Waves in a conductor

$$\vec{\nabla} \cdot \vec{D} = \rho_f = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \Rightarrow \vec{\nabla} \times \vec{B} = \mu \sigma \vec{E} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Ohm's Law

$$\vec{J} = \sigma \vec{E}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\sigma \vec{E}) = 0$$

$$\frac{\partial \rho}{\partial t} + \sigma \vec{\nabla} \cdot \vec{E} = 0$$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon}$ in a linear homogeneous dielectric.

$$\frac{\partial \rho}{\partial t} + \sigma \frac{\rho}{\varepsilon} = 0$$

$$\rho(\vec{r}, t) = \rho(\vec{r}, 0) e^{-\frac{t}{\tau}}$$

$$\boxed{\tau = \frac{\varepsilon}{\sigma}}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$; wave eqn.

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left[\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right]$$

$$\left(\frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2} - \nabla^2\vec{E}\right) = -\mu\sigma\frac{\partial\vec{E}}{\partial t}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\vec{E} = \vec{E}_0 e^{i(\tilde{k}z - wt)}$$

$$\frac{1}{v^2}(-iw)^2\vec{E} - (i\tilde{k})^2\vec{E} = -\mu\sigma(-iw)\vec{E}$$

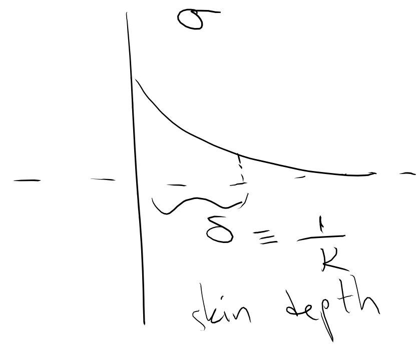
$$\Rightarrow \boxed{-\frac{w^2}{v^2} + \tilde{k}^2 = i\mu\sigma w} \Leftarrow$$

σ: conductivity

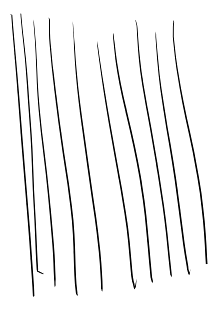
$$\boxed{\tilde{k}^2 = \frac{w^2}{v^2} + i\mu\sigma w}$$

$$\tilde{k} = k + iK$$

$$\vec{E} = \vec{E}_0 e^{-Kz} e^{i(kz - wt)}$$



Example



$$(k + iK)^2 = \frac{w^2}{v^2} + i\mu\sigma w$$

$$k^2 - K^2 + 2ikK = \frac{w^2}{v^2} + i\mu\sigma w$$

$$k^2 - K^2 = \frac{w^2}{v^2}$$

$$kK = \frac{1}{2}\mu\sigma w$$

$$K = \frac{1}{\cancel{2k}}\mu\sigma\omega$$

$$k^2 - \frac{(\mu\sigma\omega)^2}{4k^2} = \frac{\omega^2}{v^2}$$

$$(k^2)^2 - \frac{\omega^2}{v^2}k^2 - \frac{(\mu\sigma\omega)^2}{4} = 0$$

$$k^2 = \left[\frac{\omega^2}{v^2} + \sqrt{\frac{\omega^4}{v^4} + (\mu\sigma\omega)^2}\right]\frac{1}{2}$$

$$\frac{\omega}{k} \neq v$$

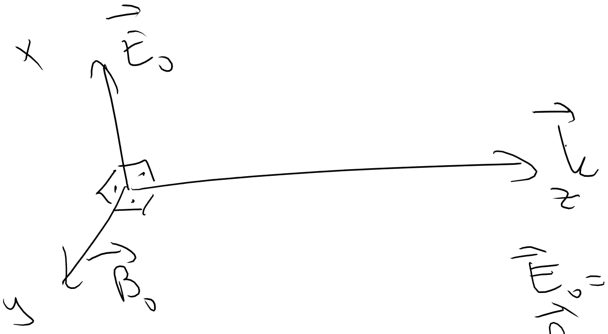
$$\frac{1}{v^2} = \left(\frac{k}{\omega}\right)^2 = \frac{1}{2v^2}\left[1 + \sqrt{1 + v^4\frac{\mu\sigma}{\omega^2}}\right]$$

$$\vec{E} = \vec{E}_0 e^{i(\vec{\tilde{k}}\cdot\vec{r} - \omega t)} \qquad \vec{\tilde{k}} = \tilde{k}\hat{z}$$

$$i\vec{\tilde{k}}\times\vec{E}_0\, e^{i(\vec{\tilde{k}}\cdot\vec{r} - \omega t)} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \vec{B} = \frac{(\tilde{k}\hat{z})\times\vec{E}_0}{\omega}\, e^{i(kz - \omega t)}$$

$$\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \vec{\nabla}\cdot\vec{E} = 0 = k\hat{z}\cdot\vec{E}_0 = 0 \Rightarrow \boxed{\hat{z}\cdot\vec{E}_0 = 0}$$



$$\vec{B} = \vec{B}_0 e^{i(\tilde{k}z - \omega t)}$$

$$\vec{E} = \vec{E}_0 e^{i(\tilde{k}z - \omega t)}$$

$$\left.\begin{aligned}\vec{E}_0 &= E_0\hat{x}\\ \vec{B}_0 &= B_0\hat{y}\end{aligned}\right\} \quad \boxed{B_0 = \frac{\tilde{k}}{\omega}E_0}$$

$$E_0 = |E_0|e^{i\delta_E} \qquad \vec{E} = |E_0|e^{-Kz}\cos(kz - \omega t + \delta_E)\hat{x}$$

$$B_0 = |B_0|e^{i\delta_B} \qquad \vec{B} = |B_0|e^{-Kz}\cos(kz - \omega t + \delta_B)$$

$$\frac{\tilde{k}}{\omega} = \left|\frac{\tilde{k}}{\omega}\right|e^{i\delta} \qquad |E_0|\left|\frac{\tilde{k}}{\omega}\right|e^{i(\delta+\delta_E)} = |B_0|e^{i\delta_B}$$

$$\delta + \delta_E = \delta_B$$

$$|B_0| = |E_0|\left|\frac{\tilde{k}}{\omega}\right|$$