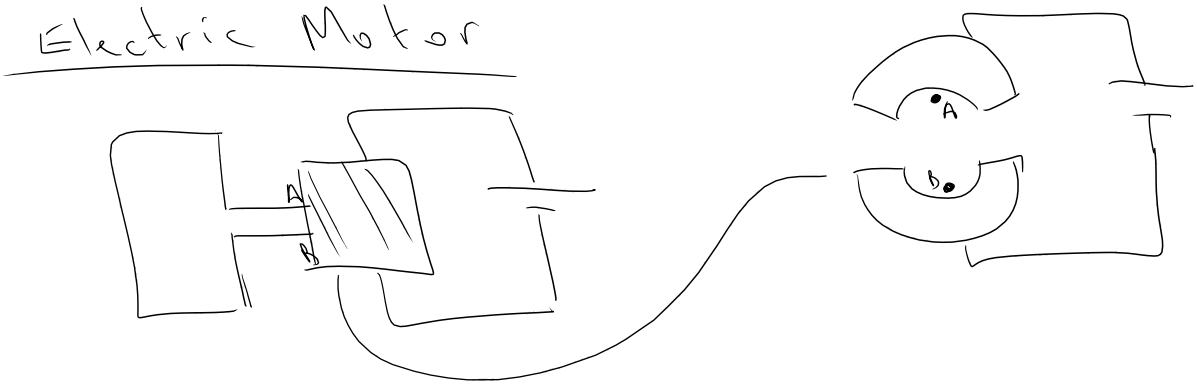


$$F = qvB$$

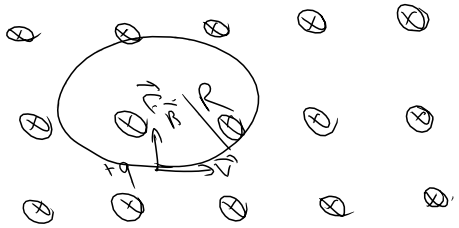
$$W = qvBL$$

$$\mathcal{E} = \frac{W}{q} = vBL$$

Electric Motor



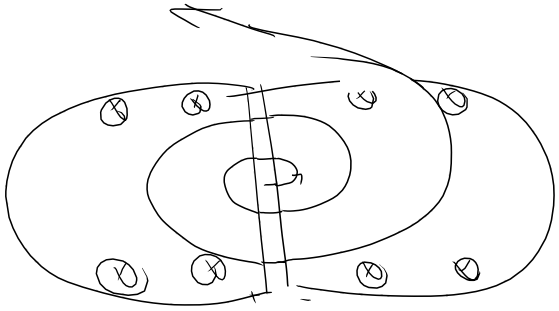
Circular Motion in a Mag. Field



$$qvB = \frac{mv^2}{R}$$

$$\boxed{qB = \frac{mv}{R}}$$

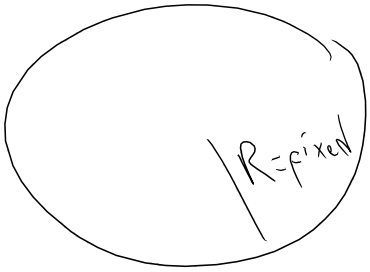
$$T = \frac{2\pi R}{v} = 2\pi\left(\frac{R}{v}\right) = 2\pi\frac{m}{qB}$$



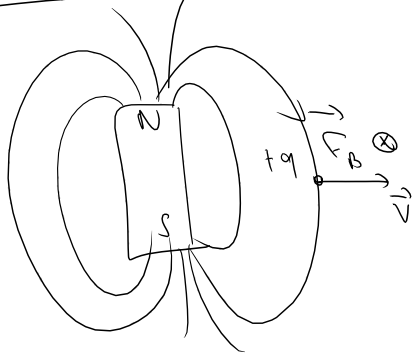
Cyclotron

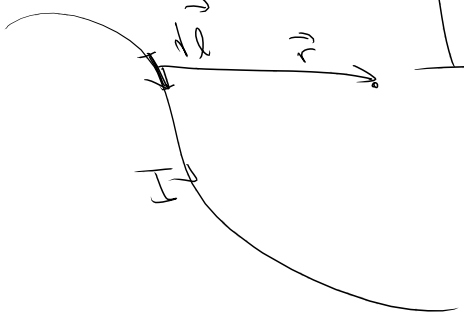
(upto energies ~50 MeV (proton)

Syncrotron (LHC)



What creates magnetic field



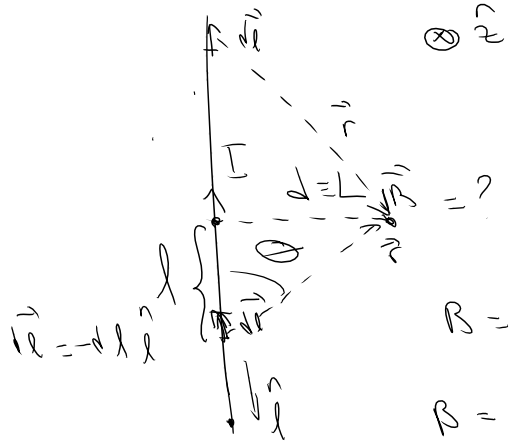


$$d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}$$

Biot-Savart Law

$\mu_0 = 4\pi \cdot 10^{-7}$ Tm/A

Example



$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi r^2} I\, d\vec{l} \times \hat{r}$$

$$d\vec{B} = dB\,\hat{z}$$

$$dB = \frac{\mu_0}{4\pi r^2} I\, dl \sin\theta$$

$$B = \int dB = \int \frac{\mu_0}{4\pi r^2} I\, dl \sin\theta$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl}{r^2} \sin\theta$$

$$r\sin\theta = d \Rightarrow r = \frac{d}{\sin\theta}$$

$$\frac{d}{l} = \tan\theta \Rightarrow l = \frac{d}{\tan\theta} \Rightarrow dl = \left(\frac{dl}{d\theta}\right) d\theta$$

$$dl = \frac{d}{d\theta}\left(\frac{L\cos\theta}{\sin\theta}\right) d\theta$$

$$= L\, d\theta\, \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta} = -L\frac{d\theta}{\sin^2\theta}$$

$$dl = -L\frac{d\theta}{\sin^2\theta}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{|dl|}{r^2} \sin\theta = +\frac{\mu_0 I}{4\pi} \int \frac{d\theta}{\sin^2\theta} \frac{\sin^2\theta}{L^2} \sin\theta$$

$$B = +\frac{\mu_0 I}{4\pi L} \int_0^{\pi} d\theta \sin\theta = +\frac{\mu_0 I}{4\pi L} \left(-\cos\theta\right)\Big|_{\theta=0}^{\pi}$$

$$= +\frac{\mu_0 I}{2\pi L}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi L}\,\hat{z}$$