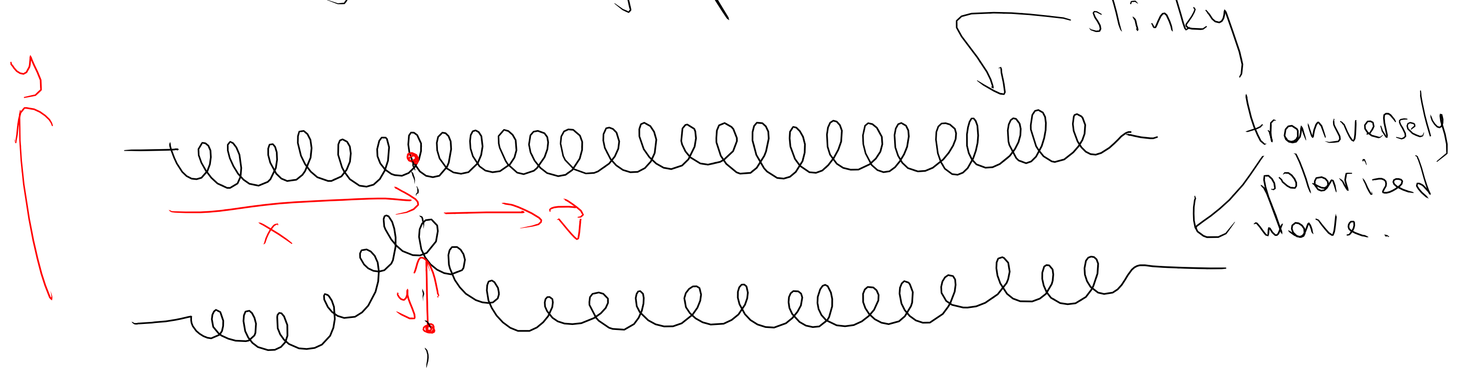
$$f(x,t) = g(x-vt) + h(x+vt)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \Longleftrightarrow \underline{\nabla}^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0$$

$$v = \sqrt{\frac{T}{\mu}}$$

- Transversely polarized wave
- Longitudinally polarized wave

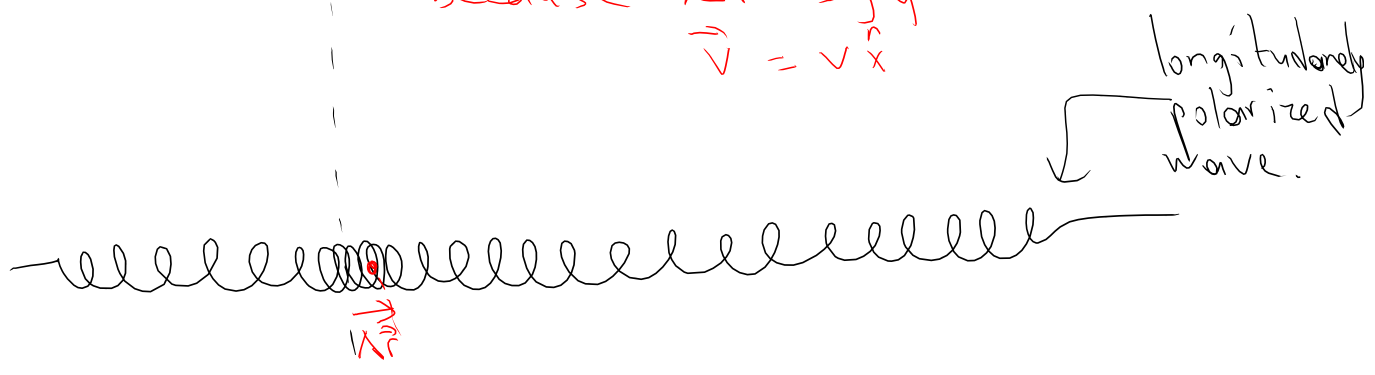


$$y = f(x,t)$$

$$\Delta\vec{r}\cdot\vec{v} = 0$$

because $\Delta\vec{r} = y\hat{y}$

$\vec{v} = v\hat{x}$



$$\Delta\vec{r}\times\vec{v} = 0$$

Electromagnetic Waves in Vacuum

$\checkmark\ \vec{\nabla}\cdot\vec{E} = 0$ $\qquad$ $\checkmark\ \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\checkmark\ \vec{\nabla}\cdot\vec{B} = 0$ $\qquad$ $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\mu_0 \varepsilon_0} \vec{\nabla} \times \vec{B}$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \vec{\nabla} \times \left(\frac{\partial \vec{B}}{\partial t} \right) = \frac{1}{\mu_0 \varepsilon_0} \vec{\nabla} \times \left(-\vec{\nabla} \times \vec{E} \right)$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{1}{\mu_0 \varepsilon_0} \vec{\nabla} \times (\vec{\nabla} \times \vec{E})$$

$$= -\frac{1}{\mu_0 \varepsilon_0} \left[\vec{\nabla} (\underbrace{\vec{\nabla} \cdot \vec{E}}_{0}) - \nabla^2 \vec{E} \right]$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \vec{\nabla}^2 \vec{E} \Rightarrow \vec{\nabla}^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \equiv \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cong 3 \cdot 10^8 \text{ m/s}$$

$$\boxed{\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{\nabla}^2 \vec{B}} \Leftarrow \qquad \vec{v} = \frac{\omega}{k} \hat{k}$$

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(\vec{k}' \cdot \vec{r} - \omega' t)}$$

$$\vec{\nabla} \cdot \vec{E} = \partial_x E_x + \partial_y E_y + \partial_z E_z$$

$$= \partial_x \left(E_{0x} e^{i(k_x x + k_y y + k_z z - \omega t)} \right) + \dots$$

$$= i k_x E_{0x} e^{i(\quad)} + \dots$$

$$= i \underbrace{(k_x E_{0x} + k_y E_{0y} + k_z E_{0z})}_{\vec{k} \cdot \vec{E}_0 = 0} e^{i(\quad)} = 0$$

$\Rightarrow$ transversely polarized

$$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{k}' \cdot \vec{B}_0 = 0$$

$$\vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right) = i \vec{k} \times \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\frac{\partial \vec{B}}{\partial t} = -i \omega' \vec{B}_0 e^{i(\vec{k}' \cdot \vec{r} - \omega' t)}$$

$$i\vec{k}\times\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = i\omega' \vec{B}_0 e^{i(\vec{k}'\cdot\vec{r}-\omega' t)}$$

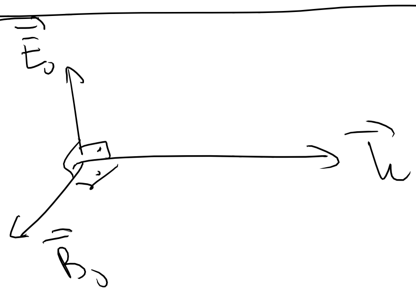
$$\Rightarrow \omega = \omega'$$

$$\vec{k} = \vec{k}'$$

$$\boxed{\vec{k}\times\vec{E}_0 = \omega'\vec{B}_0} \Rightarrow kE_0 = \omega B_0$$

$$\Rightarrow E_0 = \frac{\omega}{k}B_0 \qquad \frac{\omega}{k} = c$$

$$\Rightarrow \boxed{E_0 = cB_0}$$



$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\nabla\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$$

$$\cancel{i}\vec{k}\times\vec{B}_0 \cancel{e^{i(\vec{k}\cdot\vec{r}-\omega t)}} = \frac{1}{c^2}(-\cancel{i}\omega)\vec{E}_0 \cancel{e^{i(\vec{k}\cdot\vec{r}-\omega t)}}$$

$$\vec{k}\times\vec{B}_0 = -\frac{\omega}{c^2}\vec{E}_0$$

$$kB_0 = \frac{\omega}{c^2}E_0 \Rightarrow B_0 = \overbrace{\left(\frac{\omega}{k}\right)}^{c}\frac{1}{c^{\cancel{2}}}E_0 \Rightarrow E_0 = cB_0$$

polarization of an electromagnetic wave
= direction of the $\vec{E}$ field.