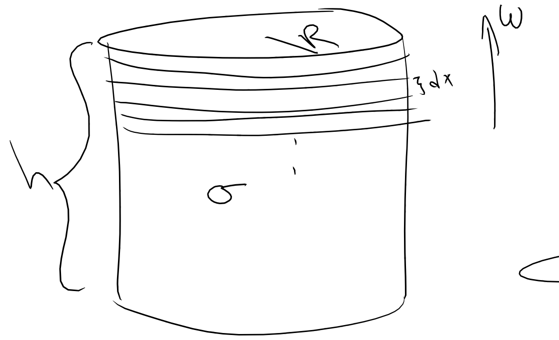
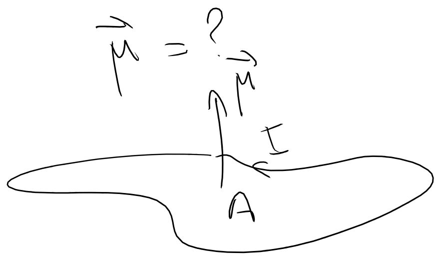
Example



σ: uniform surface charge density

$\vec{\mu} = ?$



$\mu = IA$

$dA = 2\pi R\, dx$

$dq = \sigma\, dA = 2\pi R \sigma\, dx$

$dI = \dfrac{dq}{\frac{2\pi}{\omega}} = \dfrac{\omega}{\cancel{2\pi}} \cancel{2\pi} R \sigma\, dx \Rightarrow \boxed{dI = \omega R \sigma\, dx}$

$d\mu = (dI)\, \pi R^2$

$d\mu = (\pi R^2)\, \omega R \sigma\, dx$

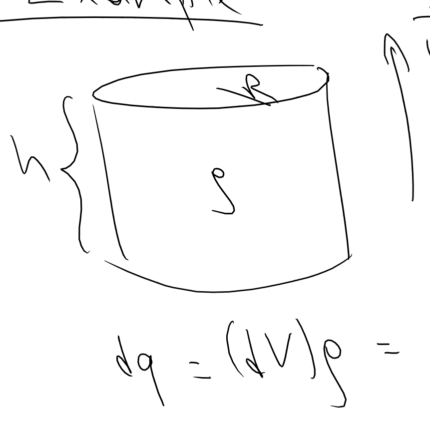
$\boxed{d\mu = \pi \omega R^3 \sigma\, dx}$

$\boxed{d\vec{\mu} = \pi R^3 \sigma\, dx\, \vec{\omega}}$

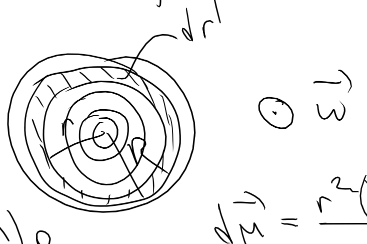
$\vec{\mu} = \sum d\vec{\mu} = \pi R^3 \sigma \vec{\omega} \sum dx$

$\boxed{\vec{\mu} = \pi R^3 h \sigma \vec{\omega}} = \cancel{\pi} R^{\cancel{3}\,2} \cancel{h} \dfrac{Q}{2\cancel{\pi} \cancel{R} \cancel{h}} \vec{\omega} = \dfrac{R^2 Q}{2} \vec{\omega}$

Example



ρ: volume charge density (charge per unit volume)



$dq = (dV)\rho = dr(2\pi r h)\rho$

$d\vec{\mu} = \dfrac{r^2 (2\pi r h \rho\, dr)}{\cancel{R}} \vec{\omega}$

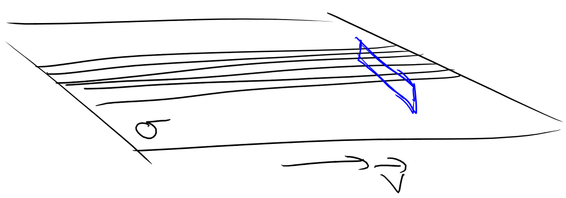
$$\vec{\mu} = \sum d\vec{\mu} = \pi h \rho \vec{\omega} \sum r^3 dr$$
$$= \pi h \rho \vec{\omega} \int_0^R r^3 dr$$
$$\vec{\mu} = \pi h \rho \vec{\omega} \frac{R^4}{4}$$
$$= \pi h \frac{R^{4}}{4} \frac{Q\vec{\omega}}{\pi R^2 h} = \boxed{\frac{R^2}{4} Q \vec{\omega} = \vec{\mu}}$$

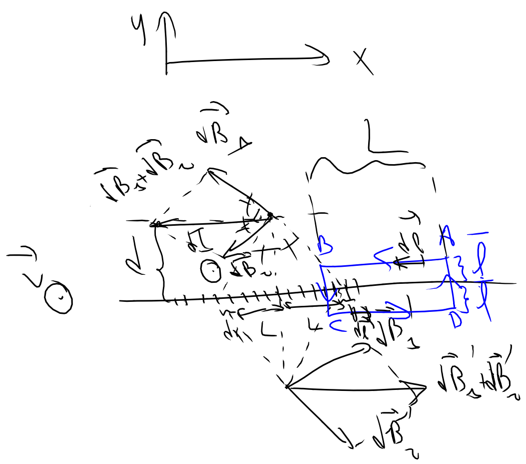
$\vec{\mu}$ $Q, R, \vec{\omega}$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Example





$$|\vec{B}| = B(d)$$
$$\vec{B}(\vec{r}) = B(|y|)(-\hat{x}) \quad \text{if } y > 0$$
$$\vec{B}(\vec{r}) = B(|y|)\,\hat{x} \quad \text{if } y < 0$$
$$\oint \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \cdot d\vec{l} + \cancel{\int_B^C \vec{B} \cdot d\vec{l}} + \int_C^D \vec{B} \cdot d\vec{l} + \cancel{\int_D^A \vec{B} \cdot d\vec{l}}$$

$\underline{A \to B}$ $d\vec{l} = dl(-\hat{x})$ $\vec{B} \cdot d\vec{l} = B(\bar{l})\, dl$

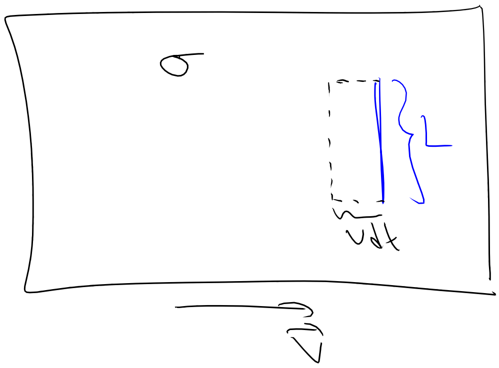
$$\int_A^B \vec{B} \cdot d\vec{l} = \int B(\bar{l})\, dl = B(\bar{l}) \int dl = B(\bar{l}) L$$

$\underline{C \to D}$ $\vec{B} \cdot d\vec{l} = B(\bar{l})\, dl \cos 0 = B(\bar{l})\, dl$

$$\int_C^D B(\bar{l})\, dl = B(\bar{l}) \int_C dl = B(\bar{l}) L$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = 2B(\bar{l}) L} = \mu_0 I_{enc}$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}}$$



$I_{enc} = \frac{dq}{dt}$

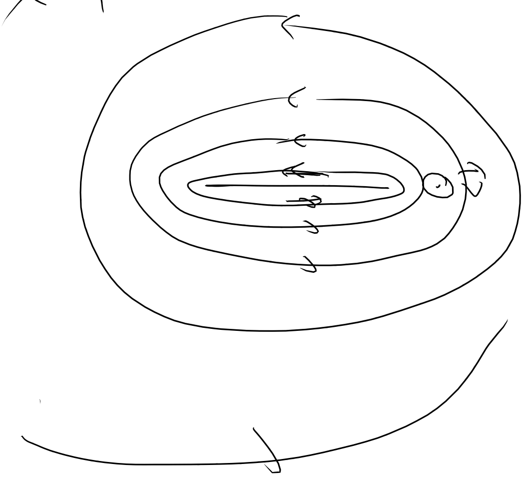
$dq = (L v\, dt)\sigma$

$I_{enc} = \frac{L v \sigma \cancel{dt}}{\cancel{dt}} = Lv\sigma$

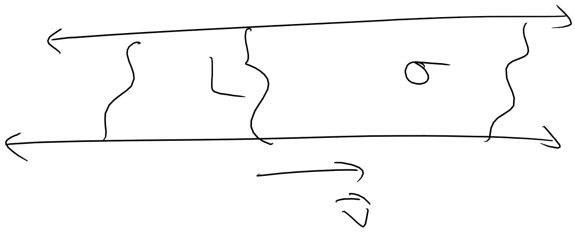
$2B(\bar{l})\cancel{L} = \mu_0 \cancel{L} v \sigma$

$$\boxed{B(\bar{l}) = \frac{\mu_0 v \sigma}{2}}$$

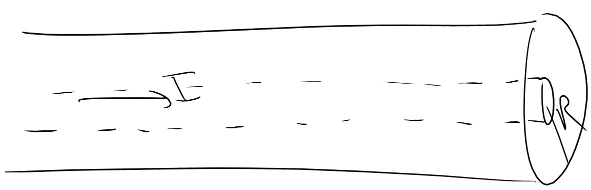
$\vec{B}$ is uniform above (below) the plane.



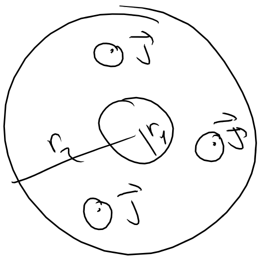
Example



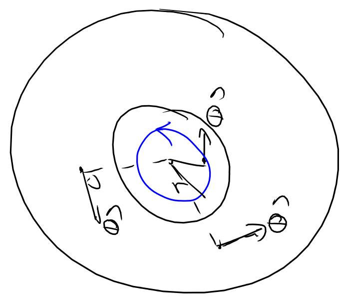
Example



I is uniformly distrubuted.



$J = \frac{I}{\pi(r_2^2 - r_1^2)}$



$$\vec{B} = B(r)\,\hat{\theta}(\vec{r})$$

$$\boxed{r<r_1}$$

$$\oint \vec{B}\cdot d\vec{l}$$

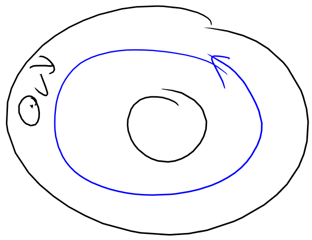
$$d\vec{l} = (dl)\,\hat{\theta}$$

$$\vec{B}\cdot d\vec{l} = B(r)\,dl$$

$$\oint \vec{B}\cdot d\vec{l} = \oint B(r)\,dl = B(r)\oint dl = B(r)\,2\pi r$$

If $r<r_1$; $I_{enc}=0 \Longrightarrow B(r)\,2\pi r = 0 \Longrightarrow B(r)=0$

If $r_1<r<r_2$



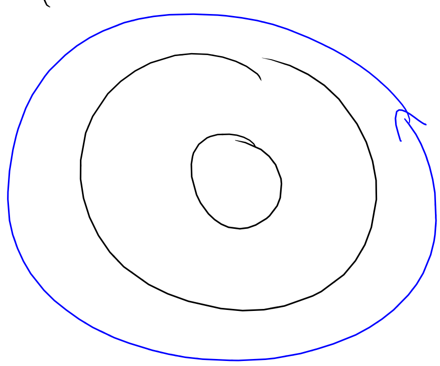
$$\oint \vec{B}\cdot d\vec{l} = B(r)\,2\pi r$$

$$I_{enc} = J\,(\pi r^2 - \pi r_1^2)$$

$$B(r)\,2\pi r = \mu_0 I_{enc} = \mu_0 \pi J\,(r^2 - r_1^2)$$

$$B(r) = \frac{\mu_0}{2\pi r}\cdot I\,\frac{r^2 - r_1^2}{r_2^2 - r_1^2}$$

If $r>r_2$



$$\oint \vec{B}\cdot d\vec{l} = B(r)\,2\pi r$$

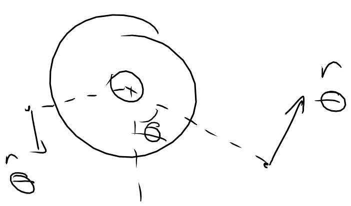
$$I_{enc} = I$$

$$B(r)\,2\pi r = \mu_0 I$$

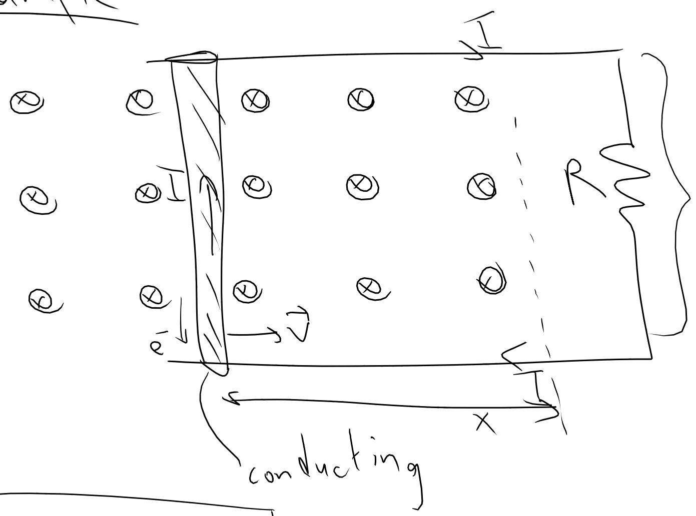
$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$J\pi(r_2^2 - r_1^2) = \underbrace{\frac{I}{\pi(r_2^2 - r_1^2)}}_{J}\,\pi(r_2^2 - r_1^2)$$

If $r>r_2$ $\quad \vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi r}\,\hat{\theta}$



Example



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB$$

$$W = FL$$

$$W = qvBL$$

$$\mathcal{E} = \frac{W}{q} = vBL$$

$$\boxed{\mathcal{E} = vBL}$$

$$v = \frac{dx}{dt}$$

$$\mathcal{E} = \left(\frac{dx}{dt}\right) BL = \frac{d}{dt}(B\,x\,L) = \frac{d}{dt}(BA) = \frac{d\Phi_B}{dt}$$

Φ_B : magnetic flux passing through an area

$$\boxed{\mathcal{E} = \frac{d\Phi_B}{dt}} \qquad \boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

$$\Phi_B = BA$$