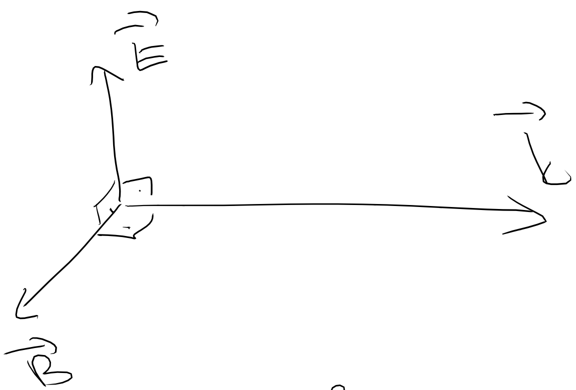


$$E_0 = v B_0 \qquad \frac{\omega}{k} = v$$

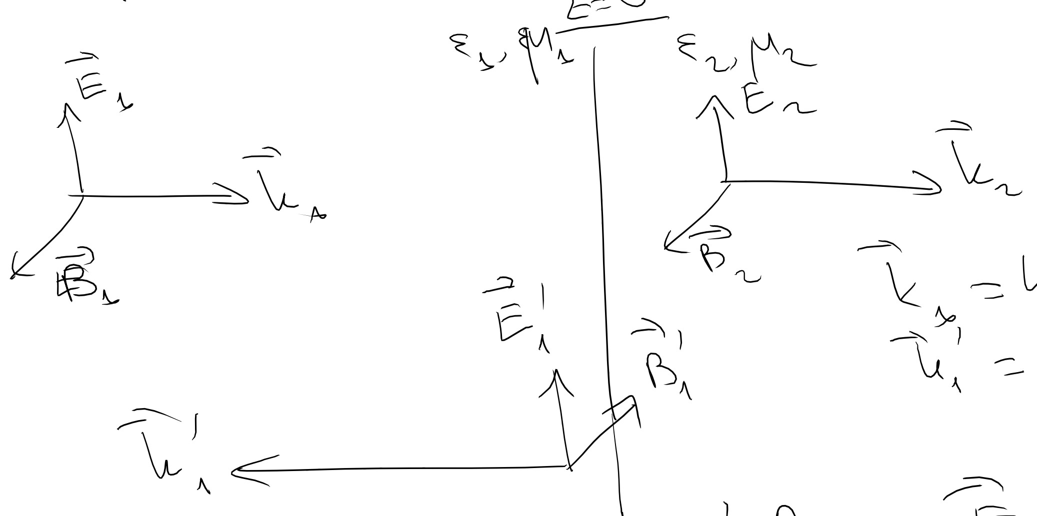
$$v = \frac{c}{n}$$

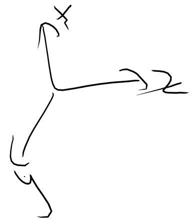
$$n = \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$$

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \rightarrow \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \qquad \varepsilon \mu \frac{\partial^2 E}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\varepsilon \mu}} = \underbrace{\frac{1}{\sqrt{\varepsilon_0 \mu_0}}}_{c} \underbrace{\sqrt{\frac{\varepsilon_0 \mu_0}{\varepsilon \mu}}}_{\frac{1}{n}}$$

Reflection & Transmission





$$\vec{k}_1 = k_1 \hat{z}$$
$$\vec{k}_1' = -k_1 \hat{z}$$

$$\vec{E}_1 = E_1 \hat{x}$$
$$\vec{B}_1 = \frac{E_1}{v_1} \hat{y}$$
$$e^{i(\pm k_1 z - \omega t)}$$

$$\vec{E}_1' = E_1' \hat{x}$$
$$\vec{B}_1' = -\frac{E_1'}{v_1} \hat{y}$$

$$\vec{E}_2 = E_2 \hat{x}$$
$$\vec{B}_2 = \frac{E_2}{v_2} \hat{y}$$
$$e^{+i(k_2 z - \omega t)}$$

$$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \Delta D_\perp = 0 \Rightarrow \Delta(\varepsilon E_\perp) = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \Delta B_\perp = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \Delta \vec{E}_{\parallel} = 0$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial D}{\partial t} \Rightarrow \Delta \vec{H}_{\parallel} = 0 \Rightarrow \Delta\left(\frac{\vec{B}_{\parallel}}{\mu}\right) = 0$$

$$\vec{H} = \frac{\vec{B}}{\mu} \;;\; \vec{D} = \varepsilon\vec{E}$$

$$\vec{E}^{<} = E_1 \hat{x} e^{i(k_1 z - \omega t)} + E_1' \hat{x} e^{i(-k_1 z - \omega t)}$$

$$\vec{E}^{>} = E_2 \hat{x} e^{i(k_2 z - \omega t)}$$

$$E_1 \cancel{e^{-i\omega t}} + E_1' \cancel{e^{-i\omega t}} = E_2 \cancel{e^{-i\omega t}}$$

$$\boxed{E_1 = E_2 - E_1'} \Leftarrow$$

$$\vec{B}^{<} = \frac{E_1}{v_1} \hat{y} e^{i(k_1 z - \omega t)} - \frac{E_1'}{v_1} \hat{y} e^{i(-k_1 z - \omega t)}$$

$$\vec{B}^{>} = \frac{E_2}{v_2} \hat{y} e^{i(k_2 z - \omega t)}$$

$$\frac{\vec{B}^{<}}{\mu_1}\Big|_{z=0} = \frac{\vec{B}^{>}}{\mu_2}\Big|_{z=0}$$

$$\frac{E_1}{v_1\mu_1} - \frac{E_1'}{v_1\mu_1} = \frac{E_2}{v_2\mu_2}$$

$$\boxed{\frac{E_1}{v_1\mu_1} = \frac{E_1'}{v_1\mu_1} + \frac{E_2}{v_2\mu_2}}$$

$$\boxed{E_1 = E_2 - E_1'} \times \frac{1}{v_1\mu_1}$$

$$\frac{2E_1}{v_1\mu_1} = E_2\left(\frac{1}{v_2\mu_2} + \frac{1}{v_1\mu_1}\right)$$

$$E_2 = E_1 \frac{2}{\cancel{v_1\mu_1}} \frac{\cancel{v_1\mu_1}\, v_2\mu_2}{v_1\mu_1 + v_2\mu_2} =$$

$$\boxed{E_2 = \frac{2 v_2\mu_2}{v_1\mu_1 + v_2\mu_2} E_1}$$

$$E_1' = E_2 - E_1 = \left[\frac{2 v_2\mu_2}{v_1\mu_1 + v_2\mu_2} - 1\right] E_1$$

$$\boxed{E_1' = \frac{v_2\mu_2 - v_1\mu_1}{v_2\mu_2 + v_1\mu_1} E_1}$$

Assume $\mu_1 \approx \mu_2 \approx \mu_0$

$$E_2 = \frac{2v_2}{v_1+v_2} E_1 = 2\frac{\frac{c}{n_2}}{\frac{c}{n_1} + \frac{c}{n_2}} E_1 = \frac{2n_1}{n_1+n_2} E_1$$

$$E_1' = \frac{n_1-n_2}{n_1+n_2} E_1$$

Intensity: energy per unit perpendicular area per unit time

$u = \varepsilon E^2$

$$I \propto \varepsilon E^2$$

$$R = \frac{I_R}{I_I} = \frac{n_1 E_1'^2}{n_1 E_1^2} = \frac{(n_1-n_2)^2}{(n_1+n_2)^2}$$

$$\vec{S} = \frac{1}{\mu} \vec{E} \times \vec{B}$$

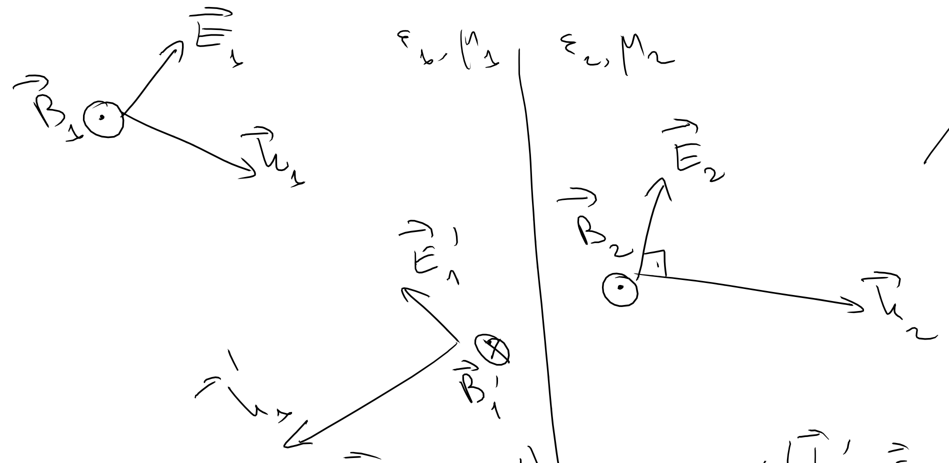
$$|\vec{S}| = \frac{EB}{\mu} = \frac{EE}{v\mu} = \frac{1}{\mu v} E^2 = \frac{n}{\mu c} E^2$$

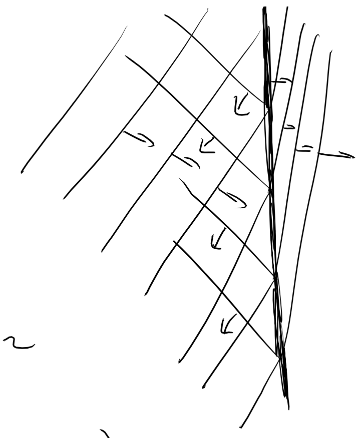
$$T = \frac{n_2 E_2^2}{n_1 E_1^2} = \frac{n_2}{n_1} \frac{4n_1^2}{(n_1+n_2)^2} = \frac{4n_1 n_2}{(n_1+n_2)^2}$$

T: transmission coefficient

R: reflection coefficient

$$T + R = \frac{4n_1n_2}{(n_1+n_2)^2} + \frac{(n_1-n_2)^2}{(n_1+n_2)^2} = \frac{(n_1+n_2)^2}{(n_1+n_2)^2} = 1$$





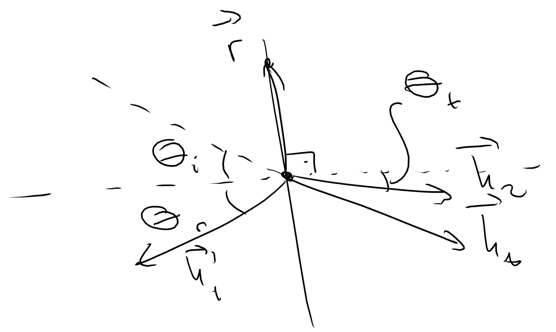
$$\vec{E}^{<} = \vec{E}_{10}\, e^{i(\vec{k}_1\cdot\vec{r}-\omega t)} + \vec{E}'_{10}\, e^{i(\vec{k}'_1\cdot\vec{r}-\omega t)}$$

$$\vec{E}^{>} = \vec{E}_{20}\, e^{i(\vec{k}_2\cdot\vec{r}-\omega t)}$$

$$\left\{\varepsilon_1\left[E_{10\perp}\, e^{i(\vec{k}_1\cdot\vec{r}-\omega t)}\right] + \varepsilon_1 E'_{10\perp}\, e^{i(\vec{k}'_1\cdot\vec{r}-\omega t)}\right\}_{z=0} = \left\{\varepsilon_2 E_{20\perp}\, e^{i(\vec{k}_2\cdot\vec{r}-\omega t)}\right\}_{z=0}$$

(*) $\vec{k}_1\cdot\vec{r}\big|_{z=0} = \vec{k}'_1\cdot\vec{r}\big|_{z=0} = \vec{k}_2\cdot\vec{r}\big|_{z=0}$ $\qquad \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$k = \frac{\omega}{v}$



$$\vec{k}_1\cdot\vec{r}\big|_{z=0} = k_1 r\cos\left(\frac{\pi}{2}+\theta_i\right)$$

$$\vec{k}'_1\cdot\vec{r}\big|_{z=0} = k_1 r\cos\left(\frac{\pi}{2}+\theta_r\right)$$

$$\vec{k}_2\cdot\vec{r}\big|_{z=0} = k_2 r\cos\left(\frac{\pi}{2}+\theta_t\right)$$

(*) $\Rightarrow k_1\sin\theta_i = k_1\sin\theta_r = k_2\sin\theta_t$

$\theta_i = \theta_r$: angle of incidence is equal to angle of reflection

$$k = \frac{\omega}{v} = \frac{\omega}{c} n$$

$$k_1\sin\theta_i = k_2\sin\theta_t$$

$$\cancel{\frac{\omega}{c}}\, n_1\sin\theta_i = \cancel{\frac{\omega}{c}}\, n_2\sin\theta_t$$

$$\boxed{n_1\sin\theta_i = n_2\sin\theta_t}$$ Snell's Law

$$\varepsilon_1 \left\{ E_{10\perp} \cancel{e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)}} \right] + \varepsilon_1 E'_{10\perp} \cancel{e^{i(\vec{k}'_1 \cdot \vec{r} - \omega t)}} \Big\}_{z=0}$$

$$= \varepsilon_2 E_{20\perp} \cancel{e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}} \Big\}_{z=0}$$

$$\boxed{\varepsilon_1 E_{10\perp} + \varepsilon_1 E'_{10\perp} = \varepsilon_2 E_{20\perp}}$$

$$\vec{B} = \frac{\hat{k} \times \vec{E}}{v}$$

$$\boxed{E_{10\parallel} + E'_{10\parallel} = E_{20\parallel}}$$

$$\boxed{\left(\frac{\hat{k}_1 \times \vec{E}_{10}}{v_1 \mu_1} \right)_\parallel + \left(\frac{\hat{k}'_1 \times \vec{E}'_{10}}{v_1 \mu_1} \right)_\parallel = \left(\frac{\hat{k}_2 \times \vec{E}_{20}}{v_2 \mu_2} \right)_\parallel}$$