$$\frac{d}{dt}\int_V \rho(\vec{r},t)\, d^3r = -\int_{\partial V} \vec{J}\cdot d\vec{A} = -\int_{\partial V} J^i\, dA^i$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = -\vec{\nabla}\cdot\vec{J} = -\partial_i v_i$$

$$\frac{\partial \rho}{\partial t} = -\partial_i J_i$$

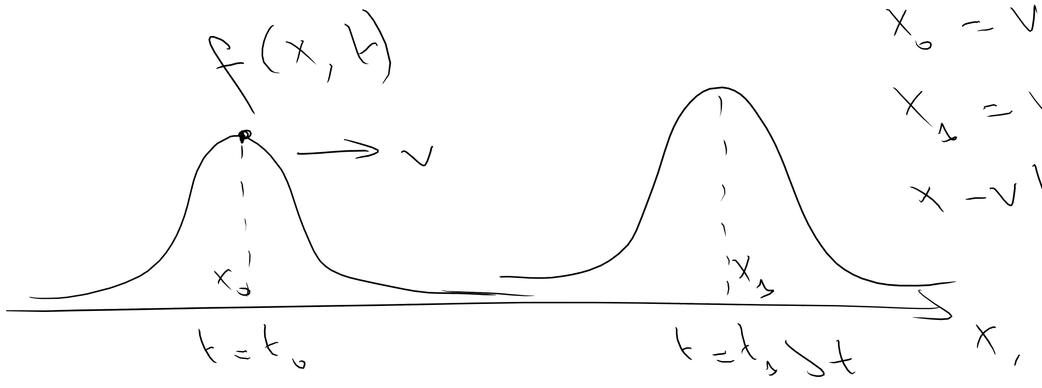
$$p = T^{ij} n^i n^j = \vec{n}\cdot\overleftrightarrow{T}\cdot\vec{n}$$

$$T^{ij} = T^{ji}$$

$$\vec{P}_{em} = \mu_0 \varepsilon_0 \frac{(\vec{E}\times\vec{B})}{\mu_0}$$

$$\frac{d}{dt}\left(\vec{P}_{mech} + \mu_0\varepsilon_0 \int dV\, \vec{S}\right) = -\int dA^i\, T^{ij}$$

## Waves

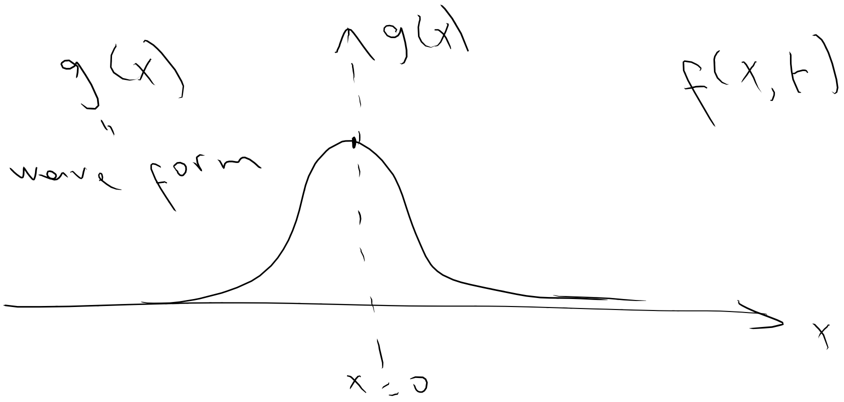



$$x_0 = v t_0$$
$$x_1 = v t_1$$
$$x - vt = 0$$

$$f(x_0,t_0) = f(x_1,t_1) = g(0) = g(x_0 - vt_0)$$
$$= g(x_1 - vt_1)$$

$g(x)$

wave form

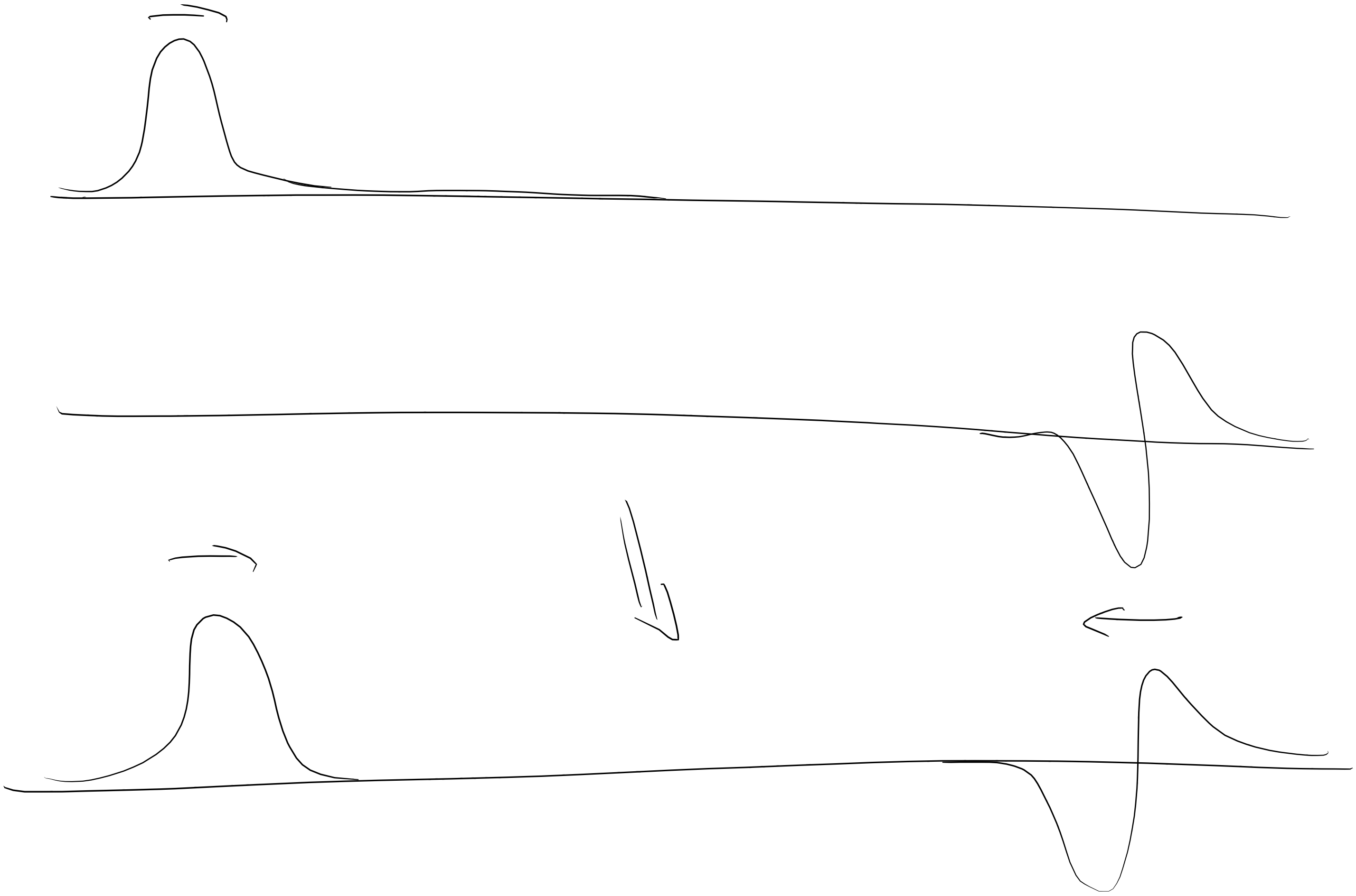



$$f(x,t) = g(x-vt)$$

$$f(x,t) = g(x-vt) + h(x+vt)$$

wave moving in the +x direction

wave moving in the -x direction.



$\vec{P} =$

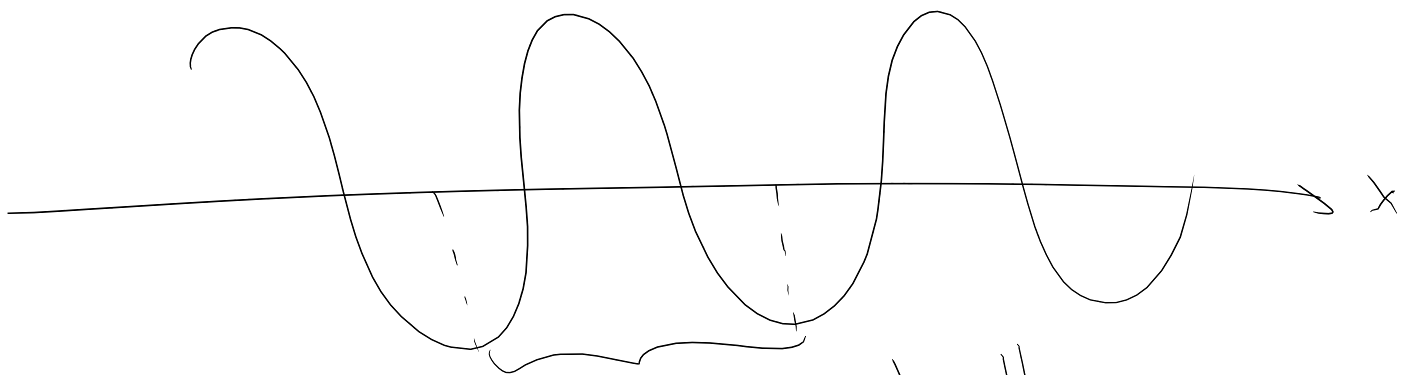
$$P_y = \varepsilon_{yx} E_x + \varepsilon_{yy} E_y + \varepsilon_{yz} E_z$$

$$P^i(\vec{E}) = \frac{\partial P^i}{\partial E_j} E_j + \frac{\partial P^i}{\partial E_j \partial E_k} \frac{E_j E_k}{2} + \cdots$$

## Harmonic waves

$$\left.\begin{array}{l} \cos\left[\underline{k}(x \pm vt)\right] \\ \sin\left[\underline{k}(x \mp vt)\right] \end{array}\right\} \quad e^{\mp i\left[k(x \pm vt)\right]}$$

k: wave vector



$\lambda$ : wave length

$$\begin{array}{c}\sin\\ \cos\end{array}\left(k\left[(x+\lambda) \mp vt\right]\right) = \begin{array}{c}\sin\\ \cos\end{array}\left(k\left[x \mp vt\right]\right)$$

$$\Rightarrow k\lambda = 2\pi$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

$$\sin\left(k\left[x \mp v(t+T)\right]\right) = \sin\left(k\left[x \mp vt\right]\right)$$

$$kvT = 2\pi \Rightarrow T = \frac{2\pi}{(kv)}$$

$$\nu = \frac{1}{T} = \frac{kv}{2\pi} \text{ : frequency}$$

$kv = 2\pi\nu \equiv \omega$ : angular frequency analogy


$v$: constant

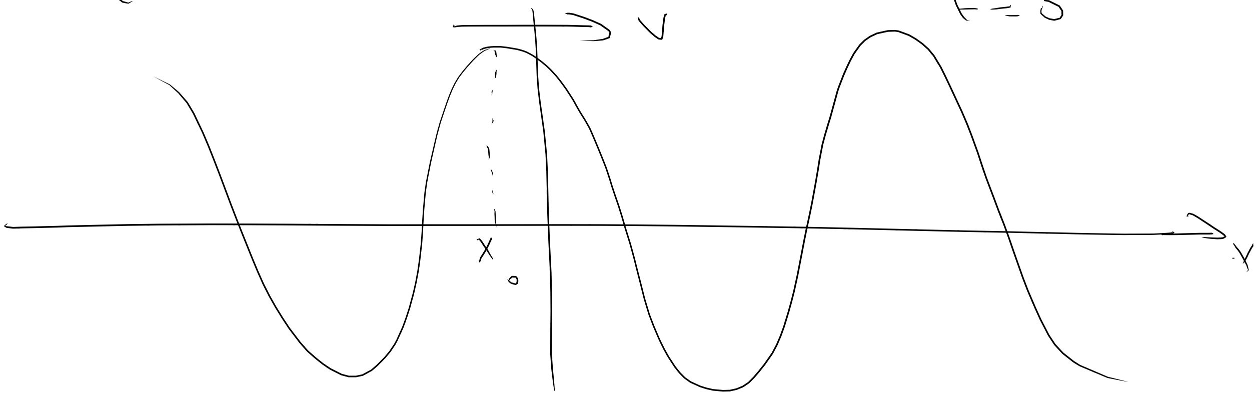
$\theta = wt$

$x = R\cos(wt)$

$$\cos(kx \mp kvt) \equiv \cos(kx \mp wt)$$

$$\cos(kx - wt + \delta)$$

$t = 0$



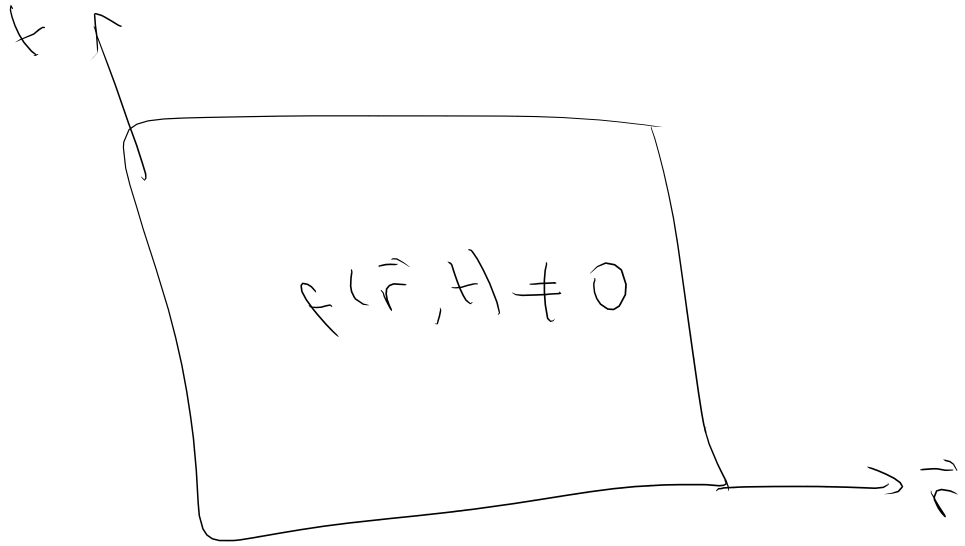

$$x_0 = -\frac{\delta}{k}$$

$\delta$: phase-shift

$$f(x,t) = \int_{-\infty}^{\infty} dk\, e^{i(kx - wt)}\, g(k)$$

$$f(\vec{r},t) = \int d^3k\, dw\, e^{i(\vec{k}\cdot\vec{r} - wt)}\, g(\vec{k},w)$$

$$g(\vec{k},w) = \int \frac{d^3r\, dt}{(2\pi)^4}\, e^{-i(\vec{k}\cdot\vec{r} - wt)}\, f(\vec{r},t)$$

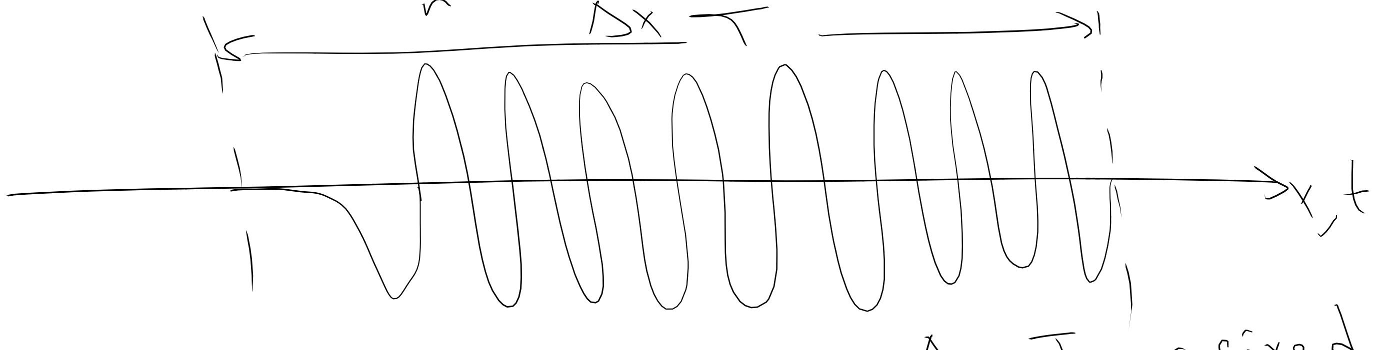




$$f(x) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{L} x\right) a_n$$

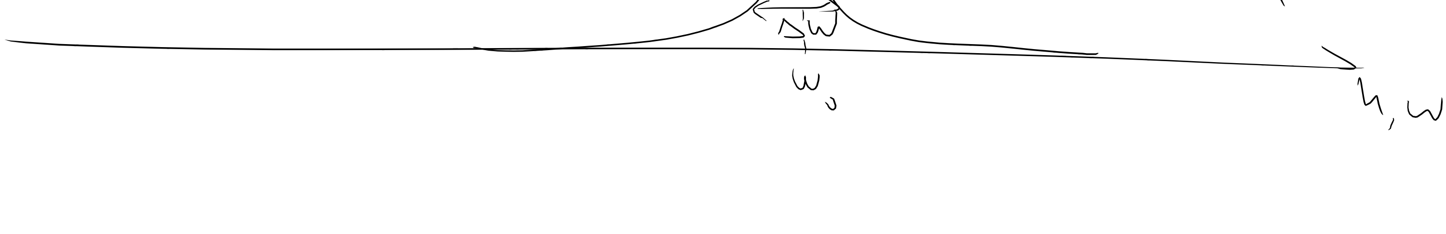
$$= \sum_{n} \cos(\quad) b_n$$

$$= \sum_{n} \sin\left(\frac{n\pi}{L'} x\right) a'_n$$




$\Delta\omega T \sim$ fixed

$\Delta k \Delta x \sim$ fixed



$$f(x,t) = g(x-vt) + h(x+vt)$$

$$\boxed{\frac{\partial f}{\partial x} = g'(x-vt) + h'(x+vt)}$$

$$g'(x-vt) = \left.\frac{dg}{dy}\right|_{y=x-vt}$$

$$\frac{\partial g(y)}{\partial x} = \left.\frac{dg}{dy}\right|_{y=x-vt} \frac{\partial y}{\partial x} = g'(x-vt)\,1$$

$$\frac{\partial f}{\partial t} = (-v)\,g'(x-vt) + v\,h'(x+vt)$$

$$\frac{\partial^2 f}{\partial t^2} = (-v)^2\,g''(x-vt) + v^2\,h''(x+vt)$$

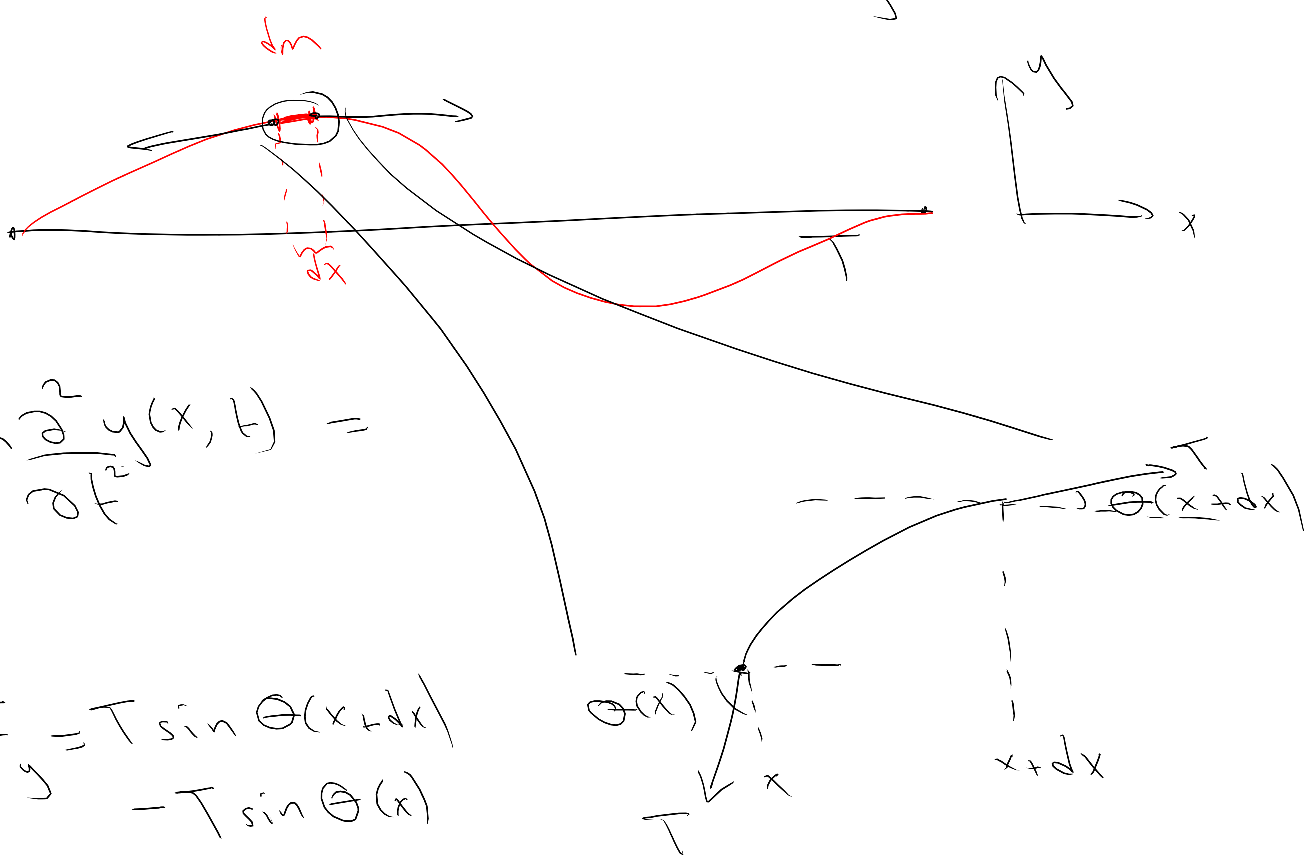
$$= v^2\left[g'' + h''\right]$$

$$\frac{\partial^2 f}{\partial x^2} = \left[g'' + h''\right] = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0}$$

$$\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) f = 0$$

Example waves on a string




$$dm \frac{\partial^2 y(x,t)}{\partial t^2} =$$

$$dF_y = T\sin\theta(x+dx) - T\sin\theta(x)$$

$$\sin\theta \sim \tan\theta = \frac{\partial y}{\partial x}$$

$$dm \frac{\partial^2}{\partial t^2} y(x,t) = T \frac{\left[ \frac{\partial y}{\partial x}\Big|_{x+dx} - \frac{\partial y}{\partial x}\Big|_{x} \right]}{dx} dx$$

$$dm \frac{\partial^2 y}{\partial t^2} = T\,dx \frac{\partial^2 y}{\partial x^2} = T\,dx \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{\frac{T}{(dm/dx)}} \frac{\partial^2 y}{\partial t^2} = 0$$

$\frac{dm}{dx} \equiv \lambda$ : mass per unit length

$$\boxed{v^2 = \frac{T}{\lambda}} \Rightarrow \frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$