$$U_{mech} = \sum_i \left(\frac{1}{2} m_i v_i^2 + U_i\right)$$

U_i: potential energy that is <u>NOT</u> due to electromagnetic forces.

$$\frac{dU_{mech}}{dt} = P_{e.m.} = \int \vec{E} \cdot \vec{J} \, dV$$

$$= -\frac{d}{dt} U_{e.m.} - \int \vec{S} \cdot d\vec{A}$$

$$\boxed{\frac{d}{dt}\left(U_{mech} + U_{e.m}\right) = -\oint \vec{S} \cdot d\vec{A}}$$

$$U_{e.m} = \int_V dV \left(\frac{1}{2} \varepsilon_0 \vec{E}^2 + \frac{1}{2\mu_0} \vec{B}^2\right)$$

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$ Poynting's Vector

$$\vec{P}_{mech} = m\vec{v} = \int \vec{F} \cdot dt$$

$$\frac{d\vec{P}_{mech}}{dt} = \vec{F}$$

$$m \cdot \!\rightarrow \vec{v} \quad \xrightarrow{\text{later time}} \quad \cdots \quad \cdot \!\rightarrow \vec{v}$$

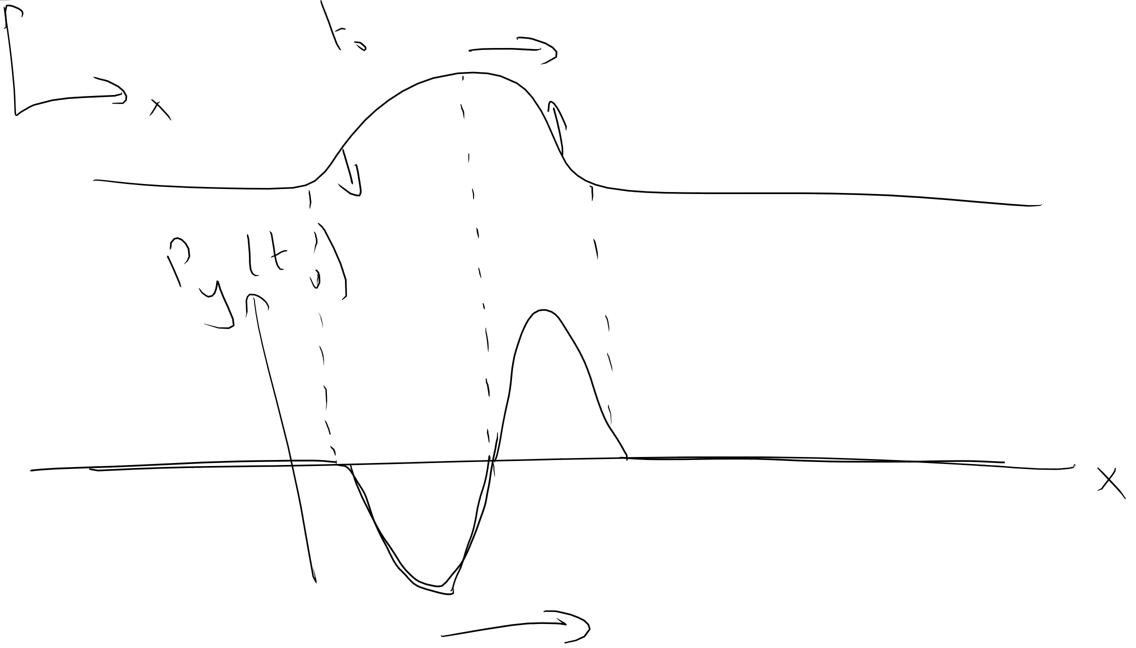
$$E = \frac{1}{2} m v^2 \qquad m \sim \frac{E}{c^2}$$

$$mc \sim \frac{E}{c}$$

$$\frac{dP^i_{mech}}{dt} = -\frac{dP^i_{e.m.}}{dt} - \oint \vec{T}^i \, d\vec{A}$$

$\vec{T}^i$ is a vector for $i = x, y, z$

Wave on a string



$$\frac{dP^i_{mech}}{dt} = F^i = \sum q_i (\vec{E}_i + \vec{v}_i \times \vec{B})^i$$
$$= \int dV (\rho \vec{E} + \vec{J} \times \vec{B})^i$$

$\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$ $\qquad$ $\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Leftarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$(\rho = \varepsilon_0 \vec{\nabla} \cdot \vec{E})$

$\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\frac{dP^i_{mech}}{dt} = \int dV \left\{ \vec{E}\left(\varepsilon_0 \vec{\nabla}\cdot\vec{E}\right) + \frac{1}{\mu_0}\left[\vec{\nabla}\times\vec{B} - \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right]\times\vec{B}\right\}^i$$

$$\frac{\partial\vec{E}}{\partial t}\times\vec{B} = \frac{\partial}{\partial t}\left(\vec{E}\times\vec{B}\right) - \vec{E}\times\frac{\partial\vec{B}}{\partial t}$$

$$= \frac{\partial}{\partial t}\left(\vec{E}\times\vec{B}\right) + \vec{E}\times\left(\vec{\nabla}\times\vec{E}\right)$$

$$\frac{dP^i_{mech}}{dt} = -\mu_0\varepsilon_0\frac{d}{dt}\int dV\left(\frac{\vec{E}\times\vec{B}}{\mu_0}\right)^i$$

$$+\int dV\left[-\varepsilon_0\vec{E}\times\left(\vec{\nabla}\times\vec{E}\right) + \frac{1}{\mu_0}\left(\vec{\nabla}\times B\right)\times\vec{B} + \varepsilon_0\vec{E}\left(\vec{\nabla}\cdot\vec{E}\right) + \frac{1}{\mu_0}\vec{B}\left(\vec{\nabla}\cdot\vec{B}\right)\right]^i$$

$$\vec{P}_{e.m} = \int \mu_0\varepsilon_0\vec{S}\,dV \qquad \mu_0\varepsilon_0 \sim \frac{1}{c^2}$$

$$\frac{d}{dt}\left(P^i_{mech} + P^i_{e.m}\right) =$$

$$\int dV\left[\varepsilon_0\left(\vec{\nabla}\times\vec{E}\right)\times\vec{E} + \varepsilon_0\vec{E}\left(\vec{\nabla}\cdot\vec{E}\right) + \frac{1}{\mu_0}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B} + \frac{1}{\mu_0}\vec{B}\left(\vec{\nabla}\cdot\vec{B}\right)\right]^i$$

$$\overset{?}{=} \int dV\left(\vec{\nabla}\cdot\vec{X}\right) \qquad \vec{X} = ?$$

$$\left[(\vec{\nabla}\times\vec{E})\times\vec{E}+\vec{E}\,(\vec{\nabla}\cdot\vec{E})\right]^i$$

$$\varepsilon_{ijk}(\vec{\nabla}\times\vec{E})_j E_k + E_i\,\partial_i E_j$$

$$= \varepsilon_{ijk}\,\varepsilon_{jlm}(\partial_l E_m)E_k + E_i\,\partial_j E_j$$

$$= -(\delta_{il}\delta_{km}-\delta_{im}\delta_{kl})(\partial_l E_m)E_k + E_i\,\partial_j E_j$$

$$= -(\partial_i E_k)E_k + (\partial_l E_i)E_l + E_i\,\partial_j E_j$$

$$= -(\partial_i E_j)E_j + (\partial_j E_i)E_j + E_i\,\partial_j E_j$$

$$= -\partial_i \frac{1}{2}(E_j)^2 + \partial_j(E_i E_j)$$

$$= -\partial_i \frac{1}{2}\vec{E}^2 + \partial_j(E_i E_j)$$

$$= -\partial_j\,\delta_{ij}\,\frac{1}{2}\vec{E}^2 + \partial_j(E_i E_j)$$

$$= -\partial_j\left[\delta_{ij}\,\frac{1}{2}\vec{E}^2 - E_i E_j\right]$$

$$\frac{d}{dt}\left(P^i_{mech}+P^i_{e.m}\right) =$$

$$\int dV\left[\varepsilon_0(\vec{\nabla}\times\vec{E})\times\vec{E}+\varepsilon_0\vec{E}(\vec{\nabla}\cdot\vec{E})\right.$$

$$\left.+\frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})\times\vec{B}+\frac{1}{\mu_0}\vec{B}\,(\vec{\nabla}\cdot\vec{B})\right]^i$$

$$= -\int dV\,\partial_j\left[\varepsilon_0\left(\delta_{ij}\,\frac{1}{2}\vec{E}^2 - E_i E_j\right)\right.$$

$$\left.+\frac{1}{\mu_0}\left(\delta_{ij}\,\frac{1}{2}\vec{B}^2 - B_i B_j\right)\right]$$

$$= -\int dV\, \vec{\nabla} \cdot \vec{T}^{i}$$

$$T^{ij} = (\vec{T}^{i})^{j} = \delta_{ij}\left(\frac{1}{2}\varepsilon_0 \vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2\right) - \varepsilon_0 E_i E_j - \frac{1}{\mu_0} B_i B_j$$

$$\frac{d}{dt}\left(P^{i}_{mech} + P^{i}_{em}\right) = -\int d\vec{A} \cdot \vec{T}^{i}$$

$$\mathrm{Tr}\begin{pmatrix} T^{xx} & T^{xy} & T^{xz} \\ T^{yx} & T^{yy} & T^{yz} \\ T^{zx} & T^{zy} & T^{zz} \end{pmatrix} = T^{xx} + T^{yy} + T^{zz} = \delta^{ij} T^{ij}$$

$$= 3\left(\frac{1}{2}\varepsilon_0 \vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2\right) - \varepsilon_0 \vec{E}^2 - \frac{1}{\mu_0}\vec{B}^2$$

$$\boxed{\mathrm{Tr}\, T = \frac{1}{2}\varepsilon_0 \vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2}$$

$$\delta_{ij}^2 = 3$$

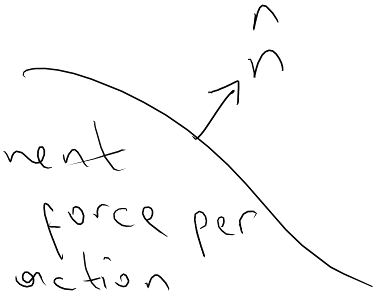
$$\vec{\nabla} \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$\frac{d}{dt}\left(P^i_{mech} + P^i_{e.m}\right) = -\int d\vec{A}\cdot\vec{T}^i$$
$$= -\int dA^j\, T^{ij}$$

T^{ij}

T^{xx}, T^{yy}, T^{zz}

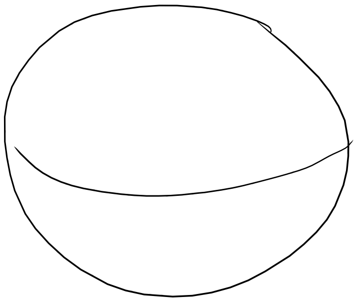
$T^{ij}\, n^j$: i component of the force per area acting on the surface.



analogy $\vec{J}\cdot\hat{n}$

e.m pressure $p \equiv T^{ij} n^i n^j$

Ex



conducting charged sphere.

Ex