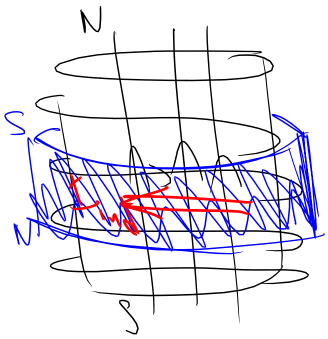
Hand in your HW.

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

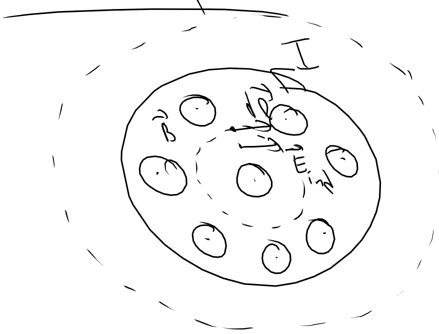


$B_i = 0$

$B_f \neq 0$

$\frac{dB}{dt} > 0$

Example



$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$\vec{E} \cdot d\vec{l} = E_{ind}\, dl$

$\oint \vec{E} \cdot d\vec{l} = \oint E_{ind}\, dl = E_{ind}\, 2\pi r$

$\phi_B = -AB = -\pi r^2 \mu_0 I n$

$E_{ind}\, 2\pi r = -\frac{d\phi_B}{dt} = \pi r^2 \mu_0 \frac{dI}{dt} n$

$$\boxed{E_{ind} = \frac{\mu_0 r n}{2} \frac{dI}{dt}}$$

$r < R$

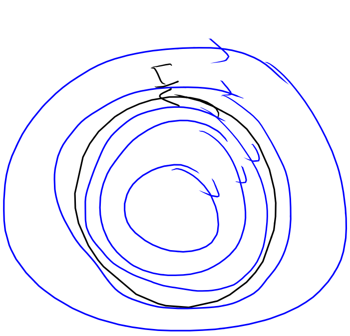
$\phi_B = \int \vec{B} \cdot d\vec{A}$

$= \int_{r<R} \vec{B} \cdot d\vec{A} + \int_{r>R} \vec{B} \cdot d\vec{A}$

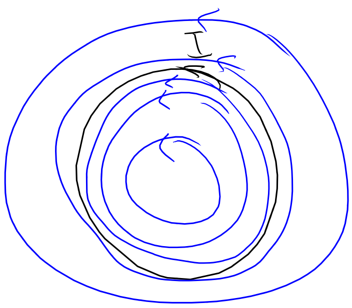
$= -B(\pi R^2)$

$E_{ind}\, 2\pi r = \pi R^2 \mu_0 n \frac{dI}{dt}$

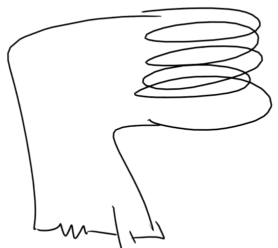
$$\Rightarrow \boxed{E_{ind} = \frac{\mu_0 R^2 n}{2r} \frac{dI}{dt}}$$



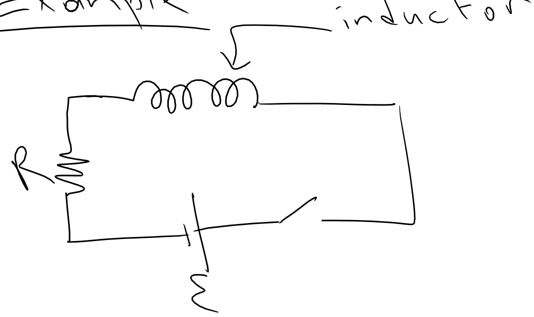
if $\frac{dI}{dt} > 0$

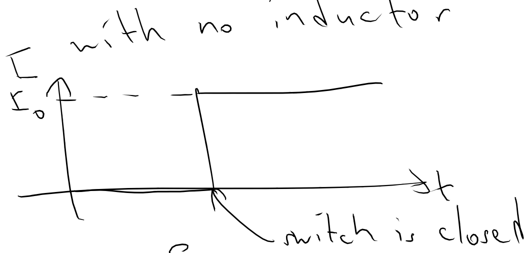


if $\frac{dI}{dt} < 0$

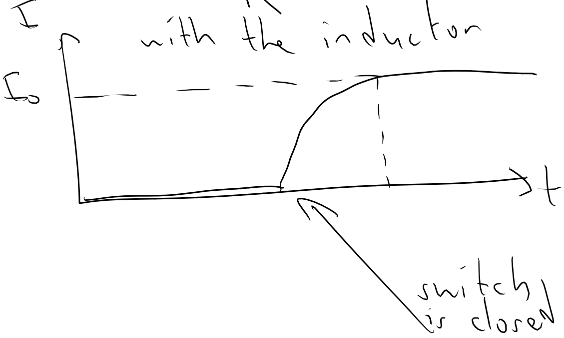


Example



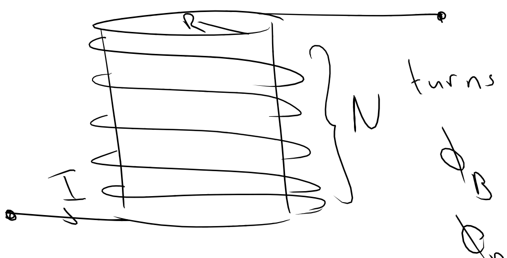


$$I_0 = \frac{\varepsilon}{R}$$



$$I_0 = \frac{\varepsilon}{R}$$

Inductor



$$\varepsilon = -\frac{d\Phi_B}{dt}$$

N turns

n: # of turns per unit length

$$\Phi_B = ?$$

$$\Phi_B = \pi R^2 B N$$

$$= \pi R^2 N (\underbrace{\mu_0 I n}_{B})$$

$$\Phi_B = \underbrace{(\pi R^2 N \mu_0 n)}_{L\text{: inductance (Henry)}} I$$

$$\Phi_B = L I$$

$$\boxed{\varepsilon = -L\frac{dI}{dt}}$$

$$P = \mathcal{E} I = L I \frac{dI}{dt}$$

$$W = \int P\,dt = \int L I \frac{dI}{\cancel{dt}} \cancel{dt} = \int_0^I L I\,dI = \frac{1}{2} L I^2$$

$$\boxed{U_{inductor} = \frac{1}{2} L I^2}$$

$$U_{capacitor} = \frac{1}{2} \frac{Q^2}{C} = \left(\frac{1}{2}\varepsilon_0 E^2\right) \text{volume of capacitor}$$

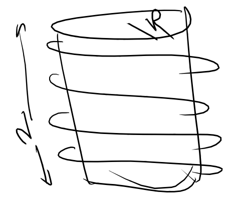
$$U_{ind} = \frac{1}{2}(\pi R^2 N \mu_0 n) I^2$$

$$= \frac{1}{2}(\pi R^2 N \cancel{\mu_0} \cancel{n}) \frac{B^2}{\mu_0^{\cancel{2}} n^{\cancel{2}}}$$

$$= \frac{1}{2} \frac{B^2}{\mu_0} (\pi R^2)\, \cancel{n} d \frac{1}{\cancel{n}}$$

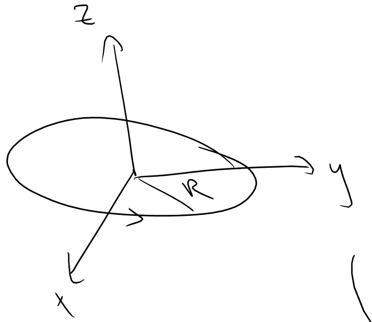
$$= \left(\frac{1}{2}\frac{B^2}{\mu_0}\right) \underbrace{\left(\pi R^2 d\right)}_{\text{volume of solenoid}}$$

$$B = \mu_0 n I$$



$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$: energy density stored in the magnetic field.

Quiz 5



$R = 5$ cm

disc lies on the xy plane

$$\vec{B} = (3\hat{x} + 4\hat{y} - 2\hat{z})\,T$$

$$\Phi_B = ?$$

(write a numerical expression)

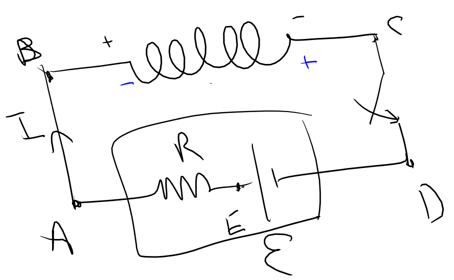
$$\vec{A} = \pi R^2 \hat{z}$$

$$\Phi_B = \vec{A}\cdot\vec{B} = (-2T)(3)(5\times10^{-2}\,m)^2$$

$$\simeq -150\ 10^{-4}\ Tm^2$$

$$\boxed{\Phi_B \simeq -1.5\ 10^{-2}\ Wb}$$

RL Circuits



swith is closed at $t=0$.

$I(t) = ?$

if $\frac{dI}{dt} > 0$; if $\frac{dI}{dt} < 0$

$$0 - \varepsilon + IR + 0 + L\frac{dI}{dt} = 0 \qquad \varepsilon = -L\frac{dI}{dt}$$

$$L\frac{dI}{dt} + IR = \varepsilon$$

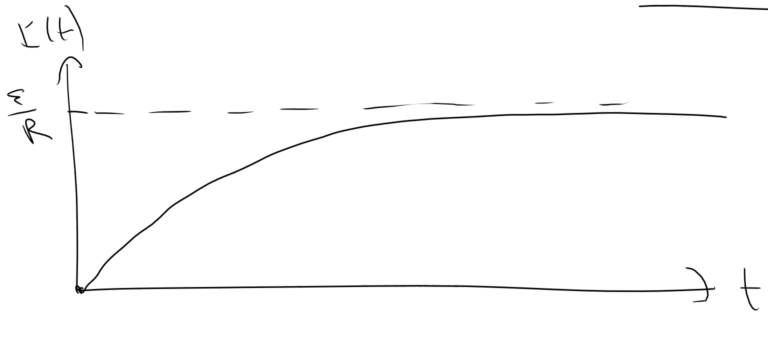
$$I(t) = \frac{\varepsilon}{R} + Ae^{-\frac{tR}{L}}$$

$$\frac{dI}{dt} = A\left(-\frac{R}{L}\right)e^{-\frac{tR}{L}}$$

$$L\frac{dI}{dt} + IR = -AR\cancel{e^{-t\frac{R}{L}}} + \left(\frac{\varepsilon}{\cancel{R}} + A\cancel{e^{-\frac{tR}{L}}}\right)\cancel{R} = \varepsilon$$

$$I(t=0) = 0$$
$$= \frac{\varepsilon}{R} + A \Rightarrow A = -\frac{\varepsilon}{R}$$

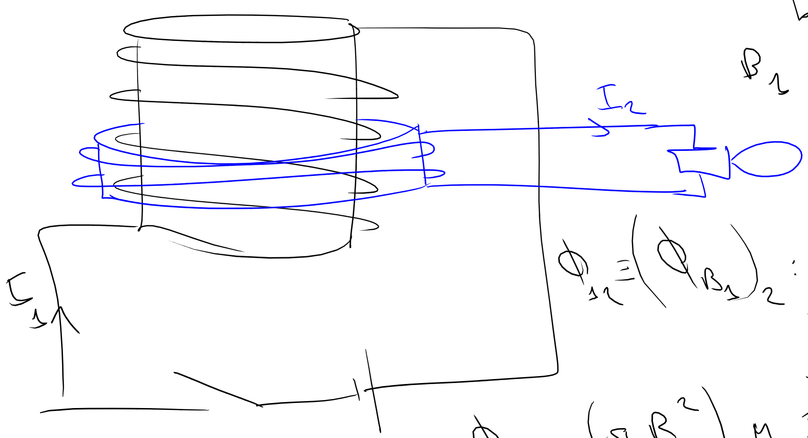
$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) \qquad \boxed{\tau = \frac{L}{R}}$$



$\tau = \frac{L}{R}$: time constant of the circuit

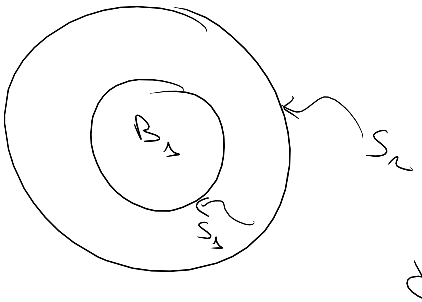
B_1: magnetic field created by I_1

$B_1 = \mu_0 I_1 n_1$



$\phi_{12} = (\phi_{B_1})_2$: flux created by B_1 in the second solenoid.

$$\phi_{12} = (\pi R_1^2)\, \mu_0 I_1 n_1 N_2$$
$$= (\pi R_1^2 \mu_0 n_1 N_2)\, I_1$$



ϕ_{12}: flux created in the second solenoid due to the current in the first solenoid.

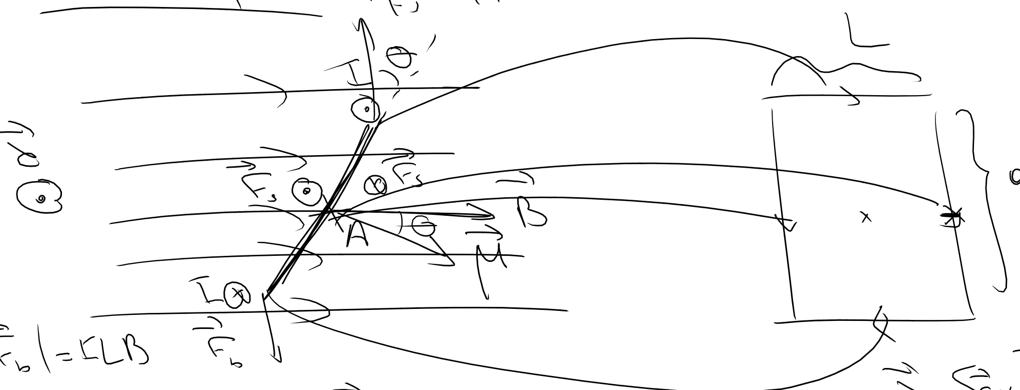
$$\phi_{12} = M I_1$$

M: mutual inductance

$$\boxed{\mathcal{E}_{2;1} = M \frac{dI_1}{dt}}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$
$$[\vec{E}] = [\vec{v}][\vec{B}]$$

Example $\vec{F}_t$, $|\vec{F}_t| = ILB$



$|\vec{F}_b| = ILB$ $\quad \vec{F}_b$

$$\vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i$$

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

if $\vec{B}$ is uniform $\quad \vec{F} = I\vec{L} \times \vec{B}$

$$\vec{\tau}_t = \vec{\tau}_b \; ; \; |\vec{\tau}_b| = \left(\frac{a}{2}\right) ILB\sin\theta = \frac{(IaL)}{2} B\sin\theta$$

$$\vec{\tau} = \vec{\tau}_t + \vec{\tau}_b = 2\vec{\tau}_b$$

$$\boxed{|\vec{\tau}| = (IaL)\, B \sin\theta}$$

$$(IaL) \equiv \mu$$

$$\boxed{\vec{\mu} = I\vec{A}}$$

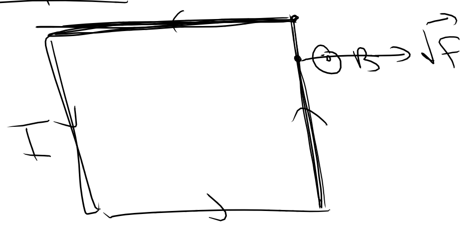
$$|\vec{\tau}| = \mu B \sin\theta$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

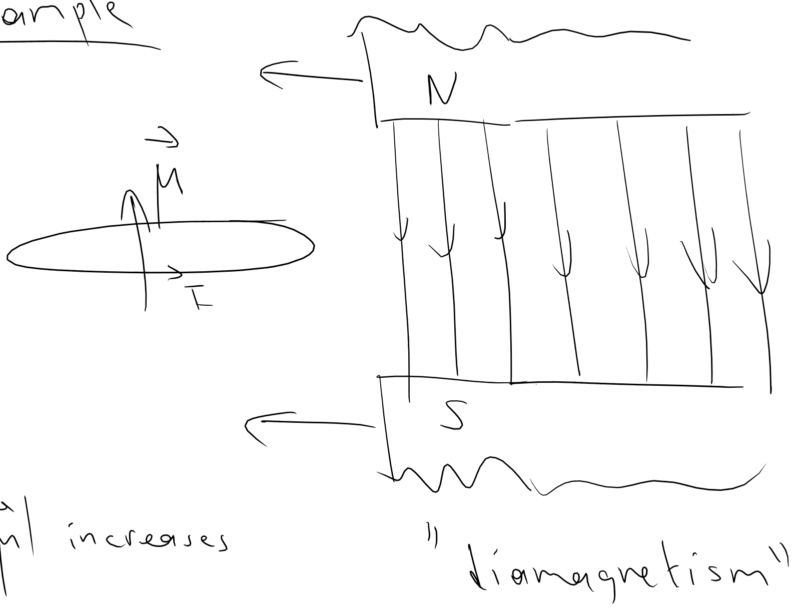
$$W = \int \tau \, d\theta \sim \int \sin\theta \, d\theta \sim \cos\theta$$

$$W \cancel{\propto} -\vec{\mu}\cdot\vec{B}$$

Example



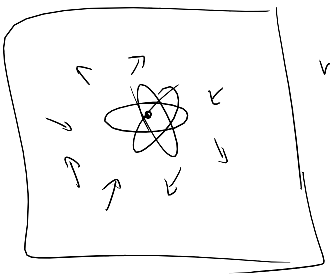
Example



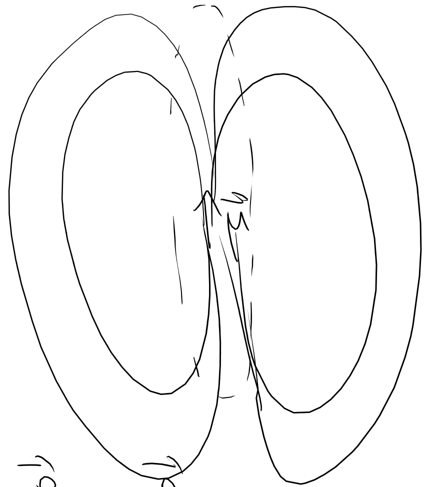
$|\vec{\mu}|$ increases

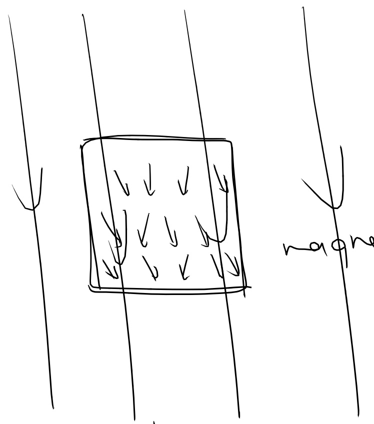
"diamagnetism"

(assume) Every atom has a non-zero mag. dipole moment



material





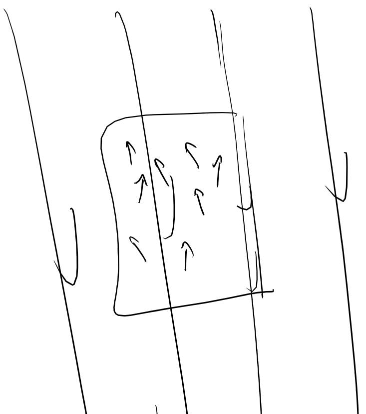
magnetized

$$\vec{B}_T^{\,inside} = \vec{B}_{ext} + \vec{B}_{dipoles}$$

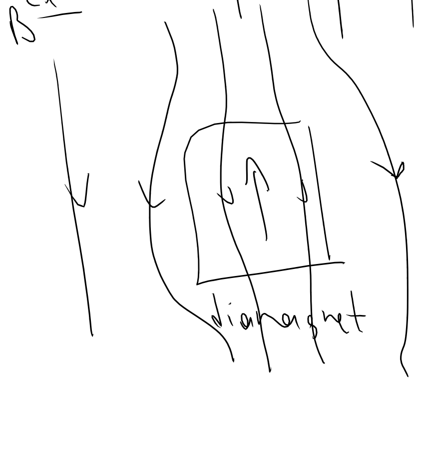
$$|\vec{B}_T^{\,inside}| > |\vec{B}_{ext}|$$

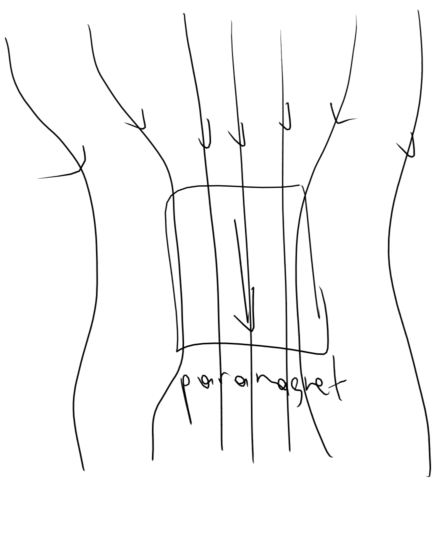
"paramagnetic material"

if the molecules/atoms have zero net mag. dipole moment

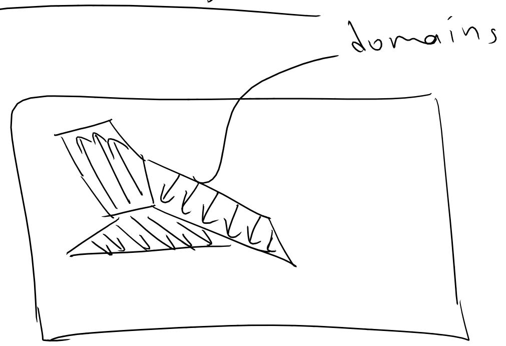


$$|\vec{B}_{inside}| < |\vec{B}_{ext}|$$





Ferromagnet



$$\frac{|\vec{B}_{dipl}|}{|\vec{B}_{ext}|} \sim 10^4$$