

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}z - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(\vec{k}z - \omega t)}$$

$$\vec{k} = k + i\kappa = \kappa e^{i\phi}$$

$$\vec{B}_0 = \frac{\vec{k} \times \vec{E}_0}{\omega}$$

$$\vec{J} = \sigma \vec{E}$$

↑ conductivity

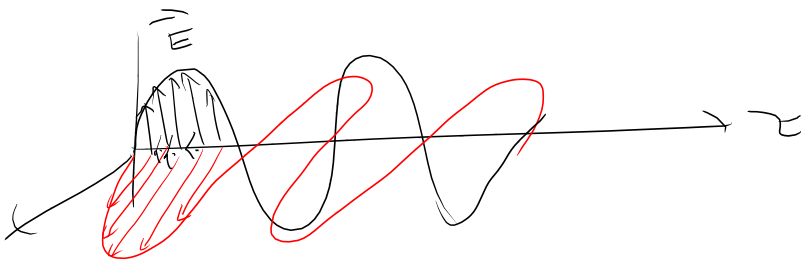
$$\vec{k} = \tilde{k}z$$

$$\vec{E} = \vec{E}_0^R \cos(kz - \omega t + \delta_E) e^{-\kappa z}$$

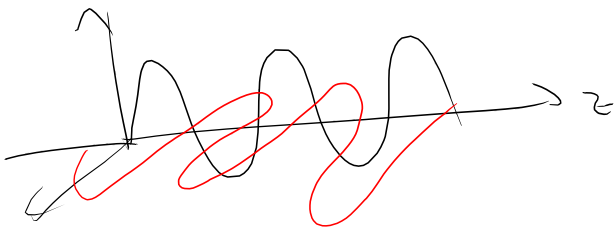
$$\vec{B} = \vec{B}_0^R \cos(kz - \omega t + \delta_B) e^{-\kappa z}$$

$$\delta_B = \phi + \delta_E$$

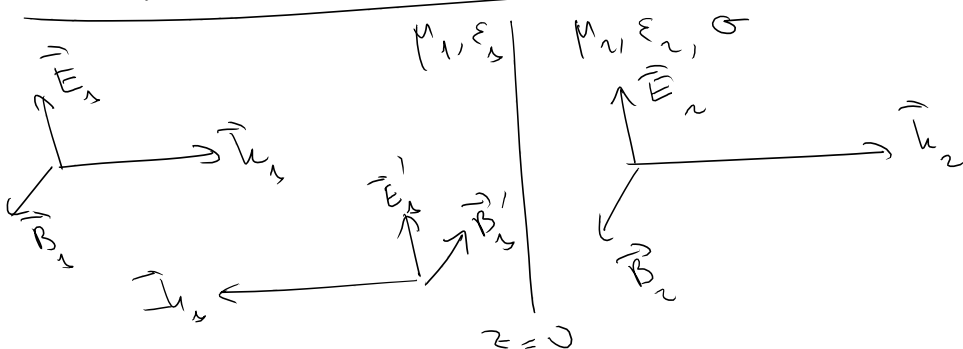
non-conducting medium



conducting medium



Reflection on a Conducting Surface



$$\vec{\nabla} \cdot \vec{D} = \rho_f \Rightarrow \epsilon_2 \vec{E}_\perp^{(2)} - \epsilon_1 \vec{E}_\perp^{(1)} = \sigma_f \quad \checkmark (\sigma_f = 0)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow B_\perp \text{ is continuous} \quad \checkmark$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{E}_\parallel \text{ is continuous}$$

$$\vec{E}_{20} e^{i(k_2 z - \omega t)} + \vec{E}'_{10} e^{i(-k_2 z - \omega t)} = \vec{E}_{20} e^{i(\tilde{k}_2 z - \omega t)} \Big|_{z=0}$$

$$\boxed{\vec{E}_{10} + \vec{E}'_{10} = \vec{E}_{20}}$$

$$\begin{aligned} \vec{E}_{10} &= E_{10} \hat{x} \\ \vec{E}'_{10} &= E'_{10} \hat{x} \\ \vec{E}_{20} &= E_{20} \hat{x} \end{aligned}$$

$$\boxed{E_{10} + E'_{10} = E_{20}}$$

$$\left. \begin{aligned} \vec{J} &= \sigma \vec{E} \\ \vec{J} &= \vec{K} \delta(z) \text{ time} \\ \vec{E} &\text{ is finite on the surface} \end{aligned} \right\} \vec{K} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \left(\frac{B_\parallel}{\mu} \right) \text{ is continuous}$$

$$\frac{B_{20} \hat{y}}{\mu_2} - \frac{B'_{10} \hat{y}}{\mu_1} = \frac{B_{20} \hat{y}}{\mu_2}$$

$$B_{10} = \frac{1}{\mu_1} E_{10}$$

$$B'_{10} = \frac{1}{\mu_1} E'_{10}$$

$$\boxed{B_{20} = \frac{\mu_1}{\mu_2} E_{20}}$$

$$\frac{E_{10}}{\mu_1 \mu_2} - \frac{E'_{10}}{\mu_1 \mu_2} = \frac{\mu_1}{\mu_2} E_{20}$$

$$\left\{ \begin{aligned} E_{10} - E'_{10} &= \tilde{\beta} E_{20} \\ E_{10} + E'_{10} &= E_{20} \\ E_{10} &= \tilde{\beta} E_{20} + E'_{10} \\ E'_{10} &= E_{20} - E_{10} \end{aligned} \right.$$

$$\tilde{\beta} = \frac{\mu_1}{\mu_2} \frac{\mu_1 \mu_2}{\mu_1}$$

$$E_{20} = \frac{2}{1 + \tilde{\beta}} E_{10}$$

$$E'_{10} = E_{20} - E_{10} = \frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} E_{10}$$

$$\beta = \frac{\tilde{k}}{\omega} \frac{v_{ph}}{v_{gr}} \frac{M_1}{M_2}, \quad \tilde{k} = k + iK, \quad K \propto \sigma$$

For a perfect conductor:

$$E_{z0} = 0$$

$$E'_{x0} = -E_{x0}$$

all the wave is reflected.

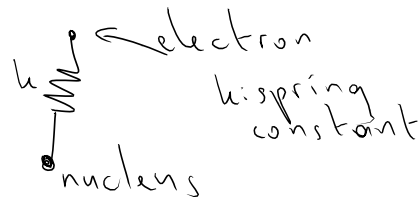
$$\tilde{k}^2 = \frac{\omega^2}{2v^2} \left[L + \sqrt{L + A(\omega)} \right]$$

$$\left(\frac{k}{\omega} \right)^2 = \frac{1}{2v^2} \left[1 + \sqrt{1 + A(\omega)} \right]$$

$\epsilon(\omega)$

↑ electric permittivity.

Freq. Dependent permittivity



x : displacement (in the \hat{x} direction) of the electron from eq. point.

$$\vec{E} = E_0 \hat{x} e^{i\omega t}$$

$$m \frac{d^2 x}{dt^2} = -kx + qE_0 e^{i\omega t} \quad \text{--- } m \frac{dx}{dt}$$

$$m \frac{d^2 x}{dt^2} + m\gamma \frac{dx}{dt} + kx = qE_0 e^{i\omega t}$$

$$x = A e^{i\omega t}$$

$$A = \frac{qE_0}{-m\omega^2 + im\omega\gamma + k}$$

$$p = qx = \frac{q^2}{-m\omega^2 + im\omega\gamma + k} E_0 e^{i\omega t}$$

$$\vec{p} = \frac{q^2}{-m\omega^2 + im\omega\gamma + k} \vec{E} \quad \leftarrow$$

$$\vec{P} = \frac{1}{V} \sum_j q_j^2 \frac{1}{in\omega \chi + k_j - m_j \omega^2} \quad \vec{P} \equiv \epsilon \chi_e \vec{E}$$

$$\epsilon = \epsilon_0 (1 + \chi_e) = \boxed{\epsilon_0 \left[1 + \frac{n q^2}{k - m \omega^2 + i n \omega \gamma} \right] = \epsilon}$$

$$\frac{k}{m} = \omega_0^2$$

$$\epsilon = \epsilon_0 \left[1 + \frac{n q^2 / m}{\omega_0^2 - \omega^2 + i \omega \gamma} \right]$$

$$\vec{E} = \vec{E}_0 e^{i(\tilde{k}z - \omega t)} \Rightarrow \tilde{k} = k + i\kappa$$

$$\frac{1}{v} = \frac{\tilde{k}}{\omega} = \sqrt{\epsilon \mu} \Rightarrow \tilde{k} = k + i\kappa$$

$$\vec{E} \propto e^{-\kappa z} \quad I \propto e^{-2\kappa z}$$

$\alpha = 2\kappa$: attenuation coefficient
absorption coefficient

phase velocity $\frac{\omega}{k} = ?$

group velocity $\frac{\partial \omega}{\partial k} = ?$

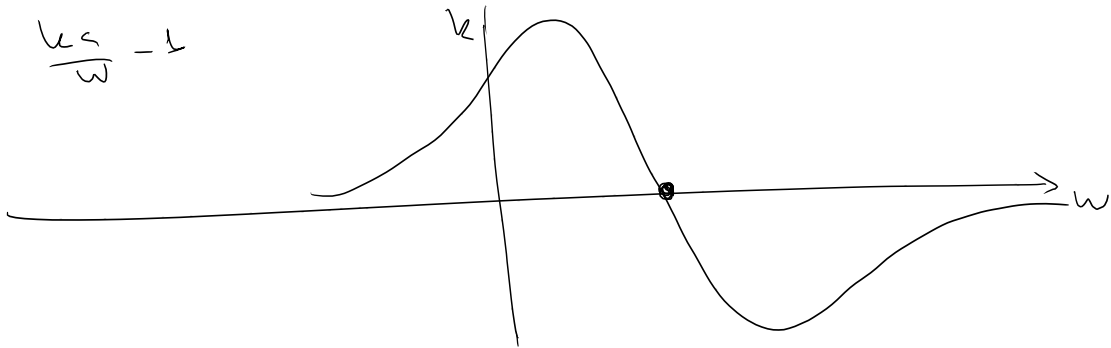
$$\frac{\tilde{k}}{\omega} = \sqrt{\epsilon \mu_0} = \sqrt{\epsilon_0 \mu_0} \left[1 + \frac{n q^2 / m^2}{\omega_0^2 - \omega^2 + i \omega \gamma} \right]^{+1/2}$$

$$\frac{\tilde{k}}{\omega} = \frac{1}{c} \left[1 + \frac{n q^2 / m}{\omega_0^2 - \omega^2 + i \omega \gamma} \right]$$

$$k = \text{Re } \tilde{k} = \text{Re} \frac{\omega}{c} \left[1 + \frac{n q^2 / m}{\omega_0^2 - \omega^2 + i \omega \gamma} \right]$$

$$k = \frac{\omega}{c} \left[1 + \frac{(n q^2 / m) (\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2} \right]$$

$$\kappa = -\text{Im } \tilde{k} = -\frac{\omega}{c} \frac{\omega \gamma n q^2 / m}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2}$$



$$\frac{k\omega}{\omega} = 1 = \frac{c}{v_p} = 1$$