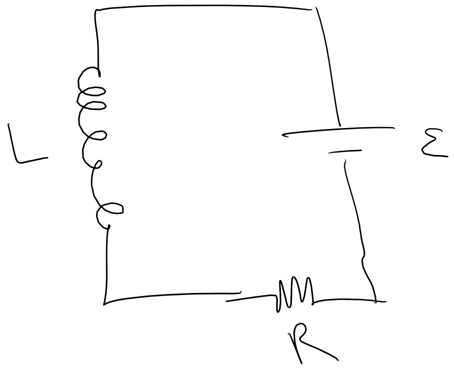
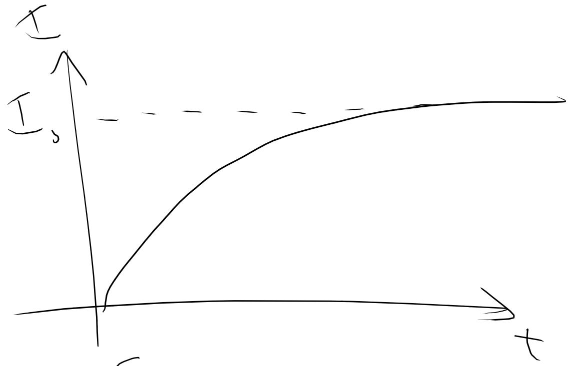
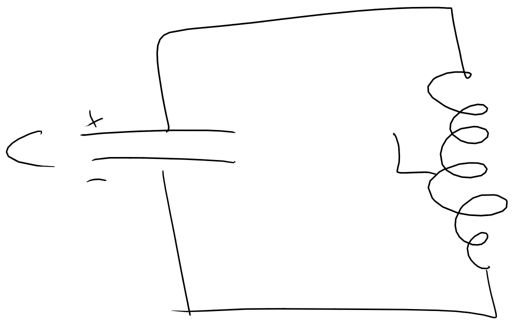
RL Circuits





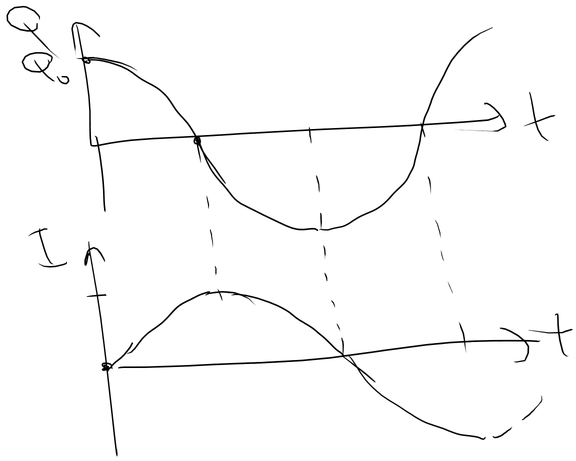
$I_0 = \frac{\mathcal{E}}{R}$

LC Circuits



$Q(t=0) = Q_0$

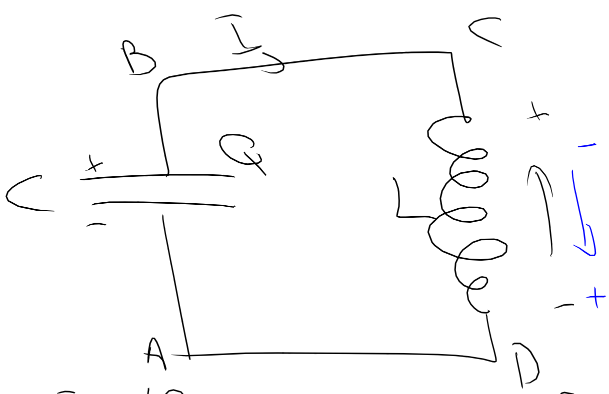
$I(t=0) = 0$



$I = -\frac{dQ}{dt}$

"sketch"

"superconducting cavities"



$-\frac{Q}{C} + L\frac{dI}{dt} = 0$

assume $\frac{dI}{dt} > 0$

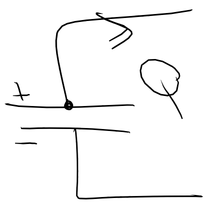
$-$ $+$ assume $\frac{dI}{dt} < 0$

$I = -\frac{dQ}{dt}$

$-\frac{Q}{C} - L\frac{d^2Q}{dt^2} = 0$

$$I = \frac{dq}{dt}$$

$$dQ = -dq$$



$$-\frac{Q}{C} - L\frac{d^2Q}{dt^2} = 0 \Rightarrow \boxed{\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q}$$

$$Q = A\sin(\omega t + \delta)$$

$$\frac{dQ}{dt} = \omega A\cos(\omega t + \delta)$$

$$\frac{d^2Q}{dt^2} = -\omega^2 A\sin(\omega t+\delta) = -\omega^2 Q$$

$$\boxed{\omega^2 = \frac{1}{LC}}$$

$$\boxed{Q(t) = A\sin\left(\frac{1}{\sqrt{LC}}t + \delta\right)} = A\sin\left(\frac{1}{\sqrt{LC}}t\right)\cos\delta + A\cos\left(\frac{1}{\sqrt{LC}}t\right)\sin\delta$$

$$Q(t=0) = Q_0$$

$$I(t=0) = 0$$

$$\parallel$$

$$-\frac{dQ}{dt}$$

$$A\sin\delta = Q_0$$

$$A\cos\delta = 0$$

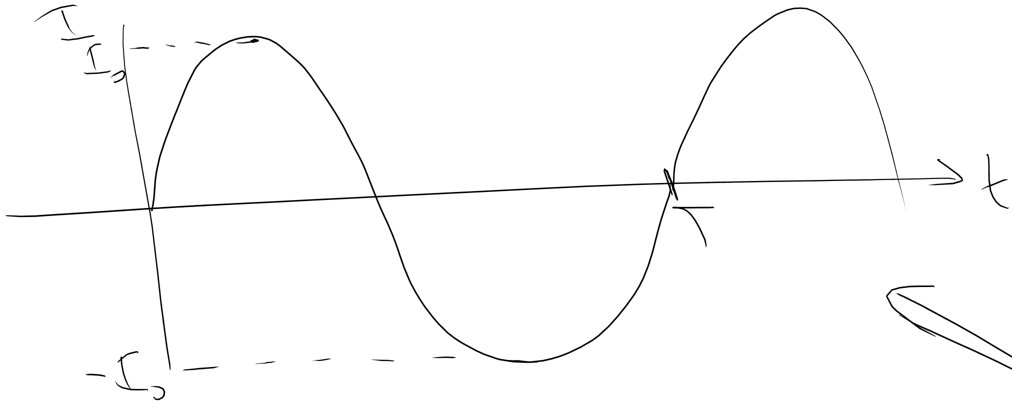
$$\cos\delta = 0 \Rightarrow \sin\delta = 1$$

$$A = Q_0$$

$$\boxed{Q(t) = Q_0\cos\left(\frac{1}{\sqrt{LC}}t\right)}$$

Electric pot "exists" if $\oint \vec{E}\cdot d\vec{l} = 0$

BUT $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$



Alternating Current (AC)

$$Q(t) = Q_0 \cos\left(\frac{1}{\sqrt{LC}} t\right)$$

$$I(t) = -\frac{dQ(t)}{dt} = \underbrace{\frac{Q_0}{\sqrt{LC}}}_{I_0} \sin\left(\frac{1}{\sqrt{LC}} t\right)$$

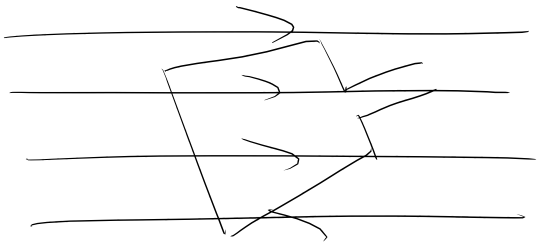
$$\boxed{T = \frac{2\pi}{\frac{1}{\sqrt{LC}}}}$$

$$U_C = \frac{Q^2}{2C} = \frac{Q_0^2}{2C} \cos^2\left(\frac{1}{\sqrt{LC}} t\right)$$

$$U_L = \frac{1}{2} L I^2 = \frac{1}{2} \cancel{L} \frac{Q_0^2}{\cancel{L}C} \sin^2\left(\frac{1}{\sqrt{LC}} t\right)$$

$$U_L = \frac{Q_0^2}{2C} \sin^2\left(\frac{1}{\sqrt{LC}} t\right)$$

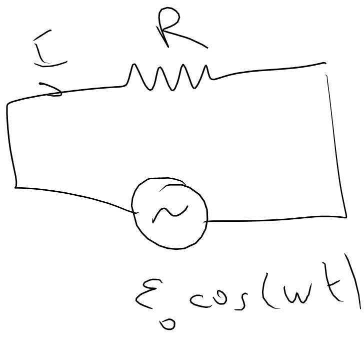
$$E = U_C + U_L = \frac{Q_0^2}{2C} = U_C(t=0)$$



$$\phi_B = \vec{B}\cdot\vec{A}$$
$$= BA\cos\theta$$
$$\theta(t) = \omega t + \delta$$
$$\phi_B(t) = BA\cos(\omega t+\delta)$$

$$\mathcal{E} = -\frac{d\phi_B}{dt} = \omega BA\sin(\omega t+\delta)$$

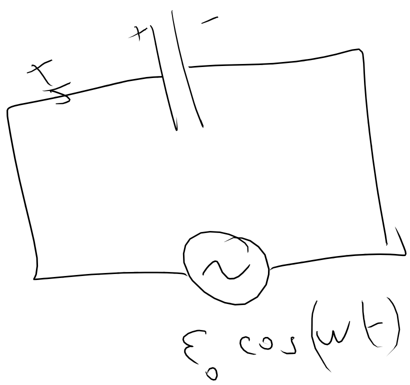
Example



$$IR + \mathcal{E}_0\cos(\omega t) = 0$$

$$\boxed{I = -\frac{\mathcal{E}_0}{R}\cos(\omega t)}$$

Example



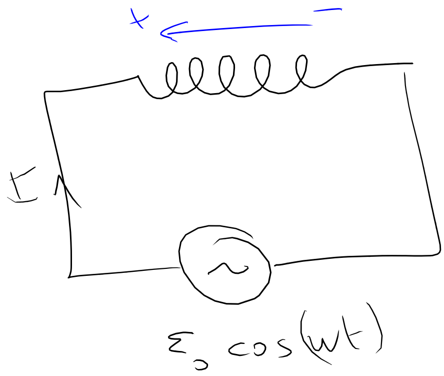
$$\frac{Q}{C} + \mathcal{E}_0\cos\omega t = 0$$
$$Q = -\mathcal{E}_0 C\cos\omega t$$
$$I = \frac{dQ}{dt} = +\mathcal{E}_0 C\omega\sin\omega t$$
$$I = +\frac{\mathcal{E}_0}{\left(\frac{1}{\omega C}\right)}\sin\omega t$$

$$I_{max} = \frac{\mathcal{E}_0}{\left(\frac{1}{\omega C}\right)} = \frac{\mathcal{E}_0}{X_c}$$

X_c : capacitative impedance.

$$I = -\frac{\mathcal{E}_0}{\left(\frac{1}{\omega C}\right)}\cos\left(\omega t + \frac{\pi}{2}\right)$$

Example



$$+L\frac{dI}{dt} + \mathcal{E}_0 \cos\omega t = 0$$

assume $\frac{dI}{dt} > 0$

$$\frac{dI}{dt} = -\frac{\mathcal{E}_0 \cos\omega t}{L}$$

$$I = -\frac{\mathcal{E}_0}{\omega L}\sin\omega t = -\frac{\mathcal{E}_0}{X_L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$I_{max} = \frac{\mathcal{E}_0}{\omega L} \equiv \frac{\mathcal{E}_0}{X_L}$$

$X_L = \omega L$: inductive impedance

$$I \neq \frac{\mathcal{E}}{X_L} \qquad I_{max} = \frac{\mathcal{E}_{max}}{X_L}$$

resistor

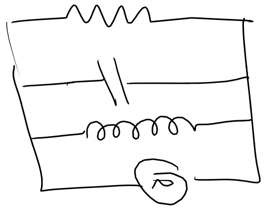
$$I = -\frac{\mathcal{E}_0}{X_R}\cos(\omega t + 0) \qquad X_R = R$$

capacitor

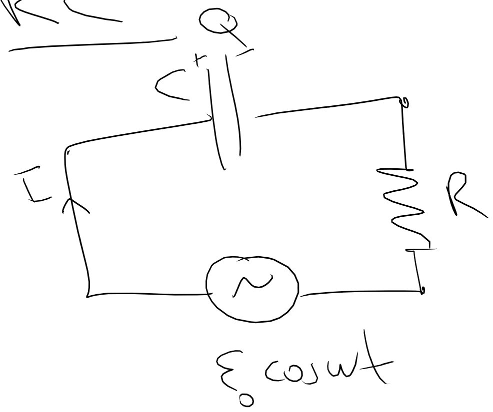
$$I = -\frac{\mathcal{E}_0}{X_C}\cos\left(\omega t + \frac{\pi}{2}\right) \qquad X_C = \frac{1}{\omega C}$$

inductor

$$I = -\frac{\mathcal{E}_0}{X_L}\cos\left(\omega t - \frac{\pi}{2}\right) \qquad X_L = \omega L$$



RC



$$\frac{Q}{C} + IR = \varepsilon_0 \cos\omega t$$

$$I = \frac{dQ}{dt}$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{\varepsilon_0}{R} \cos\omega t$$

$$Q = A\cos(\omega t + \delta)$$

$$-A\omega \sin(\omega t + \delta) + \frac{A\cos(\omega t + \delta)}{RC} = \frac{\varepsilon_0}{R}\cos(\omega t)$$

$$\sin(\omega t + \delta) = \sin\omega t \cos\delta + \cos\omega t \sin\delta$$

$$\cos(\omega t + \delta) = - - - -$$

$\omega t = 0$ $$-A\omega \sin\delta + \frac{A\cos\delta}{RC} = \frac{\varepsilon_0}{R}$$

$\omega t = \frac{\pi}{2}$! $$-A\omega\cos\delta - \frac{A\sin\delta}{RC} = 0$$

$$A = \frac{\varepsilon_0}{\sqrt{X_R^2 + X_C^2}} \equiv \frac{\varepsilon_0}{Z}$$