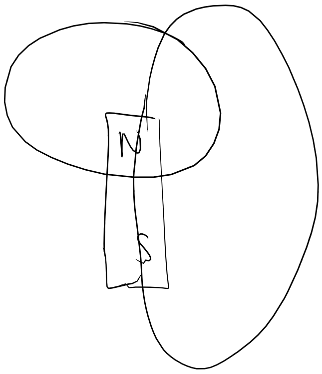
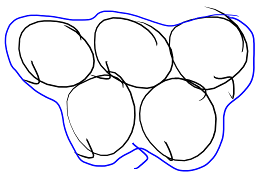
$\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$

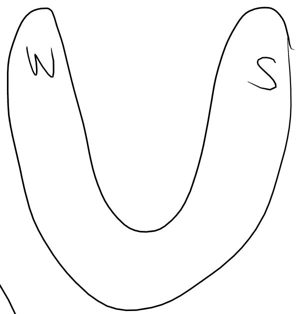
$d\vec{A} = d\vec{S}$ infinitesmall surface element

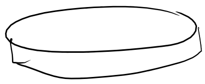
$\oint \vec{B}\cdot d\vec{S} = 0$

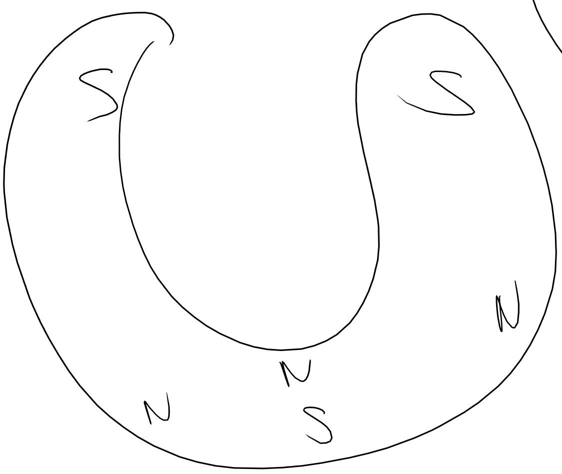
magnetic monopoles do not exist











$$\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

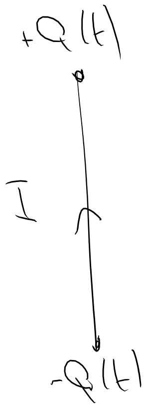
∂S: edge of the surface S (which is a closed loop)

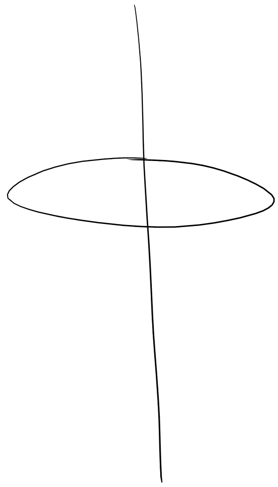
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

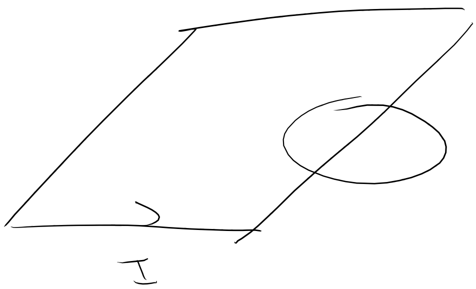
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\, d\vec{l}\times\hat{r}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow |\vec{B}| = \frac{\mu_0}{2\pi r} I$$

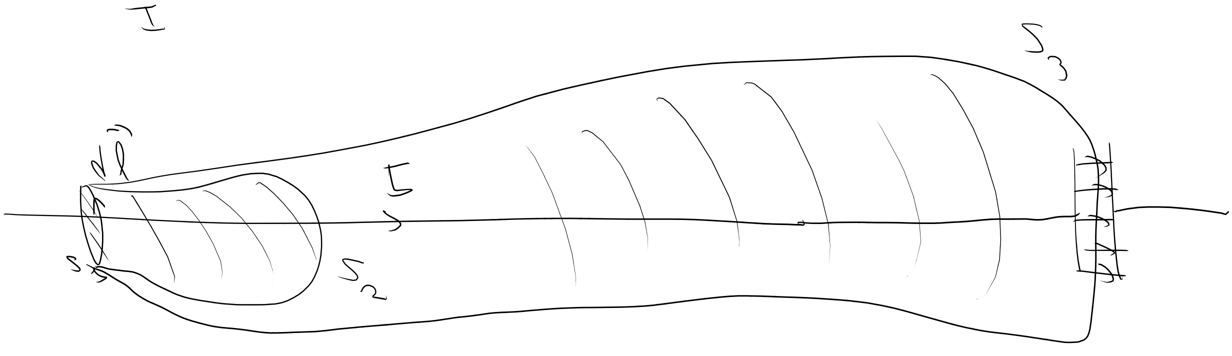
$$\oint \vec{B}\cdot d\vec{l} = 2\pi r B$$







$$\oint \vec{B}\cdot d\vec{l} \neq 2\pi r B$$



$$\oint_{\partial S} \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 \int_S \vec{J}\cdot d\vec{A}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

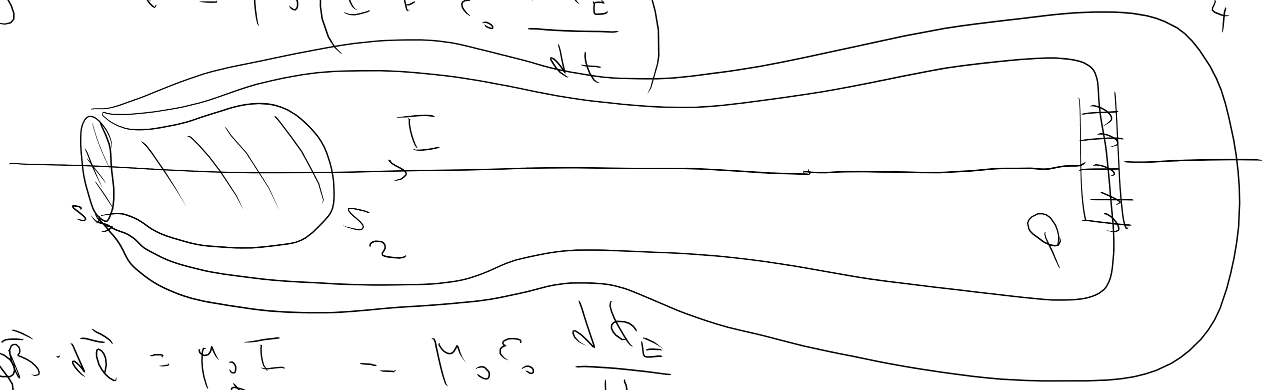
$$Q = (AE)\,\varepsilon_0$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$I = \frac{dQ}{dt} = \varepsilon_0 \frac{d}{dt}(AE)$$

$$I = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$



$$\oint \vec{B}\cdot d\vec{l} = \underset{S_1, S_2}{\mu_0 I} \quad \text{--} \quad \underset{S_3}{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \underset{\substack{\text{passing}\\ \text{through}\\ \text{surface}}}{I} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Faraday-Maxwell's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{A} + \mu_0 \varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

$\parallel$

$$\mu_0 I = \mu_0 I + \mu_0 \varepsilon_0 \cancel{\frac{d\Phi_E^{part}}{dt}} + \cancel{\mu_0 I_{plate}}$$

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint \vec{B}\cdot d\vec{S} = 0$$

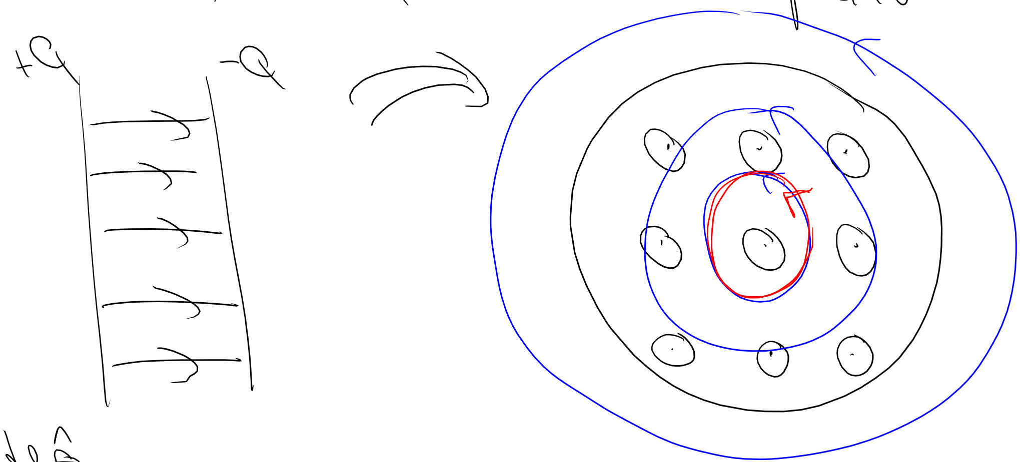
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

displacement current

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

<u>Example</u> Magnetic field inside the capacitor



$$d\vec{l} = dl\,\hat{\theta}$$
$$\vec{B} = B(r)\,\hat{\theta}$$

$$\oint \vec{B}\cdot d\vec{l} = \oint B(r)\,dl = B(r)\,2\pi r$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{\text{passing through the surface}} + \varepsilon_0 \frac{d\Phi_E}{dt}\right) = \mu_0 \varepsilon_0 \pi r^2 \dot{E}$$

$$\boxed{B(r) = \frac{\mu_0 \varepsilon_0 \pi r^2}{2\pi r}\frac{dE}{dt} = \frac{\mu_0\varepsilon_0}{2}\left(\frac{dE}{dt}\right) r}$$

$r < R$

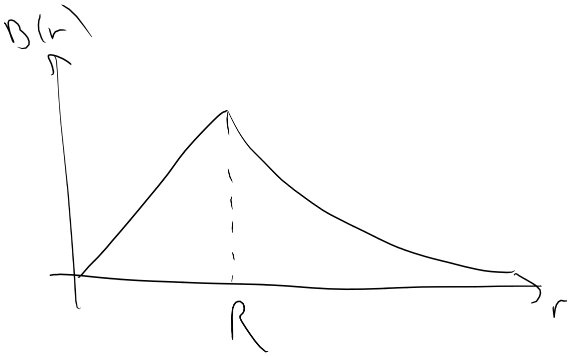
$r > R$

$$2\pi r\, B(r) = \mu_0 \varepsilon_0 \frac{d}{dt}(A E) \qquad A = \pi R^2$$

$$\boxed{B(r) = \frac{\mu_0 \varepsilon_0}{2\pi r} A \left(\frac{dE}{dt}\right)}$$

$$= \frac{\mu_0}{2\pi r} \frac{d}{dt}\left(\cancel{\varepsilon_0} A \frac{Q}{\cancel{\varepsilon_0}}\right)$$

$$= \frac{\mu_0}{2\pi r} \frac{dQ}{dt}$$

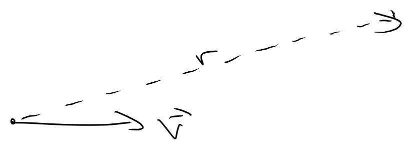


$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt} \oint \vec{E} \cdot d\vec{S}$$

$$= \int \underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\vec{J}_d} \cdot d\vec{S}$$

Example

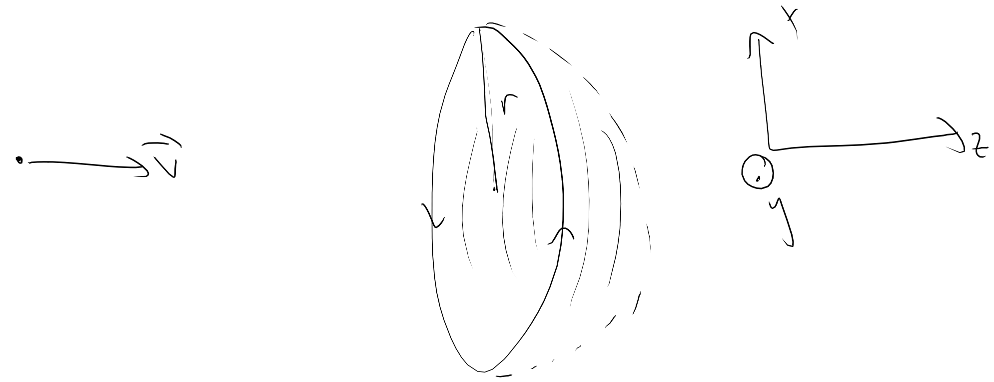


$$\vec{E}^{(0)} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

$$\vec{E} = \vec{E}^{(0)} + \vec{E}^{(1)} + \vec{E}^{(2)} + \cdots \qquad \vec{B}^{(n)} \propto v^n$$

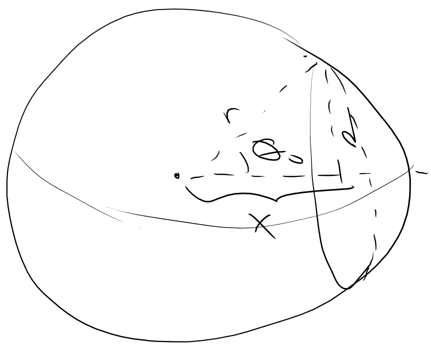
$\vec{B} = \vec{B}^{(0)} + \vec{B}^{(1)} + \vec{B}^{(2)} + \dots$

$\vec{B}^{(0)} = 0$



$\oint \vec{B} \cdot \vec{dl} = B(r,z) \int dl = B(r,z)\, 2\pi r$

$\Phi_E = ?$



$\vec{E} \parallel d\vec{A}$

$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \int dA$

$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} 2\pi \int_0^{\theta_0} \sin\theta \, d\theta$

$\Phi_E = \frac{q}{2\varepsilon_0} (1 - \cos\theta_0)$

$r = \sqrt{x^2 + d^2}$

$\frac{d\Phi_E}{dt} = \frac{q}{2\varepsilon_0} \sin\theta_0 \frac{d\theta_0}{dt}$

$\sin\theta_0 = \frac{d}{r}$

$\cos\theta_0 \frac{d\theta_0}{dt} = -\frac{d}{r^2} \frac{dr}{dt} = -\frac{d}{r^2} \frac{x}{\sqrt{x^2+d^2}} \overbrace{\frac{dx}{dt}}^{-v}$

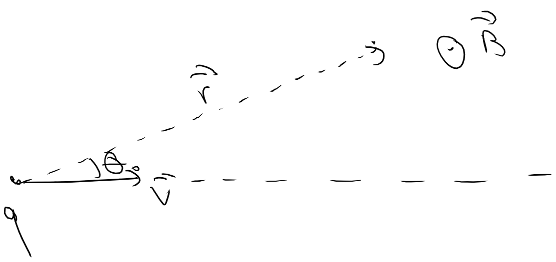
$\cancel{\cos\theta_0} \frac{d\theta_0}{dt} = \frac{dv}{r^2} \cancel{\cos\theta_0}$

$$\boxed{\frac{d\Phi_E}{dt} = \frac{q}{2\varepsilon_0} \frac{vd}{r^2} \sin\theta_0}$$

$$\frac{d\Phi_E}{dt} = \frac{q}{2\varepsilon_0}\frac{v\,d}{r^2}\sin\theta_0$$

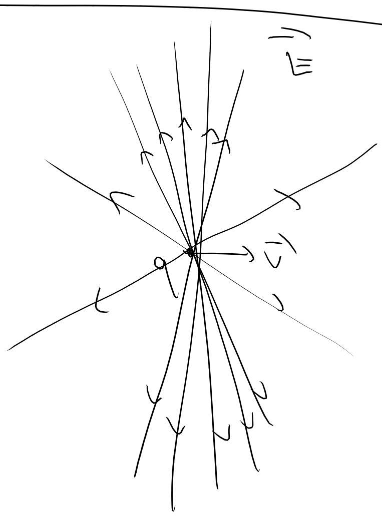
$$B\,2\pi d = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} = \mu_0\cancel{\varepsilon_0}\frac{q v}{2\cancel{\varepsilon_0}}\frac{d}{r^2}\sin\theta_0$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{q v \sin\theta}{r^2}}$$

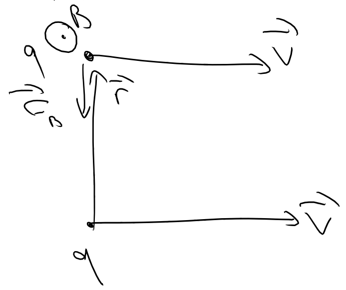


$$|\hat{r}\times\vec{v}| = 1\,v\sin\theta_0$$

$$\boxed{\vec{B}^{(1)} = \frac{\mu_0}{4\pi}\,q\,\frac{\vec{v}\times\hat{r}}{r^2}}$$



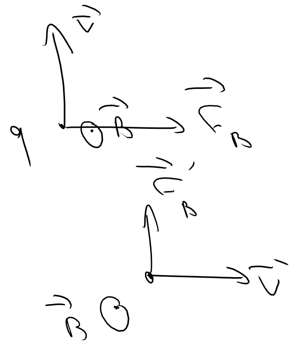
Example



$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$

$$\vec{B}^{(1)} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}$$

Example



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$