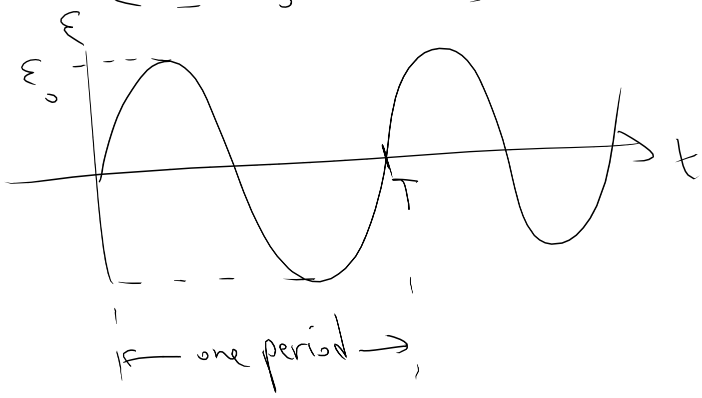
$v_d \sim 10^{-6}$ m/s

$t \sim \frac{1}{100}$ s

$x = v_d \cdot t \sim 10^{-8}$ m

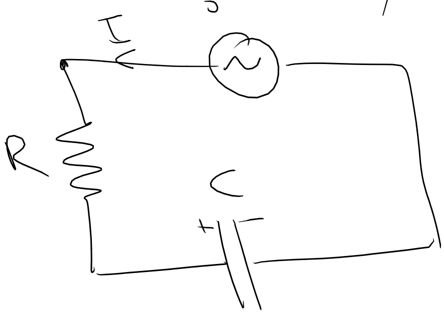
$\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$



$T = \frac{2\pi}{\omega}$

ω: angular speed

RC



$$\boxed{IR + \frac{Q}{C} = \mathcal{E}_0 \cos(\omega t)}$$

$$I = \frac{dQ}{dt}$$

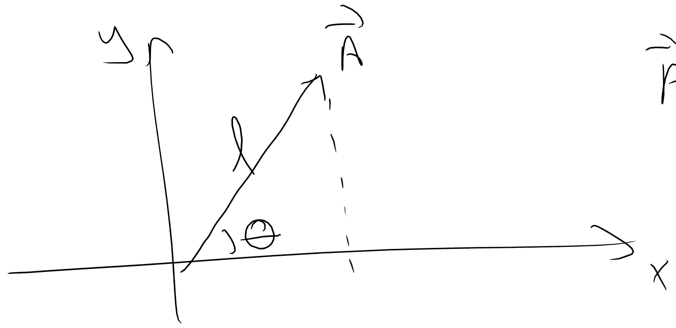
$$R\frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E}_0 \cos(\omega t)$$

$$Q = Q_0 \cos(\omega t + \delta) = Q_0 \cos\delta \cos\omega t - Q_0 \sin\delta \sin\omega t$$

$$-R\omega \sin(\omega t+\delta) Q_0 + \frac{Q_0}{C} \cos(\omega t+\delta) = \mathcal{E}_0 \cos(\omega t)$$

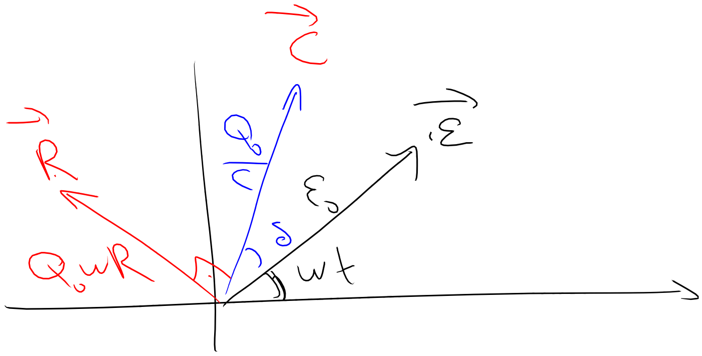
$t=0$: $-\omega R (Q_0 \sin\delta) + \left(\frac{Q_0}{C} \cos\delta\right) = \mathcal{E}_0$

$\omega t = \frac{\pi}{2}$: $-\omega R (Q_0 \cos\delta) - \frac{(Q_0 \sin\delta)}{C} = 0$



$$\vec{A} = l\cos\theta\,\hat{x} + l\sin\theta\,\hat{y}$$

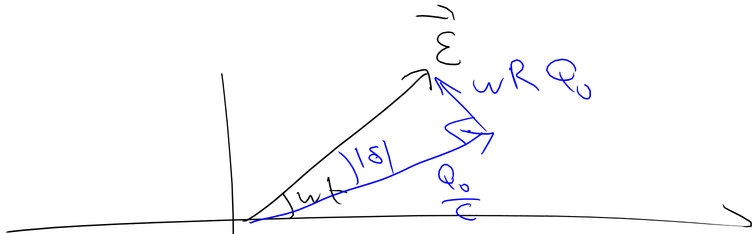
$$-R\omega \sin(\omega t+\delta)\, Q_0 + \frac{Q_0}{C}\cos(\omega t+\delta) = \mathcal{E}_0 \cos(\omega t)$$



$$\vec{R} + \vec{C} = \vec{\mathcal{E}}$$

$$\Rightarrow \delta < 0$$

$$-\sin(\omega t+\delta) = \cos\left(\omega t+\delta+\frac{\pi}{2}\right)$$



"phasor diagram"

$$\mathcal{E}_0^2 = \left(\frac{Q_0}{C}\right)^2 + (\omega R Q_0)^2 \Rightarrow Q_0^2\left(\frac{1}{(\omega C)^2} + R^2\right)\omega^2 = \mathcal{E}_0^2$$

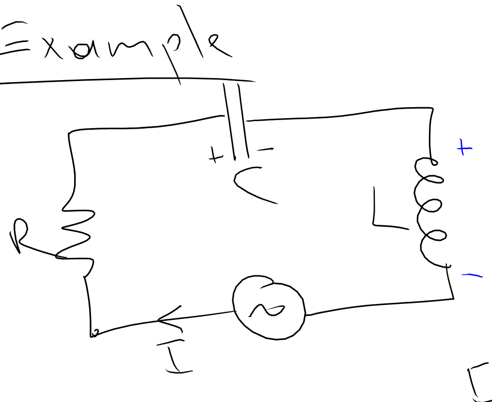
$$\boxed{Q_0 = \frac{\mathcal{E}_0}{\omega}\left(R^2 + \frac{1}{(\omega C)^2}\right)^{-1/2}}$$

$$\tan|\delta| = \frac{\omega R Q_0}{Q_0/C} = \omega R C$$

$$Q(t) = \frac{\mathcal{E}_0}{\omega}\frac{1}{\sqrt{R^2+X_c^2}}\cos(\omega t+\delta)$$

$$\boxed{I(t) = -\mathcal{E}_0 \frac{1}{\sqrt{R^2+X_c^2}}\sin(\omega t+\delta)}$$

Example



$$IR + \frac{Q}{C} + L\frac{dI}{dt} = \mathcal{E}_0 \sin(\omega t)$$

assume $\frac{dI}{dt} > 0$

$$I = \frac{dQ}{dt}$$

$$\boxed{\frac{I}{C} + \frac{dI}{dt}R + L\frac{d^2 I}{dt^2} = \omega\mathcal{E}_0 \cos(\omega t)}$$

$$\boxed{kx + \gamma\dot{x} + m\frac{d^2x}{dt^2} = F}$$

forced oscillator with damping

$$I = I_0 \cos(\omega t + \delta)$$

$$\frac{I_0}{C}\cos(\omega t + \delta) - RI_0\omega \sin(\omega t + \delta) - L\omega^2 I_0 \cos(\omega t + \delta) = \omega\mathcal{E}_0 \cos(\omega t)$$

$$-L\omega I_0 \cos(\omega t + \delta) - RI_0 \sin(\omega t + \delta) + \frac{I_0}{\omega C}\cos(\omega t + \delta) = \mathcal{E}_0 \cos(\omega t)$$

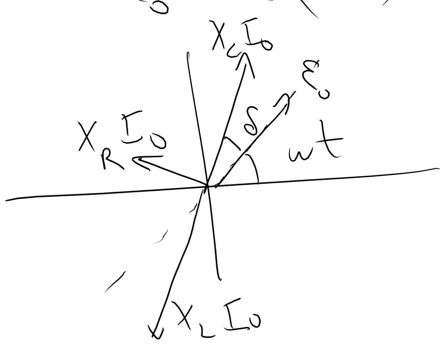
$$X_L = \omega L \qquad X_C = \frac{1}{\omega C} \qquad X_R = R$$

$$I_0\left(-X_L \cos(\omega t + \delta) - X_R \sin(\omega t + \delta) + X_C \cos(\omega t + \delta)\right)$$

$$= \mathcal{E}_0 \cos(\omega t)$$

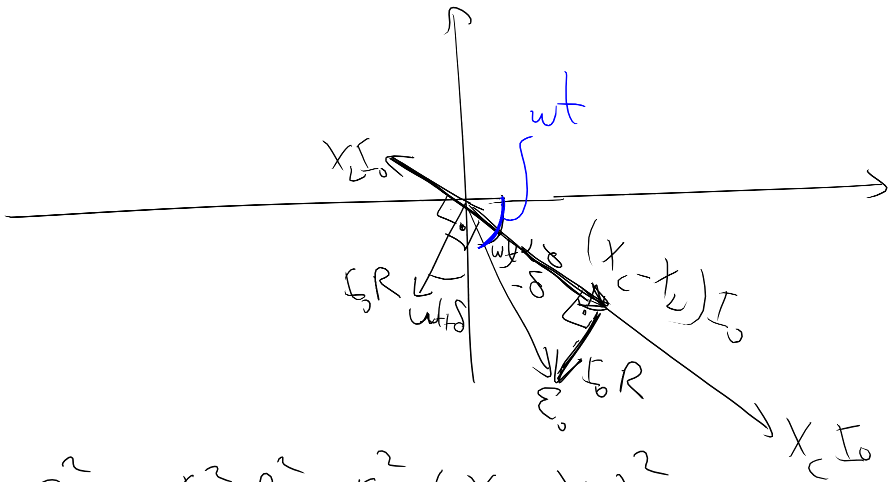
$$-X_R \sin(\omega t + \delta)$$

$$= X_R \cos\left(\omega t + \delta + \frac{\pi}{2}\right)$$



$$-L\omega I_0 \cos(\omega t+\delta) - R I_0 \sin(\omega t+\delta) + \frac{I_0}{\omega C}\cos(\omega t+\delta) = \mathcal{E}_0 \cos(\omega t)$$

$$L\omega I_0 \sin(\omega t+\delta) - R I_0 \cos(\omega t+\delta) - \frac{I_0}{\omega C}\sin(\omega t+\delta) = -\mathcal{E}_0 \sin \omega t$$



$$\mathcal{E}_0^2 = I_0^2 R^2 + I_0^2 (X_C - X_L)^2$$

$$I_0 = \frac{\mathcal{E}_0}{Z} \; ; \; Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\tan(-\delta) = \frac{I_0 R}{I_0 (X_C - X_L)} = \frac{R}{X_C - X_L}$$

$$\boxed{\tan\delta = \frac{R}{X_L - X_C}}$$

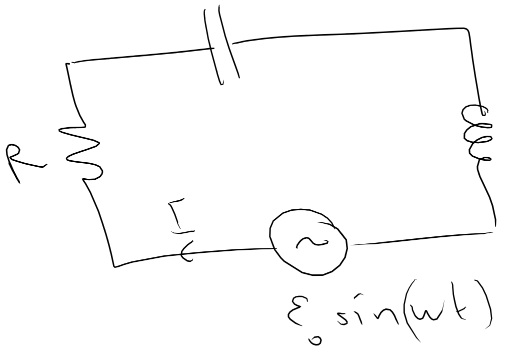
what happens when $X_L = X_C$?

$$I = I_0 \cos(\omega t + \delta) \Leftarrow$$

$$I_0(\omega) = \frac{\varepsilon_0}{Z} \; ; \; Z = \sqrt{R^2 + (X_C - X_L)^2}$$

R indep of ω; $X_C = \frac{1}{\omega C}$, $X_L = \omega L$

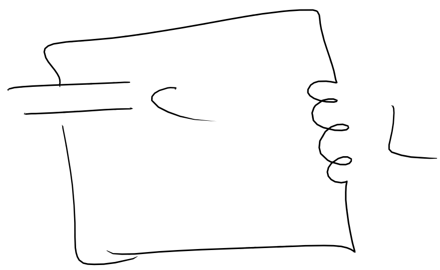
$$\tan\delta = \frac{R}{X_L - X_C}$$



Z is a min when
$X_C = X_L$: resonance condition

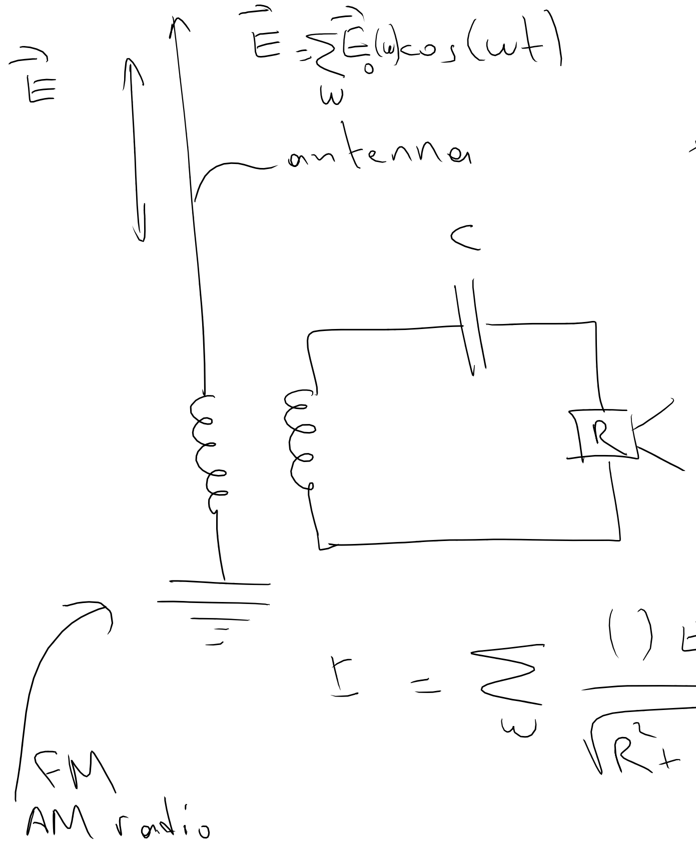
$$\frac{1}{\omega_R C} = \omega_R L$$

$$\Rightarrow \omega_R = \frac{1}{\sqrt{LC}}$$

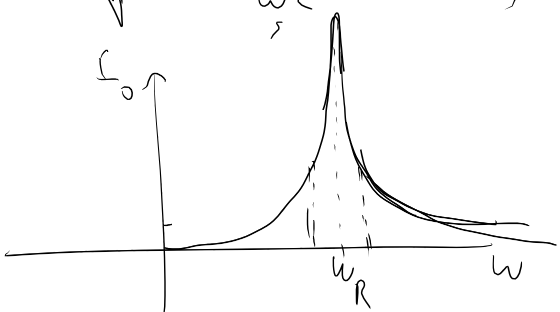


$$Q = Q_0 \cos\left(\frac{1}{\sqrt{LC}} t\right)$$

$$\vec{E} = \sum_{\omega} \vec{E}_0(\omega)\cos(\omega t)$$



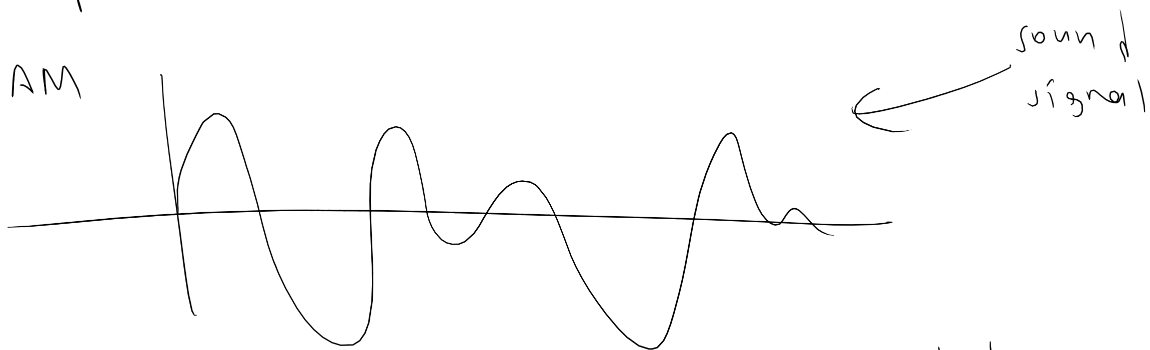
$$I_0 \propto \frac{1}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

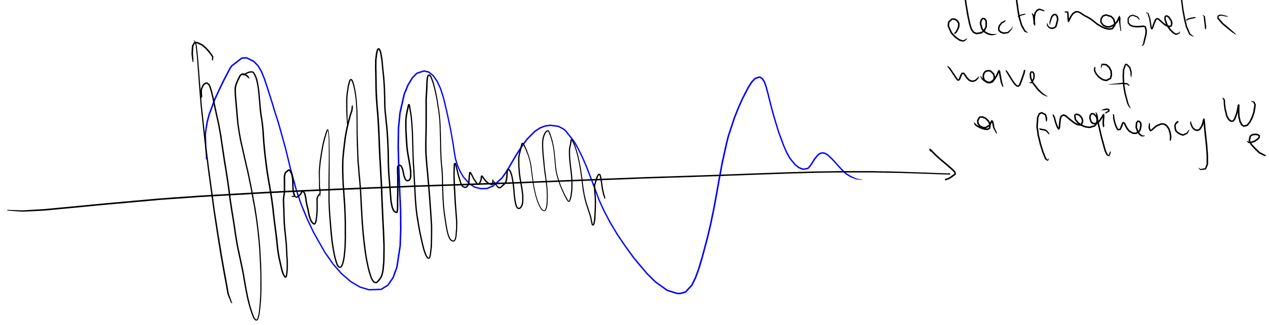


$$I = \sum_{\omega} \frac{(\ )\,E_0(\omega)\cos(\omega t+\delta)}{\sqrt{R^2 + (X_L - X_C)^2}} \simeq (\ )\,\frac{E_0(\omega_R)\cos(\omega_R t+\delta_R)}{R}$$

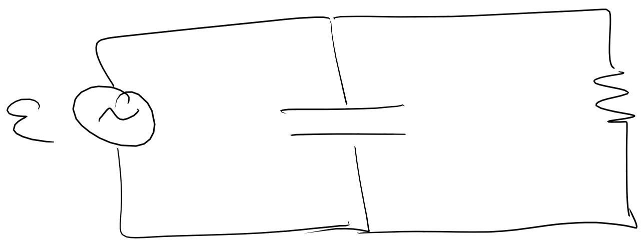
FM: frequency modulation

AM: amplitude modulation.





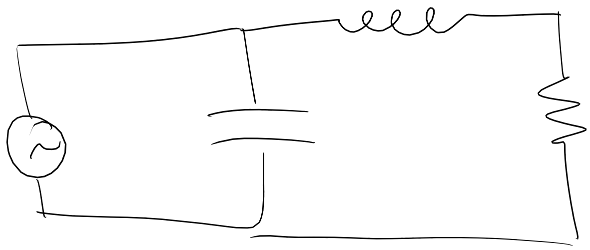
Example



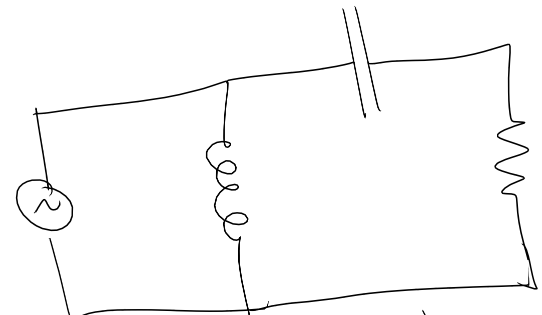
low frecuencies will arrive at the resistor.
high frecuencies will not reach the resistor

$$\varepsilon = \sum_{\omega} \varepsilon(\omega) \cos(\omega t)$$

Low pass filter



better Low pass filter



low frecuencies will not reach the resistor
high frecuencies will reach the resistor

high pass filter.

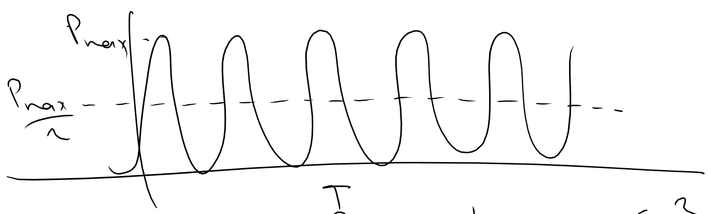
Loud speaker

Power

$$P = I^2 R$$

$$I = I_0 \cos(\omega t + \delta)$$

$$P(t) = I_0^2 R \cos^2(\omega t + \delta)$$



$$\bar{P} = \frac{1}{T} \int_0^T P(t)\,dt = \frac{I_0^2 R}{2} = I_{rms}^2 R$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$ rms : root (of) mean (of) squared

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$ $$f_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

$$\frac{1}{T} \int_0^T \cos^2(\omega t + \delta)\,dt = \frac{1}{2} \;;\; T = \frac{2\pi}{\omega}$$

$$\bar{P} = I_{rms}^2 R = I_{rms} V_{rms}$$

Power Delivered by the $\mathcal{E}$ —

$$P(t) = I\mathcal{E} = I_0 \cos(\omega t + \delta)\, \mathcal{E}_0 \sin(\omega t)$$

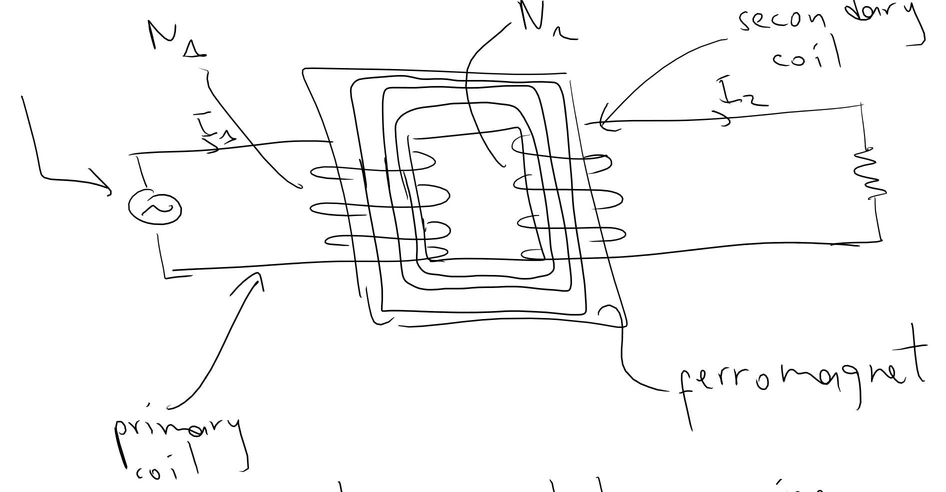
$$= I_0 \mathcal{E}_0 \cos^2(\omega t) \cos\delta - I_0 \mathcal{E}_0 \cos\omega t \sin\omega t \sin\delta$$

$$\bar{P} = \frac{I_0 \mathcal{E}_0}{2} \cos\delta = I_{rms} \mathcal{E}_{rms} \sin\delta$$

$$\text{if } R = 0 \Rightarrow \sin\delta = 0$$

$$\tan\delta = \frac{R}{X_L - X_C}$$

Transformers (not the optimus prime)



$$\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2$$

$$P_R = I^2 R$$

Φ_0 : flux created per ring.

Φ_1 : flux through the primary coil

Φ_2 : flux through the secondary coil.

$$\Phi_0 = \frac{\Phi_1}{N_1} = \frac{\Phi_2}{N_2}$$

$$\frac{1}{N_1}\frac{d\Phi_1}{dt} = \frac{1}{N_2}\frac{d\Phi_2}{dt}$$

$$\boxed{\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2}} \qquad \mathcal{E}_2 = \frac{N_2}{N_1}\mathcal{E}_1$$

Example

$\mathcal{E}_1 = 220\,V$

$\mathcal{E}_2 = 12\,V$

$$\frac{N_2}{N_1} = \frac{12}{220}$$

laminated