A CRITICAL SURVEY OF FUZZY LOGIC AS A BASIS FOR APPROXIMATE REASONING

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Prof. Dr. Bahattin Akşit

Director

I certify that this thesis satisfies all the requirements for the degree of Master of Science.

Prof. Dr. Ahmet İnam

Chairman of Department

This is to certify that we read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. Akın Ergüden

Supervisor

Examining Committee Members

Prof. Dr. Akın Ergüden

Prof. Dr. Ahmet İnam

Prof. Dr. Ersin Tulunay

ABSTRACT

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KÜÇÜKÖNCÜ, Tansu
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Classical (bivalent) logical systems are inefficient to deal with lots of propositions used in daily life. They, generally, put these outside of their interest area. The aim of this thesis is to present a critical survey of fuzzy logic as a basis for approximate reasoning, and to show that it promises, even already gives, a better, than those of bivalent ones, way of understanding and expressing human reasoning.

In this study, first, basic concepts of fuzzy logic is introduced. How fuzziness occurs from a human's point of view, and how it is stated in linguistic terms, especially by using hedges, is explicated with examples. Then, a critique that how a fuzzy logician, or any user of fuzzy logic, as a pragmatist, interprets the concepts of truth and logic (hence, fuzzy logic, too) is given. This also gives a picture of how truth is used daily life, and even, for various cases, in scientific environments. Finally how to reason with fuzzy concepts, namely approximate reasoning, is explained.

Key Words : Fuzzy Logic, Approximate Reasoning, Fuzzy Reasoning, Fuzziness, Pragmatism, Approximate Truth, Hedges

ÖZ

YAKLAŞIMSAL USLAMLAMANIN TEMELİ OLARAK BULANIK MANTIĞA ELEŞTİREL BİR BAKIŞ

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Klasik (iki değerli) mantık sistemleri, günlük yaşamda kullanılan bir çok önermeyle ilgilenebilmek için yetersizdirler. Onlar, genelde, bunları ilgi alanlarının dışında bırakırlar. Bu tezin amacı, yaklaşımsal uslamlamanın temeli olarak bulanık mantığı eleştirel bir bakış açısıyla ele almak, ve onun insan uslamlamasını anlamak ve açıklayabilmek için, iki değerli olanlardan daha iyi bir yol vaat ettigini, hatta halen vermekte olduğunu göstermektir.

Bu çalışmada, ilk olarak, bulanık mantığın temel kavramları tanıtılır. İnsan açısından bulanıklık nasıl oluşur, ve dilsel terimlerle, özellikle dilsel sınırlayıcılarla, nasıl ifade edildiği örneklerle açıklığa kavuşturulur. Ardından, bir bulanık mantıkçının, veya herhangi bir bulanık mantık kullanıcısının, bir yararcılıkçı olarak, doğruluk ve mantık kavramlarını (böylece, bulanık mantığı da) nasıl yorumladığı verilir. Bu, aynı zamanda doğruluğun günlük yaşamda, hatta pek çok durum için bilimsel ortamlarda da, nasıl kullanıldığının bir resmini verir. Son olarak bulanık kavramlarla nasıl uslamlacağı, yani yaklaşımsal uslamlama açıklanır.

Anahtar Sözcükler : Bulanık Mantık, Yaklaşımsal Uslamlama, Bulanık Uslamlama, Bulanıklık, Yararcılık, Yaklaşık Doğruluk, Dilsel Sınırlayıcılar

To My Parents

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CHAPTER 1
INTRODUCTION

Consider the sentence "x is a bird", where x is, for example, canary, eagle, chicken, penguin, or bat. We want to assign truth values to this sentence for different x's. At first sight, it seems that we can assign 'true' to all. In fact, doing so makes most people to feel disturbance. This is because that canary, eagle, and bat fly, chicken can fly only a few meters, but penguin prefers to swim instead. All gives egg, but bat gives birth, since it's a mammal, others are not. Hence, for canary, and eagle, this sentence seems more true than others. For each, it seems true at a different degree [Lakoff-73: 459].

Consider the classical syllogism :

Socrates is a man.
All men are mortal.

∴ Socrates is mortal.

If we change it as below :

Socrates is very healthy.
Healthy men live a long time.

∴ Socrates will live a very long time,

it is not easy to represent it in terms of classical logics. Moreover, many of the classical logical systems put this kind of sentences out of their interest. But these are of kind of sentences which we use in daily life most of the time [Gaines-76: 407].

The aim of this thesis is to represent, with critics, fuzzy logic as a basis reasoning with these and other kinds of ambiguous sentences, namely approximate reasoning, which is commonly used in daily life.

Systems using fuzzy logic run subways, tune tv.s and computer disc heads, focus and stabilise cameras and camcorders, adjust air conditioners and washing machines and vacuum sweepers, defrost refrigerators, schedule elevators and traffic lights, and control automobile motors, suspensions, and emergency breaking systems, control cruises, cement mixers, guide robot-arm manipulators, recognise characters, select golf clubs, even arrange flowers.

Fuzzy systems are trainable dynamical systems. Unlike statistical estimators, they estimate a function without a mathematical model of how outputs depend on inputs. They are model-free estimators. They 'learn from experience' with numerical and, sometimes, linguistic sample data. Adaptive fuzzy systems learn to control complex processes as much as we do [Kosko-92, 94].

CHAPTER 2
CONCEPT OF FUZZINESS

"Troy is a tall man". "Lykia is a beautiful woman". "100 is a much greater number than 1". "This leaf is red". These are sentences to which classical (bivalent) logical systems can assign truth values with great difficulty, since tall, beautiful, greater, and also much are imprecisely defined terms. They have uncertainties in their meanings. But, such imprecisely defined concepts play an important role in human thinking [Zadeh-65: 338]. The essence and power of human reasoning is in its capability to grasp and use inexact concepts directly [Gaines-76: 625].

Classical logical systems only deals with propositions which are assumed to occur in certain conditions, and to have strict truth values (either true or false). They are not interested in uncertainty. Then, how can we assign reasonable truth values to this kind of sentences?

The answer is using a fuzzy truth, that is truth in a continuous or graded form. The notion of fuzzy truth provides a convenient point of departure for the construction of conceptual framework which parallels in many respects the framework used in the case of ordinary truth, but is more general than the latter and, potentially, have a much wider scope of applicability. Such a framework provides a natural way to dealing with problems in which the source of imprecision is the absence of sharply defined criteria of truth [Zadeh-65: 339].

Fuzzy becomes a modern term replacing previous usage in the literature of terms such as 'inexact' or 'vague'.

Mathematically fuzziness means multivaluedness or multivalence and finds its origins in the Heisenberg position-momentum uncertainty principle (1) in quantum mechanics. Three-valued fuzziness corresponds to truth, falsehood, and indeterminacy, or to presence, absence, and ambiguity. Multivalued fuzziness corresponds to degrees of indeterminacy or ambiguity, partial occurance of events or relations.

2.1. A Brief History

Logical paradoxes and the Heisenberg uncertainty principle led to the development of multivalued logics in the 1920s and 1930s. Quantum theorists allowed for indeterminacy by including a third or middle truth value in the bivalent logical framework. The next step allowed degrees of indeterminacy, viewing true and false as the two limiting cases of spectrum of indeterminacy.

The Heisenberg uncertinty principle, with its continuum of indeterminacy, forced multivaluedness on science, though few Western philosophers have accepted multivaluedness, Lukasiewicz, Gödel, and Black did accept it and developed the first multivalued or fuzzy logic and set systems.

Polish logician Jan Lukasiewicz first formally developed a three-valued logical system in the early 1930s. Lukaziewicz, finally, extended the range of truth values to all numbers [Kosko-92].

In the 1930s quantum philosopher Max Black applied continuous logic componentwise to sets or lists of elements or symbols. Historically, Black drew the first fuzzy-set membership functions. Black called the uncertainty of these structures "vagueness". Anticipating Zadeh's fuzzy-set theory, each element in Black's multivalued sets and lists behaved as a statement in a continuous logic.

In 1965 systems scientists Lotfi Zadeh formally developed a multivalued set theory, and introduced the term fuzzy into the technical literature [Zadeh-65].

Notes

(1) The famous principle says that when observing an electron, it is not possible to measure its position and velocity at the same time

correctly. Errors occurring when measuring both quantities at the same time can not be reduced to reasonable limits [Heisenberg-69: 85-97].

CHAPTER 3
FUZZY LOGIC

Logic had shown development from ancient times to present (1). It had tried to adopt itself to the requirements of the time. Fuzzy logic is one of the last steps it moved.

In classical logics, a proposition (2) is assumed to be either true or false. It is assumed that the third case is impossible, and most of the time such a case is called as a paradox. In other words, truth is a function which relates propositions to values in the set {False, True}, or equivalently, numerically {0, 1} (3).

The basic idea behind fuzzy logic is that a proposition may be 'true', 'false', 'very true', 'very false', 'very very true', 'very very false', 'nearly true', 'nearly false', etc. [Zadeh-73, 75a]. In other words, truth is a function (4) which relates propositions to values in the set including infinitely many truth values between classical false and true, or numerically in the domain [0, 1] (real numbers between 0 and 1) (5). This is a consequence of Zadeh's first work on fuzzy sets [Zadeh-65] (See "Relation Between Logic And Sets").

Perhaps the simplest way of characterising fuzzy logic is to say that it is a logic of approximate reasoning. As such, it is a logic whose distinguishing features are

a. fuzzy truth-values expressed in linguistic terms, e.g. 'true, very true, more or less true, rather true, not true, false, not very true, and not very false, etc., (or numerically in the interval, [0,1]);

b. imprecise truth tables; and

c. rules of inference whose validity is approximate rather than exact. In these respects, fuzzy logic differs significantly from standard logical systems ranging from the classical Aristotelian logic to inductive logics and many-valued logics with set-valued truth-values [Zadeh-75a: 407].

The truth tables and the rules of inference in fuzzy logic are

a. inexact, and

b. dependent on the meaning associated with the primary truth value 'true' as well as the modifiers 'very, quite, more, or less, etc.' [Zadeh-75a: 408].

The problem of inference is less straightforward in fuzzy-valued logic than in multivalent logic, i.e. to find v(q) when you know v(p) and v(p |=> q), where |=> is some implication connective, and v(.) is the truth value of proposition(s) which is/are its parameter(s). We notice that in a chain of approximate inferences, truth and precision progress in the same sense, conclusions are always less precisely true than premises [Dubois, Prade-80: 170-174]. I will deal with the inference in detail in Chapter 6.

3.1. Basic Operations

Disjunction (OR-ing)	: $v(p \vee q) = \max(v(p), v(q))$
Conjunction (AND-ing)	: $v(p \wedge q) = \min(v(p), v(q))$
Negation	: $v(\sim p) = 1 - v(p)$

where v(p) is the truth value of proposition p, and max and min are functions which return the maximum and minimum value of its arguments, respectively, as their results [Zadeh-75a: 410].

In bivalent logics, the negation cannot be distinguished from the antonym. In most approaches to fuzzy logic, this charecteristic of bivalency seems to dominate. Linguistic fuzzy systems can take the difference between antonyms and negations into consideration. Hence, not true can be interpreted different from false. It, sometimes, may plausible to assume that not true is closer to the truth than false [Niskanen-93: 7].

As it can be seen easily, the formulas above give the usual results when the valuation set is reduced to {0,1}, that's, the borders of [0, 1].

A justification of the choice of max and min was given by Bellman and Giertz (1973). Fung and Fu (1975) also found max and min to be the only possible operators [Dubois, Prade: 11-12]. Beside its mathematical justification, effect of max and min functions seem to represent what happens in human reasoning. Consider anyone should reason using n propositions each having graded truth values. For 'or' case, he wants to be nearer to truth condition, and chooses the best, that's having the greatest truth value, proposition's truth value as n or-ed propositions' truth value. For 'and' case, he wants to know the worst case, and chooses the truth value of the proposition with the lowest truth value as the truth value of n and-ed propositions.

3.2. Properties Of Fuzzy Logical Connectives : 'And', 'Or', 'Not'

1. Commutativity : $p \vee q = q \vee p$; $p \wedge q = q \wedge p$
2. Associativity : $p \vee (q \vee r) = (p \vee q) \vee r$;
   $p \wedge (q \wedge r) = (p \wedge q) \wedge r$
3. Idempotency : $p \vee p = p$; $p \wedge p = p$
4. Distributivity : $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$;
   $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
5. $p \vee F = p$; $p \wedge T = p$

6. Absorption : $p \vee (p \wedge q) = p$; $p \wedge (p \vee q) = p$

7. De Morgan's Laws : $\sim(p \vee q) = \sim p \wedge \sim q$; $\sim(p \wedge q) = \sim p \vee \sim q$,

8. Involution : $\sim(\sim p) = p$

9. Equivalence Formula : $(\sim p \vee q) \wedge (p \vee \sim q) = (\sim p \wedge \sim q) \vee (p \wedge q)$

10. Symmetrical Difference Formula (Exclusive Disjunction) :

$$(\sim p \wedge q) \vee (p \wedge \sim q) = (\sim p \vee \sim q) \wedge (p \vee q)$$

[Zadeh-65] [Dubois, Prade-80].

The only law of ordinary logic theory that is no longer true is the excluded-middle law (and, hence, non-contradiction) :

$v(p \vee \sim p)$!= T ; $v(p \wedge \sim p)$!= F [Zadeh-65: 342].

where != means 'not equal', and F, and T corresponds to 'false', and 'true', respectively.

Fuzziness arises from the ambiguity or vagueness between a proposition and its opposite ~p. If we do not know v(p) with certainty, we do not know v(~p) with certainty either. Else by double negation we would know v(p) with certainty. This ambiguity produces non-degenerate overlap: v(p Λ ~p) != F, which breaks the law of non-contradiction. Equivalently, it also produces non-degenerate underlap: v(p V ~p) != T, which breaks the law of excluded middle [Kosko-92: 269].

3.3. Relation Between Logic And Set Theories

Logic provides a strong link between philosophy and sciences, especially via mathematics.

There are usually considered to be three principal present-day school of mathematics: the intituonist, formalist, and logistic schools. The thesis of the logistic school is that mathematics is a branch of logic. Mathematical concepts are to be formulated in terms of logical concepts, and all the theorems of mathematics are to be developed as theorems of logic [Johnson-72: 73].

Set theory (6) forms the foundations of arithmetic (7), logic, and indeed the major part of mathematics and formal reasoning. Hence, there is a strong relation between basic set theoretic operations (union and inclusion) and basic logical operators (or connectors; 'and' and 'or') [Gaines-76: 664]. It is common for logicians to give truth conditions for predicates in terms of set theory [Lakoff-73: 458].

Consider the famous example, "Socrates is a man". We know that man is a common name for some having certain properties, that is a set. Hence to say that "Socrates is a man" is true, is equivalent to say that "Soctares is an element of the set called man".

3.4. Fuzzy Sets

The concept of a fuzzy set grew out of Zadeh's dissatisfaction with the mathematical techniques of classical system theory that enforced an artificial precision inappropriate to discussion of many real world systems, particularly complex systems involving human-beings.

Zadeh proposes that such predicates as tall, red, and stable, do not define classical sets with a binary membership function but instead fuzzy sets with graded membership [Zadeh-65].

The theory of fuzzy sets is a formulisation of vagueness, and ambiguity, and ambivalence. It is a sort of multi-valued logic, but its operations differ from those traditional for multi-valued logics [Goguen-68-69] [Giles-76] [Baldwin, Pilswoth-79].

Each entity is regarded as a member to some degree, not simply as a member or not as in bivalent set theories [Gaines-76: 647]. For example, a man with height of 1.90 m. is a member of the set tall men. A man of 2.00, and of 2.10 are, too. For some purposes it may not be enough to classify them simply as a member or not. In such cases, to define their membership values, gradually, according to their height will be convenient.

The concept of a 'fuzzy set' may be seen as providing a new tool, more appropriate than that of classical set theory, for a programme of precisiation. It allows the inherent imprecision of the concepts that we actually use, and wish to use, to be neither discarded nor introduced explicitly in the explicatum, but rather to be subsumed in the (universal) concept of a degree of membership to a fuzzy set centered on that explicatum [Gaines-76: 629].

3.5. Measures Of Fuzziness

How much fuzzy is a fuzzy proposition? Various authors have proposed scalar indices to measure the degree of fuzziness of a fuzzy proposition. The degree of fuzziness is assumed to express on a global level the difficulty of deciding which elements belong and which do not belong to a given fuzzy set [Dubois, Prade-80: 32]. Measures of fuzziness evaluate v(p) and v(~p) at the same time.

A measure of fuzziness is a mapping d from fuzzy universe, X, to [0, 1] satisfying the conditions :

1. d(p) = 0, iff p is an ordinary proposition in X;
2. d(p) is maximum, iff v(p) = 1/2, $\forall p \in X$;
3. d(p) = d(~p), (~p is as fuzzy as p) [Dubois, Prade-80: 32].

One of the most common measures of fuzziness is entropy [Weber-84]. Entropy measures the uncertainty of a system or message. Its uncertainty equals its fuzziness. A proposition, p, can be ambiguous, if it's truth value is not equal to true (1) or false (0), or clear, otherwise. The fuzzy entropy of p, E(p), generally varies from 0 to 1. For an event, represented by a set of propositions, imagine a space constituted by truth values of propositions, with the assumption that they're orthogonal (that's, independent of each other). The shape obtained is called an n-dimensional hyper-cube, of which false for all propositions is one vertex, and trues are the other vertexes. For such a case, vertexes are clear, that's have zero entropy since non-fuzzy propositions are unambiguous, all the rest is ambiguous. Ambiguity is maximum, generally 1 is choosed to numerically represent this, at the midpoint of the hyper-cube. Such a geometric consideration simplifies the calculation of degree of fuzziness [Kosko-92: 275-278].

Instead of using a quantitative measure of fuzziness, we may simply employ a qualitative typology, as suggested by Kaufman (1975), in order to classify fuzzy sets in rough categories such as 'slightly fuzzy', 'almost precise', 'very fuzzy' [Kosko-92: 275].

3.6. Fuzziness Versus Randomness

Is uncertainty the same as randomness? Many people, trained in probability and statistics, belive so. Especially people in Bayesian camp of statistics, where probabilists view probability not as a frequency or other objective testable quantity, but as a subjective state of knowledge, defends this idea [Kosko-92: 265].

Randomness and fuzziness differ conceptually and theoretically. But, they also have similarities [Zadeh-68, 78] [Dale-79] [Hisdal-88] [Dubois, Prade-91] [Dubois, et al.-91]. Both systems can describe uncertainty with numbers in the unit interval [0,1]. Both systems combine propositions associatively, commutatively, and distributively. The key distinction concerns how the systems jointly treat a proposition p and its opposite ~p. Classical logic theories demands $v(p \Lambda \sim p) = F$, and probability theory conforms :

$$P(p \Lambda \sim p) = v(p \Lambda \sim p) = P(F) = 0.$$

where P is probability of a proposition to be true. So $p \Lambda \sim p$ represents a probabilistically impossible event. But fuzziness begins when $v(p \Lambda \sim p) != F$.

Fuzziness describes event ambiguity. It measures the degree to which an event occurs, not whether it occurs. Randomness describes the uncertainty of event occurrence. Whether an event occurs is random; that's it may occur, but also it may not occur. To what degree it occurs is fuzzy. While fuzziness generally is deterministic, randomness is stochastic.

Consider parking your car in a parking lot with pinted parking spaces. You can park in any space with some probability. Your car will totally occupy one space and totally unoccupy all other spaces. The probability number reflects a frequency history or Bayesian brain state that summarises which parking space your car will totally occupy. Alternatively, you can park in every space to some degree. Your car will partially, and deterministically, occupy every space. In practice your car will occupy most spaces to zero degree. Finally, we can use numbers in [0,1] to describe, for each parking space, the occurrence probability of each degree of partial occupancy - probabilities of fuzzy events.

Fuzziness is a type of deterministic uncertainty. Ambiguity is a property of physical phenomena. Unlike fuzziness, probability dissipates with increasing information [Kosko-92: 265-267].

3.7. Paradoxes And Fuzzy Logic

Basically, a paradox is a proposition having both true and false, or neither true nor false as its truth value.

Paradoxes, such 'sorites' (the 'heap' that remains one even if an object is removed) and 'falakros' (the 'bald man' who remains so even if he grows one additional hair), first time, had been noted by ancient Greek philosophers [Gaines-76: 627].

Logical systems follow two methods when dealing with paradoxes; avoiding them (by treating them as special cases which may be impossible for that system), or assigning truth values to them. Fuzzy logic applies the latter.

In terms of truth value assignment, paradoxes, mainly, can be grouped into two :

a. for which a third truth value is enough, such as Russell's barber, Cretian liar, the card that says on one side "The sentence on the other side is true", and says on the other side "The sentence on the other side is false".

b. for which more than three truth values are required, as in heap, or bald man cases.

The paradoxes in (a) are more dangerous than those in (b). They have the same form. A statement p and its negation ~p have the same truth-value, that's $v(p) = v(\sim p)$. This violates the laws of non-contradiction and excluded middle ($v(p) = 1 - v(\sim p)$, and $v(\sim p) = 1 - v(p)$). But in fuzzy interpretation :

$$v(p) = 1 - v(\sim p), \text{ and } v(\sim p) = 1 - v(p),$$

hence $v(p) = v(\sim p) = 1/2$. So the paradoxes reduce to half-truths [Kosko-92: 4-7].

Fuzziness also resolves the paradoxes of type (b). As an example, consider a heap of sand. Is it still a heap if we remove one grain of sand (a paradox of Sorites type)? How about two grains? Three? The transition is gradually, not abruptly, from a thing to its opposite. We arrive again at the degrees of truth [Kosko-92: 4]. We can imagine it as a road from p to ~p. Beginning from p, with the removal of each grain, we leave p, and approach ~p step by step. On this road, we can answer the question with "it is a heap", "it is almost a heap", "it is like a heap", etc. Or, assign truth values to the proposition "it is a heap", such as 'true', 'almost true', 'nearly true'. Effect of degrees of being true can be given numerically in a more detailed way, such as "it is a heap to a degree of 0.999..., or 0.875..., or 0.764...", etc. [Kosko-94: 94-97].

Notes

(1) Haack gives classification of logical systems as below [Haack-78: 4] :

traditional logic	Aristotelian syllogistic
classical logic	2-valued sentence calculus, predicate calculus
extended logics	modal logics, tense logics, deontic logics, epistemic logics, preference logics, imperative logics, erotetic (interrogative logics)
deviant logics	many-valued logics, intuitionist logics, quantum logics, free logics
inductive logics	

This also shows, roughly, a chronological ordering.

(2) By a proposition is usually meant that which is asserted in the making of a declarative sentence, or "what is common to a set of synonymous (e.g. in different languages) declarative sentences" [Haack-78: 75], the content of act of thinking (not the way of explaining it [Pitcher-64: 5].

(3) It is called as the valuation set [Dudois, Prade-80 : 10].

(4) A function is associated with a domain of objects and it assigns, for each object in its domain taken as argument, a unique object called the value of the function for that argument. Set theoretically, it's a special kind of relation [Lemmon-68: 81].

(5) In fact, any interval of real numbers can be used as valuation set for fuzzy sets, but, since it is assumed that the interval [0, 1] can represent all of them, it is more convenient to use it for practical purposes [Zadeh-65].

(6) Basically, a set can be defined as a collection of objects, generally, which have similar properties or attributes. The main concept related to sets is elementhood, or equivalently membership. Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. Membership in a classical subset A of X is often viewed as a characteristic function m(x) from X to {0,1} such that

$$m(x) = \begin{cases} 1, & \text{iff } x \in A \\ 0, & \text{iff } x \notin A \end{cases}$$

The birth of the set theory can be recognised in Cantor's paper in 1874 in Crelle's Journal (in German) [Johnson-72]. One of the first attempts to axiomatize set theory was performed by Frege. But his system was failed by the invention of Russel's paradox. Then a first powerful axiomatised set theory was constituted by Zermaelo, including a solution for avoiding Russell's paradox [Fraenkel-66].

(7) Natural numbers are defined in terms of empty set and sets of empty sets. Then operations like addition and subtraction are explained in terms of the union and intersection operations between the sets representing natural numbers [Lemmon-68 : 92-120].

CHAPTER 4
STATING FUZZINESS

4.1. Sources Of Fuzziness

How fuzziness appears when we play the game of natural languages? These, at least, as I have seen in the literature, which fuzzy logic deals with, can be summarised as follows [Hisdal-86b] :

a. A subject may recognise that under non-exact conditions of observation, a person may make errors in the estimation of the attribute (e.g. height) values of objects. Assigned truth values may change in worse or better conditions of observation.

b. For an attribute whose assignment should be performed in a under dimensional universe, the subject uses a universe of lower dimension for purposes of assignment, but tries to compensate this with a graded truth. Here, dimensions corresponds to properties which should be taken into consideration during truth value assignment.

c. There can be some quantitative variations between different subjects in the choice of partitioning for assigning truth values (e.g. for an elderly person may think that "Age of 50 year is a medium age" is true, but a young person may think that it is false, unless 50 is replaced with 35. Then it becomes true).

d. Assignment in an over-dimensional universe. For example, suppose that a subject is to assign truth values to sentences about persons concerning the label 'tall'. Let p1, p2 be two persons with the same height value. However, p1 is a very self-assured person, while p2 is very unsure of himself. It may then happen that the subject will say that "p1 is taller than p2" is true. The same may happen when p1 is very thin and p2 is very stout.

e. When taking into account the possible errors during assigning truth values, different subjects may assume different levels of error.

f. There may exist, equally acceptable, different criteria for truth value assignment. In fact, error has at least three meanings, viz. ignorance or incompleteness, falsity, and uncertainty [Niskanen-93: 5].

g. There may exist overlapping of quantisation intervals. For example, at the same time "heights between 1.70 and 1.90 m. are tall", and "heights between 1.60 and 1.75 are medium " may be assumed as true.

h. There may exist lack of specification of the situation. An expression in a natural language with a given 'meaning' (as specified, for example, in a dictionary) has, at most, cases, different 'interpretations', depending on the situation to which it refers.

i. Adjectives are, in most cases, context dependent upon the noun to which they are explicitly or implicitly attached. For example, the truth value of the sentence "it is old" depends upon whether the object (it) is a dog, or a fly, or a house, or a human.

j. There may exist lack of specification of the labels. For example, consider a person, p1, with a very large height value. A subject will assume that "p1 is tall" is true, if he uses only small, medium, and tall as labels. Later, he will assume that "p1 is very tall" is true, but the previous one is not true, if he uses very small, small, medium, tall, and very tall.

k. There may exist lack of clarity as to the inclusive or exclusive meaning of 'or'. For example, "a or b is tall" may mean "a or b or both is tall" (inclusive), or "either a or b is tall, but not both " (exclusive) [Hisdal-86a: 95-97].

The works on fuzzy logic in the literature do not cover ambiguities related to indirect speech, and relative clauses (with or without that, which, etc.). For example, "she saw the man with the telescope" (who had the telescope?), "the girl that chased the cat that sat on the tree gave Mary an apple" (who was on the tree?), "John saw the man in the garden" (who was in the garden?). To deal with the ambiguities contained in these sentences, the sentences should be pre-processed first, and be transformed into propositions to which present fuzzy logical operations can be applied.

4.2. Fuzzy Predicates

A word is an atom of meaning; a sentence is a molecular structure -it is a set of words which we can use to say something [Bastable-75: 16]. The truth or falsity of a proposition is, generally, functionally dependent upon definitions of the words (or terms). What creates problem here is that any definition of a word that can be offered is essentially of the nature of a verbal convention, and therefore theoretically quite arbitrary; that is to say, theoretically, any one such definition is as good as any other, if it is clear and free from contradictions [Ducasse-68: 134] [Machina-72].

Hence, any proposition may be fuzzy according to, for example, the accepted definition of its predicate for any purpose. Consider the sentence "x is tall", where x is a person. For a Pygmy, this sentence may be true, for example, for x's of height between 1.30 m. and 1.50 m., obviously at different degrees. For a Turk, it is true, also at different degrees, for example, for x's of height between 1.75 m. and 1.90 m. Lets look at another example [Machina-72]; "x is color-y", where x is an object, and color-y is any color name, e.g. red, blue, green, etc. For color-y equals red, this sentence is true for x's emitting light of wavelength between, say 6000 nm. and 7000 nm. Truth value for x's emitting light of different wavelengths changes gradually, again.

From truth valuation point of view, fuzzy predicates have similar characteristics. To handle them, it is required, basically, five things to be defined. These are types of criteria that should be taken into consideration, effects of different criteria to its meaning, lower and upper boundaries for these criteria, and steps of degree between these boundaries.

4.3. Hedges

The most important types of words used to express fuzziness are hedges. Hedges are linguistic notions than can weaken or strengthen other notions, when used in combination with them [Hellendoorn-92: 31]. Hedges form the class of adverbs. They are called hedges because they replace the boundaries of the original notions. Hedges can also reveal a great deal more about meaning [Lakoff-73: 473]. Hedges constitutes an important portion of words used to represent fuzziness in linguistic terms. They fuzzify non-fuzzy propositions, and increase the fuzziness of fuzzy propositions.

Below, there are some examples to hedges and related phonemena [Lakoff-73]:

hedge	related phenomena
sort of	in a real sense
kind of	in an important sense
loosely speaking	in a way
more or less	mutatis mutandis
roughly	details aside
pretty (much)	so to say
relatively	a veritable
somewhat	a true
rather	a real
mostly	a regular
technically	virtually
strictly speaking	all but technically
essentially	practically
in essence	all but a
basically	anything but a
principally	a self-styled
particularly	nominally
par excellence	he calls himself a ...
largely	in name only
for the most part	actually
very	really
especially	(he as much as ...
exceptionally	-like
quintessential(ly)	-ish
literally	can be looked upon as

often	can be viewed as
almost	crypto-
typical(ly)	(he's) another (Achilles)
as it were/	(Troy) is the
in a sense/	(Paris) of
in one sense	(ancient times)

This section, partially, will clarify how fuzziness is represented in daily life. I will show this with examples which includes some hedges which are a bit more complex than we often meet (e.g. very, almost, etc.).

Consider the example "Easther Williams (EW) is a fish". It is false, since EW is a human being, not a fish. But "Easther Williams is a regular fish" would seem to be true, at least to some degree, since it says that EW swims well and is at home in water. It presupposes that EW is not literally a fish and asserts that she has certain other characteristic properties of a fish. Here, 'regular' picks out certain 'metaphorical' properties. Another example, consider a married man, Alexandros. "Alexandros is a bachelor" would be false, but "Alexandros is a regular bachelor" might be true, at least to some degree again, since it might be said of a married man who acts like a bachelor.

It is usually assumed that the connotations of words are part of pragmatics. Truth is usually taken to involve literal or denotative meaning alone. Yet in sentences with 'regular', as in the above examples, the truth value of the sentence as a whole depend not upon the literal meaning of the predicates involved, but strictly upon their connotations. What this indicates is that semantics cannot be independent of pragmatics.

We can meet more complex situations if we use some hedges that are opposites of 'regular', i.e. ones that picks out literal meaning alone; for example 'strictly speaking', and 'technically'.

a. A whale is technically a mammal.

b. Strictly speaking, a whale is a mammal.

Technically and strictly speaking seem to have the same effects in the examples given above. However, in other sentences they produce radically different results.

a. Richard Nixon is technically a Muslim.

b. Strictly speaking, Richard Nixon is a Muslim.

The sentence (a) might be true to some degree, but (b) is false. The example above show that, technically picks out some definitional criterion, while strictly speaking requires both the definitional criterion and other important criteria as well. Richard Nixon may be a Muslim in some definitional sense, but he does not have the religious and ethical views characteristics of Muslims. He meets the definitional criterion, but not other important criteria.

"Technically, Diogenes was a racist". In this sentence, technically seem to mean only technically, that is it asserts that the definitional criteria are met but some important criterion is not met.

Strictly speaking contrasts interestingly with loosely speaking.

a. Strictly speaking, a whale is a mammal.

b. Loosely speaking, a whale is a fish.

These sentences show the need for distinguishing between important or primary properties on the one hand and secondary properties on the other hand. (a) says that whales are mammals if we take into account important criteria for distinguishing mammals from fish. For example, they give live birth and breathe air. (b) seems to say that we can call whales as fish if we ignore the primary properties and take into account certain secondary properties, for example their general appearance and the fact that they live in water.

However, 'loosely speaking' still differs sharply from 'regular', as the following examples show.

a. A whale is a regular fish.

b. Loosely speaking, Anaxagoras is a fish.

Both are false, since (a) presupposes that a whale is not a fish to any extent, and (b) says that Anaxagoras is a fish to a small degree. Calling some properties as characteristic-though-incidental will help to distinguish between metaphoric and non-metaphoric usages [Lakoff-73: 473-477].

By looking at just four hedges we can say that we must distinguish at least for types of criteria when assigning truth values to sentences with hedges; definitional, primary, secondary, characteristic though incidental.

4.3.1. Problems with hedges :

1. Dependence upon context. For example, "Technically, this TV set is a piece of furniture". But, there is no generally recognised technical definition accepted that will tell you whether a particular TV set is or is not a piece of furniture.

2. Modifiers that affect the number of criteria considered. Consider the propositions :

a. Troy and Assos are similar.

b. Troy and Assos are very similar.

In judging (a) to be true a certain degree, one might take into account merely their records as ancient city. One might then want to go on to assert (b) by taking into other criteria, for instance, geographical position, settlement, people, language, etc.

3. Perhaps values should not always be linearly ordered, but sometimes only partially ordered. There may exist 'true to a certain degree', but also 'true in a certain respect'. For example,

a. In some respects, Nixon helped the country.

b. In a sense, Priapos was a great man.

c. In a real sense, Nixon is a murderer.

But very often, sentences without such hedges are meant to be taken in the same way.

d. Nixon is a murderer and he's not a murderer.

The usual sense of (d) is not either a statement of contradiction nor a statement that Nixon is a murderer to a degree. Rather it would usually be understood as saying that if you take into account certain criteria for being a murderer, Nixon qualifies, while if you give prominence to other criteria, he doesn't qualify.

What would have to be done in fuzzy logics to incorporate the notion of 'truth in some respect' still is an open question.

4. Problems with 'very'

a. Strictly speaking, Çanakkale is the kind of city I want to live in.

b. Very strictly speaking, Çanakkele is the kind of city I want to live in.

One of the things that 'very' does, when applied to 'strictly speaking', is further restrict the number of categories considered most important : this can be viewed as changing the weights assigned to various criteria at the upper end of the spectrum. This is the opposite of what it does when applied to 'similar'. 'Very' affects also 'loosely speaking' in the opposite of what happened in the case of 'strictly speaking'. It increases the number of criteria considered -or at least increase the weights assigned to the lower end of the spectrum.

Another problem is whether 'very' shifts the threshold (lower and upper) values for being true, or not.

5. Restrictions on the occurrences of modifiers. Some modifiers can apply to other modifiers, but the combinations are quite limited. We get 'very strictly speaking', but not 'very rather' [Lakoff-73: 484-490].

6. Interaction of hedges with performatives. Consider the example "Technically, I said that Sam was a bastard". What this sentence would generally be taken to mean is that I said it but I didn't mean it. That is, 'technically' seems to be cancelling the implication that if you say something, you mean it [Lakoff-73: 490].

4.4. A Few Words More

To systematise the relations between language and truth values, it seems that an appropriate formal language, which is a simulation of a subset of a natural language -the ideal case is to extend it to whole of the natural language; in fact, this bears great problems, at least, for now-, is required. Selection of linguistic values, modifiers, connectives, and quantifiers, sentence formation rules for practical purposes should be given in this language. From the standpoint of philosophy of science, this presupposes concept analysis, examination of theory formation and argumentation analysis, at least, within the context of fuzzy systems [Niskanen-93: 1].

As a result, what should be noticed is that linguistic approaches to fuzzy logic seem inappropriate in various applications [Niskanen-93: 1]. This is not strange, because it should not be expected that anyone can solve all logical problems using only linguistic techniques. He needs language of numbers sometimes when he meets complex problems.

CHAPTER 5
PHILOSOPHY OF FUZZY REASONING

5.1. On True And False

Lots of definitions of truth have been given until today. Since ancient times, along the centuries, the philosophers tried to reach the link between existence and thought. Truth theories were, generally, influenced by the philosophical fashions of the days.

Unfortunately, existing theories of truth bear lots of problems. For example, they do not provide a satisfactory answer to 'types of truth' [Rorty-82: xvi]. Gobar in [Gobar-87] tries to explain this using three different approaches, e.g. logical, epistemological, dialectical (phenomenological, empirical, mathematical, logico-philosophical). Similarly, Kaufmann in [Kaufmann-48] studies on different meanings of truth, and takes our attention that we must clearly distinguish between three different meanings of truth, which are related to three types of judgement in the logic of science, viz.

a. judgements concerning internal relations among propositions,

b. judgements concerning the acceptability of propositions in terms of given standards of validation,

c. judgements concerning principles for the establishment of standards of validation. [Kaufmann-48: 345]

Subjectivity and time are other important problems. As for subjectivity, we are subject to various social and moral pressures, most of the time, when assigning truth values to propositions in daily life, and we sometimes have strong personal interests in doing so. Added to this we all have obsessions, or belief fixations, which prevent us from drawing appropriate inferences; moreover bad habits of reasoning have sometimes been reinforced [Ellis-90: 169]. To handle this Davidson sees the truth as a function of sentence, person, and time, and suggest that a truth bearer should be an ordered triple consisting of them [Davidson-67]. Some suggests, in a similar way, but from a different point of view, that truth is a function of three factors; standard, method, and observer. Since the truth or falsity of the proposition is dependent, among other things, upon the meaning of its predicate, and meaning can be changed. But it is not to say that yesterday's true proposition shall no longer have been true yesterday [Ducasse-68: 135].

As it can be expected, the problems of truth bearers go parallel with those of truth. First of all, what are the truth bearers? Sentences (1) ? Statements (2) ? Propositions? Or, anything else? A detailed discussion can be found in [Pap-54] and [Ducasse-43]. I prefer the term proposition (see Notes in Chapter 2), which generally is preferred by logicians, without paying much attention to deepness it has. Proposition is the technical word used for that which is said, the content of meaning. The same proposition can be expressed in different sentences (e.g. Turkish, English, etc.), and the same sentence can express different propositions (e.g. "I was born in Çanakkale" as uttered by various people). In fact, the science of logic is concerned with the proposition not the sentence [Bastable-75: 16].

No one can deny that we live in a perplexing atmosphere. The traditional concepts of knowledge, and rationality, as that of truth are often argued to have outlived their usefulness. This results in an interesting situation; contemporary philosophy now seems to face a problem of identity. Under the influence of pragmatism, also with the effect of 'linguistic turn', philosophy is now viewed by some as having lost its normative character to a great extent. Broadly speaking, thinkers of recent times, as we have seen during this section, have attacked especially the once-alleged sovereignty of a particular kind of 'reason' stripped of subjective elements. They rightly noticed that such a delusively idealised reason was actually non-existent. Their arguments, taken together, can also be interpreted as a devastating battle against universality (and positively with good reasons).

Marx says that, it is in practice that man must demonstrate the truth, that is, the reality and power, the this-sideness of his thinking. And he continues, the philosophers have only interpreted the world in different ways; but what has become more urgent is to change it [Scheffler-74: 198]. In some respect, he lightens the way of pragmatists.

Rorty, as a pragmatist, joins the debate by saying 'truth' is just the name of a property which all true statements share [Rorty-82: xiii]. This is the idea which fuzzy logicians apply in their works [Niskanen-93: 24].

5.2. Truth As A Tool For Reasoning

From the perspective that logic is a basis for reasoning, there can be only one concept of truth, by which values can be assigned to propositions [Gobar-87: 315]. Unless we have any information about the truth value of a proposition, it is, logically, no sense. Valuation gains more importance in case of fuzzy logic. Hence, from fuzzy logic's point of view, truth, whatever it is, should be measurable, either qualitatively (e.g. in linguistic terms such as very, more or less, almost, etc.), or numerically (e.g. 0.4, 0.7, etc.).

In the literature, most of the time, several kinds of mathematical functions [Baets, Kerre-93; etc.], and metric structures [Niskanen-93] are used for truth valuations. Truth value obtained using metric structures generally is called as metric truth. As for linguistic valuation, relativisation or ordering of truth values is of much importance. This is valid not only for 'true to some degree', but also for 'true in some respect'. The structure or method used for truth valuation should give an answer to both kind. In general, given a proposition p, what is expected from the model or structure on p is that it should give a distance between possible truth values, or more generally a distance between propositions having the same predicate, but different subjects. The general idea is that truth has a continuous form. But continuity is generally restricted with our needs, and with 'low level' apparatus that determines the number and distribution of the perceived values depending on the shape of the curve and various contextual factors [Lakoff-73: 483]. For some case, we really want a continuum of truth values, with an ordering relation defined on it. Fortunately, the assignment of exact values, usually, doesn't matter much for deciding on logical relations between vague propositions; what is of importance instead is the ordering relation between the truth values of various propositions [Machina-76 : 60-61].

Like 'good' and 'bad', the terms 'true', and 'false' are valuations and, from fuzzy logic's point of view, as we have seen in earlier chapters, have reference to 'purpose'.

There are six distinct tests of truth that are today accepted in various quarters as providing a court of last appeal. Indeed there are no doubt more; but at any rate these are the most widely accepted. They are [Lewis-64(2): 216] :

1. correspondence with fact

2. self-evidence
3. coherence
4. 'working' as defined by the pragmatist
5. the peculiar warrant that attaches to mystical intuition
6. the voice of authority

The one that the fuzzy logicians follow is the pragmatist point of view.

5.3. Pragmatism : The Philosophy Behind Fuzzy Logic

Pragmatism, itself, is a vague, ambiguous, that's fuzzy, and overworked word [Rorty-82: 160]. The term is derived from the same Greek word, meaning action, from which our words 'practice' and 'practical' come. It was first introduced into philosophy by Charles Peirce in 1878 [James-72: 43]. Peirce hoped to develop logic as something that would unify science and life. But logic, for him, was scientific logic and not merely dialectic [Thayer-68: 18].

The basics of today's pragmatic understanding was founded by W. James. According to James, philosophy should answer that what concrete difference will anything's being true make in any one's actual life? How will the truth be realised? These are the usual questions of pragmatism. His answer is that true ideas are those that we can assimilate, validate, corroborate and verify. False ideas are those that we cannot. That is the practical difference it makes to us to have true ideas; that, therefore, is the meaning of truth is known-as. [James-72: 133] You can say of if then either that "it is useful because its true" or that "it is true because it is useful". True is the name for whatever idea starts the verification process, useful is the name for its completed function in experience [James-72: 135].

He sees truth as one of the species of good. True is the name of whatever proves itself to be good in the way of belief, and good, too, for definite assignable reasons [James-72: 59]. This means that there is nothing deeper to be said: truth is not the sort of thing which has an essence; it is no use being told such a thing that truth is 'correspondence to reality' [Rorty-82: 162]. Truth, for James, is simply a collective name for verification-processes, just as health, wealth, strength, etc. are names for other processes connected with life, and also pursued because it pays to pursue them. Truth is made, just as health, wealth and strength are made, in the course of experience [James-72: 143].

According to James, truth happens to an idea (that is, an idea may at one time be true, at another time not, depending on whether or not, up to the time in question, the idea has been verified (3)). The truth of an idea is not a stagnant property inherent in it. It becomes true, is made true by events [James-72: 133]. Its verity is in fact an event, a process, the process namely of its verifying itself, its verification. Its validity is the process of its validation [James-72: 196]. The pragmatists tell us, it is the vocabulary of practice rather than of theory, of action rather than contemplation, in which one can say something useful about truth [Rorty-82: 162]. He says that "ideas become true just insofar as they help us to get into satisfactory relation other parts of our experience" [Rorty-82: 205].

Dewey is the second important contributor to the foundation of pragmatism. Truth, for Dewey, seems to refer in general to those conditions which make the difference between what a is a problem and what a solution; truth refers to just that set of conditions and operations which renders a problematic situation unproblematic [Scheffler-74: 199]. According to Dewey, the hypothesis that works is the true one; and truth is an abstract noun applied to the collection of cases, actual, foreseen, and desired, that receive confirmation in their works and consequences [Horwitch-91: 1]. Dewey often used the language of James, saying that "the truth is that which works", or that which proves 'successful' or 'satisfactory' [Scheffler-74: 202]. Whereas Dewey had once tended to define truth as the 'working' or 'satisfactory' procedure of thought, 'the verified' idea or hypothesis, he later preferred to speak of 'warranted assertability', where warranted assertion means an answer produced as a result of inquiry, the interpretation and analysis of the problem [Scheffler-74: 194].

Dewey, then also Kuhn, suggests that we can give up the notation of science travelling an end called 'correspondence with reality', and instead say merely that a given vocabulary works better than another for a given purpose [Rorty-82: 193].

As anyone can expect, 'succes' in validating a 'truth' is a relative term, 'relative to the purpose' with which the truth is claimed. The 'same' predication may be 'true' for me and 'false' for you, if our purposes are different. The determination of truth or falsehood, then, is as various as the purposes and interests that inspire divers human interests and actions. [Scheffler-74: 297].

The latest and most powerful defender of pragmatism is R. Rorty. Pragmatism appeared at the beginning of this century. It had taken strong criticisms, even blamed for being non-philosophical. It has been seen for a long time, as a result of misunderstanding its pioneers, up to 70's, as outdated, until Rorty reactivated it. In this chapter, my aim is only to present, not to discuss, what pragmatism is; especially, how a pragmatist interprets truth. I do this to give a better understanding of how a fuzzy logician, or any user of fuzzy logic, as a pragmatist, interprets truth, and hence fuzzy logic itself. See Note (4) for a brief summary of critiques made against pragmatism before Rorty studied on it. As long as we see James and Dewey as having 'theories of truth', or 'theories of knowledge', or 'theories of morality' we shall get them wrong, he says in [Rorty-82: 160]. They asked us to give up the Cartesian quest for certainty. They had things to say about truth, knowledge, and morality, even though they did not have theories of them [Rorty-82: 161].

He, as a pragmatist, tells us that it is useless to hope that objects will constrain us to believe the truth about them, if only they are approached with an unclouded mental eye, or a rigorous method, or a perpicious language [Rorty-82: 165]. For the pragmatists, the sentences are not true because they correspond to reality, and so there is no need to worry what sort of reality, if any, a given sentence corresponds to -no need to worry about what 'makes' it true [Rorty-82: xvi].

No matter how dark the time, we shall no longer turn to philosophers for rescue as our ancestors turned to priests. We shall turn instead to the poets and engineers, says Rorty (5) in [Rorty-91: 143].

Rorty's characterisation of pragmatism can be summarised as follows [Rorty-82: 162 - 165] :

1. it is simply anti-essentialism applied to notions like 'truth', 'knowledge', 'language', 'morality', and similar objects of philosophical theorising.

2. there is no epistemological difference between truth about what ought to be and truth about what is, nor any metaphysical difference between facts and values, nor any methodological difference between morality and science [Rorty-82: 163].

3. It is the doctrine that there are no constraints on inquiry, save conversational ones - no wholesale constraints derived from the nature of the objects, or of the mind, or of language, but only those

retail constraints provided by the remarks of our fellow-inquirers [Rorty-82: 165].

The pragmatists will tell us, what matters is our loyalty to other human-beings clinging together against the dark, not our hope of getting things right [Rorty-82: 166].

To say, as Dewey wants to, that to gain knowledge is to solve problems, one does not need to find 'continuities between' nervous system and people, or between 'experience' and 'nature' [Rorty-82: 92]. True sentences work because they correspond to the way things are [Rorty-82: xvii].

For the pragmatist, the pattern of all inquiry -scientific as well as moral- is deliberation concerning the relative attractions of various concrete alternatives. The idea is that in science or philosophy we can substitute 'method' for deliberation between alternative results of speculation is just wishful thinking [Rorty-82: 164].

In science, it was discovered that, the decisions of the scientist were not generated from absolute principles, but from the interest conditions they suffered; in other words, the scientific community was the determinant by pragmatic motives. Fuzzy systems used in different disciplines of science are good and popular examples of effects of pragmatism in science.

Pragmatism offer no theory of truth [Rorty-91: 128]. But, it helps enormously to show us where to look for the truth that is reliably attainable, and how to know it when we see it [Armstrong-81: 443]. As can be expected, pragmatists deny bivalence for all statements [Rorty-82: xxvii]. Pragmatism takes the practical use of the truth into consideration, and combines it with vitalism by giving all activities and motions of life to the thought. Pragmatism is very strong in expressing the true in practical life. Without doubt, the rationale of pragmatism is a tenable one. I think that especially contemporary pragmatists have correctly diagnosed sicknesses of the mind. Pragmatists sustained a pluralistic conversation, defined our epistemological goals within possible experience, and turned to our actual problems, which may be called as social, instead of getting submerged in an intangible cloud of 'eternal', 'timeless' philosophical arguments, or grand systems, and so on.

Our critical decisions in general will be pragmatic, granted that in particular cases decisions over what is most useful or needed in our rational endeavours are relative to some given point of view and purposes [Scheffler-74: 431]. Fuzzy logicians and their followers apply this beginning from truth valuation (e.g. choosing thresholds for truth, type of truth function, etc.) to the end, at each step, of a logical process, e.g. in choosing the type of the implication operator (it depends upon properties we desire our inference to have) [Yager-80: 337], etc.

What is more important; what to look at, or where to look from? Pragmatists prefer the latter one as answer.

Today applicability is regarded as the ultimate criterion for a fuzzy system. Hence, within the context of fuzzy logic, the advantages and the disadvantages of the methods applied also be ultimately weighted against each other in practical conditions [Niskanen-93: 24] [Machina-76: 55].

5.4. Approximate Truth

Quantitative information is almost never perfectly accurate. As Einstein says, so far as the laws of mathematics refer to reality, they are not certain; and so far as they are certain, they do not refer to reality [Kosko-92: 263]. Results of measurement, and of most numerical calculations by machine are at best good approximations. Even the best confirmed scientific theories invariably turn out to be inexact. In other words, even 'successful' measurements, calculations, and theories yield results which are false, but near the truth. In fact, the truth value lies in a specific interval, which does not affect the result, hence they generally call it as true [Weston-87: 203]. Truth valuation is subject to the critical objective of maximum usefulness in serving our needs [Scheffler-74: 431]. James takes our attention to an important point by saying that: the greatest enemy of any one of our truths may be the rest of our truths [Thayer-68: 107].

I believe, as Popper does [Gaines-76: 630], that precision and certainty are false ideas. They are impossible to attain, and therefore dangerously misleading if they are uncritically accepted as guides. Precision is analogous to certainty. An increase in the precision of, a prediction, or even a formulation, may sometimes be highly desirable. But, it is always undesirable to make an effort to increase precision for its own sake. Even, it may be dangerous to assume certain forms of precision when they do not exist. One should never try to be more precise than the problem situation needs. The complexity of additional precision should only be introduced when we are forced to 'make new distinctions, for the purpose in hand [Gaines-76: 631]. In general, complexity and precision bear an inverse relation to one another in the sense that, as the complexity of a problem increases, the possibility of analysing it in precise terms diminishes.

Natural language concepts have vague boundaries and fuzzy edges, and natural language sentences will often be neither true, nor false, nor nonsensical, but rather true to a certain extent and false to a certain extent, true in certain respects and false in other respects. By the very nature of the imprecise concepts involved one cannot expect a clear-cut distinction between a statement being true, and its not being true [Gaines-76: 643]. Any attempt to limit conditions for natural language sentences to true, false, 'nonsense' will distort the natural language concepts by portraying them as having sharply defined rather than fuzzily defined boundaries [Lakoff-73: 458].

Degree of a truth may change with increasing of knowledge about it. There is no method for knowing when one has reached the truth, or when one is closer to it than before [Rorty-82: 166]. As being parallel to the concept of truth, knowledge is also a matter of degree. However, degree of knowledge about it is not the only reason for a proposition's being fuzzy. Even though knowledge about a proposition is complete, it may still be fuzzy. Consider the example "Homeros is tall". Suppose that what is required for being tall is to have a height between 1.70 m. and 1.90 m. We know that Homeros is of height 1.75 m.; that's, our knowledge to be able to assign truth value to this sentence is complete, unless we don't need more precision (is it 1.754, or 1.757, etc.?). Then, our example seems true. But, we know that Heredot is 1.87 m. The sentence "Heredot is tall" also seems true. When it is needed to distinguish between their degrees of truth, as in lots of cases in daily life, say, just 'true' for the previous one, and 'very true' for the latter may be used for valuation.

5.5. On Logic

We are symbol-using beings. It is this basic human fact that which has created the opportunity and need for logic. We think and talk symbolically. The language of logic is not a first-order one referring to the world in which we live, but a second-order language which describes and refers to our own symbolic thinking and talking [Bastable-75: 5]. The laws of logic are indeed the expression of 'thinking habits', but also of the habit of thinking. That is to say they can be said to show : how humanbeings think, and also what humanbeings call 'thinking'. The propositions of logic are 'laws of thought', "because they bring out the essence of human thinking"; to put more correctly, because they bring out, or show, the essence, the technique, of thinking. They show what thinking is and also show kinds of thinking [Wittgenstein-56: 41].

There are many sciences working in the world of symbols as there are many natural sciences for first-order reality. Besides logic, there are, e.g. semiotics, semantics, linguistic, grammar, psychology. What is the specific interest and perspective of logic? We can say, broadly, that 'logic' is concerned with the most basic facts about the structuring of meaning and with necessities inherent in thought and speech which arise from the very nature of meaning [Bastable-75: 5]. In fact, whether or not a formal system should count as a logic is itself a question which involves quite deep and difficult philosophical issues [Haack-78: 9].

The following are some basic philosophical questions, which are still open, raised by the enterprise of logic :

Which formal systems count as logics, and why?

Is there one correct logic? and what 'correct' mean here?

How does one recognise a valid argument or a logical truth?

What does it mean to say that an argument is valid? that one statement follows from another? that a statement is logically true?

Is validity to be explained as relative to some formal system?

Or is there an extra-systematic idea that formal systems aim to represent?

What has being valid got to do with being a good argument?

How do formal logical systems help one to assess informal argument?

How like 'and' is conjunction, for instance, and what should one think of 'p' and 'q' as standing for? etc. [Haack-78: 1][Bastable-75: 3-5].

A fuzzy logician answers these questions from a pragmatic point of view. He is against universalism and idealism. He applies this also to his notion of logic. Wittgenstein says that for the crystalline purity of logic (has got us) on to slipperly ice where there is no friction and so in a certain sense the conditions are ideal, but also, just because of that, we are unable to walk. We want to walk; so we need friction. Back to the rough ground.

The existential phenomenologies of both Merleau-Ponty and Wittgenstein eschew clear and simple language-games which ignore friction and air resistance : their goals are to lead philosophy back to the rough ground of a life-world, a world conditioned by history and society. V. Rossvaer states that Wittgenstein's most important insight is that logic must accomodate itself with the world and not the world to logic. Merleau-Ponty contends that : "the phenomenon of truth is known only through the praxis which creates it" and that "truth is another name for cultural sedimentation" [Gier-81: 199]. Wittgenstein believe that grammar is more fundemantal than traditional logic. Traditional concepts of logical necessity have ignored the 'role of thinking and inferring in our life' [Gier-81: 184].

A central concern of logic is to discriminate valid from invalid arguments; and formal logical systems, such as the familiar sentence and predicate calculi, are intended to supply precise canons, purely formal standards of validity. The claim of a formal system to be a logic, Haack thinks, upon its having an interpretation according to which it can be seen as aspiring to embody canons of valid argument. She counts many-valued 'logics' as logics, for example, because they have interpretations according to which their values are 'truth-values', their variables sentences, their operators negation, conjunction etc. (6).

The intention is to distinguish between formal logics and system of, say, arithmetic or geometry, or axiomatisations of biology, physics, and so forth. The demarcation is not based on any very profound ideas about 'the essential nature of logic', indeed, the doubts that there is any such 'essential nature'. But it is not wholly arbitrary; it corresponds reasonably well, she hopes, to what writers on philosophy of logic have in mind when they speak of logics; and it has, at least, some pragmatic rationale.

Deviations of classical logic, i.e. systems with the same vocabulary but different (usually more restricted) axioms or rules : their similarity -formal, in purpose, and intended interpretation- to classical logic makes it natural to regard these systems as logics.

The traditional idea that logic is concerned with the validity of arguments as such, irrespective, that is, of their subject matter -that logic is 'topic-neutral'- could be thought to offer a principle on which to delimit the scope of logic. On this account these systems which are applicable to reasoning irrespective of its subject-matter would count as logics (7). It is suggested, again, that logic applies to reasoning irrespective of its subject-matter because it is concerned with the form of arguments rather than their content [Haack-78: 1-5].

Above all, the logician is concerned with the phenomenon of imlication : the logician is interested in the fact that evidence has force, gravitating our intelligence to a conclusion, that a conclusion is folded up in evidence and falls out when we scrutinize the evidence sufficiently. Historically, this phenomenon, one way or another, occupied a central position in logic and it would not be a bad introductory description to say that logic is concerned, principally, with observing and analysing varieties of implication and with formulating a theory of implication that will elucidate the phenomenon in general and will provide a logical foundation for reasoned conclusions in every area of life and science. The greatest logicians of modern times have taken this as central theme, and it seems reasonable to say that everything else in the corpus has its place there because of its connection with the main enterprise of classifying and articulating the principles of formally valid inference. What does inferring really consist in? In the transition from one assertion to another? During inferrence, there occurs a transition from one proposition to another via other propositions, that is, a chain of inferrences. There is nothing occult about this process; it is a derivation of one sentence from another 'according to a rule'; a comparison of both with some paradigm or another, which represents the scheme of the transition, or something of the kind. The conclusion may also be drawn in such a way that one proposition is uttered after the other, without any such process; or the process may consist merely in our saying 'therefore', or 'it follows from this', or something of the kind. We call it a 'conclusion' when the inferred proposition can in fact be derived from the premise [Bastable-75 :5-6]. According to Wittgenstein, logic is a kind of ultra-physics, the description of the 'logical structure' of the world, which we perceive through a kind of ultra-experience (with the understanding e.g.). What we call 'logical inference' is a transformation of our expression. We must only infer what really follows. The mind carries out the special activity of logical inference according to some laws. Does the mind always infer according to these laws? What does the specific activity of inferring consist in? It is supposed to be a fundamental law of logic that it is correct to run in this way; and this fundamental law should presumably be self-evident [Wittgenstein-56: 6-9] (8). When we say "this proposition follows from that one", that is to accept a rule. The acceptance is based on the proof [Wittgenstein-56: 13]. We might also say : when we follow the laws of inference (inference-rules) then following always involves interpretaion too [Wittgenstein-56: 34].

In an article 'The Triumph of Ambiguity', J.E. Marsh states that whereas earlier thinkers, Russell, Wittgenstein, himself in his

Tractatus, and Husserl, tried to overcome ambiguity; the later Wittgenstein and Merleau-Ponty accept ambiguity, and make equivalent moves away from meaning conceived as concrete, pluralistic, contextualistic, implicit and tentative. In the later works Wittgenstein distinguishes carefully between philosophical clarity and exactness. He completely rejects the Socratic method of exact definition. He says that we mistakenly want to say that there cannot be any vagueness in logic, and continues that he strieves not after exactness, but after a synoptic view. Wittgenstein says that some concepts are essentially (and not accidentally) inexact. Both Wittgenstein and James are aware of the fact and importance of 'vagueness' in human communication, and both offer a way out of 'vagueness' by insisting that we look to the practical circumstances of contextual use. Wittgenstein told his class that philosophy is not analysis if that means taking something apart and discovering something new. He also strongly rejects logico-deductive methods, which start with certain self-evident propositions from which other propositions are deduced, which is very sound and a way in which we do work, and whose goal is to find out where ambiguities in our language are. The logic of vagueness of Merleau-Ponty and Wittgenstein is not at all descriptive; and at least it conforms to the ambiguities of lived experience. These ambiguities are grasped by a tacit knowing which only becomes problematic when one attempts to make it explicit. S.B. Mallin phrases it : the experience of evidence will not, therefore, be of a propositional truth but of a situational truth, and it will be ambivalent ambiguous to thetic powers of reflection.

According to Wittgenstein, philosophy is therefore the art of seeing connections, seeing synoptically, rather than seeking causal connections, proposing historical explanations, or making speculative claims about the nature of reality [Gier-81: 73-78].

Wittgenstein believes that internal relations are in the world not exclusively in the mind or mere linguistic connections, and not exculisively in things themselves [Gier-81: 87]. Wittgenstein is interested in the logic (the grammar, the essence) of a phenomena; therefore his philosophy is a trancendental phenomenology (a logos of phenomena) and not a mere phenomenalism on physics [Gier-81: 94]. Wittgenstein, life-philosophers, plus some pragmatists offer a radical alternative to traditional, as well as contemporary philosophical methods, which at least agree in a self-justifying reason and prescriptive logic [Gier-81: 201].

Both Husserl and Wittgenstein believe that there is a philosophical grammar more fundamental than traditional logic. Wittgenstein calls his grammar 'a theory of logical types'. Grammar shows the possibility of constructing true or false proposition, but not the truth or falsehood of any particular proposition. Therefore, grammar describes logical forms or types. The logical possibility of p depends on whether p has sense; if p has sense (where sense is anything that grammar allows), then this shows that p is possible. From 1929 onwards, Wittgenstein revises his views of grammar considerably. What does now have significance for Wittgenstein is the concept of 'systems' of language, language-games, which represent different linguistic worlds. Wittgenstein calls the rules of grammar 'arbitrary' and 'conventional'. Meanings are no longer atemporal, ideal entities-meaning is use.

Husserl accept many different kinds of sense, he never would have embraced a pluralism of 'truths'. For him, there are, of course, 'everyday practical situational truths', but only one objective truth for Husserl's logic of truth, which remains purely formal. For Wittgenstein, however, there are, e.g., true somethings in Euclidian system that are false in another system. As we have seen, what is true and false depends on one's Weltbild (9), or an agreement in Lebensformen (10). For Wittgenstein to state that the rules of grammar are arbitrary is not to say that we can change them at will-we must remain within our given Weltbild and Lebensformen. We remain because grammatical rules are arbitrary, but their application is not. What 'arbitrary' does mean is that there is no reason, no justification, why some propositions make sense and others do not. There is no such thing as a justification and are ought simply to have said : that's how we do it. As Wittgenstein elaborates : "the language in which we might try to justify the rules of grammar of our language would have to have a grammar itself. No description of the world can justify the rules of grammar".

For the later Wittgenstein the rules of grammar have no justification and we can give none : we must begin with the distinction between sense and nonsense. Nothing is possible prior to that. We cannot give it a foundation. This means that grammar as a pure logic of ideal meanings built on a strict distinction between Sinn and Unsinn (11) is gone. We begin in a world already interpreted by a language, where the bounds of 'sense' are already set [Gier-81: 99-103].

The foundations of logic are pre-epistemological; they are found deep in our nature, in our needs, our purposes, and our forms of life. Logic is an 'anthropological phenomenon', it is invention and not discovery. The laws of logic are the expression of thinking habits, and as long as we are in the habit of not thinking otherwise, then we will continue to infer as we do, and to hold some concepts as a priori and others as emprical [Gier-81: 195].

Wittgenstein generates his own attack on modern logic. Heidegger reveals that 'the idea of logic' itself disintegrates in the turbulame of a more original questioning. Heidegger's intent was not to abolish logic, but simply to show that traditional logic was inadequate, not incorrect [Gier-81: 185]. Wittgenstein believes that the laws of logic are like laws of society. The inexorability of logic lies in us and our forms of life [Wittgenstein-56: 36]. A.B. Levison sums up the point nicely : "these laws are not inexorable; rather it is we who are inexorable in applying them -because we wait them to do a job and because we see that in order to get the job done by everyone has to do it by the same rules". Wittgenstein's conception of logic is essentially that of an evolving structure and pattern of developing uses, customs, criteria and standards. What is distinctive of Wittgenstein is neither the abolition of logic, nor its absolute severance from the emprical conditions of developing inquiry, but its 'temporalization'.

Wittgenstein declares that Aristotelian logic is a language-game, and in later works, agrees that "the rules of logical inference are rules of the language-game". Language-game have a historical basis and therefore can change. If Aristotelian logic is a language-game, then it is just one of many ways of looking at the world, of making sense of it, and it is conceivable that more primitive (for example, in which negation and double negation are done in a way quite baffling to us but gramatically correct for them) 'logics' can serve just as well without following Aristotle' s laws of inference. In our own culture it is easier to conceive of someone accepting the idea of a man-god in one context, but rejecting the idea of a round-square in another, while many would find the two concepts logically analogous. If what belongs to a language-game belongs to logic, then there could be as many logics as there are language-games. Like Wittgenstein, Mauthner believed that logic and grammar are historical products and that there would be as many logics as there were languages with different structures. He would have agreed with Wittgenstein' s claim that "whether a thing is a blunder or not -it is blunder in a particular system in a particular game".

The reason why we persist in believing that there is only one logic, i.e. the strict reason of traditional logic, is because of the dominance of this particular language-game in the history of western thought. Indeed, each culture will insist that its way of putting the 'world' together is the connect one, primarily because, as Mauthner put it, "it is against logic, against the feel of language, to construct a plural for word logic".

Wittgenstein believes that language-games and forms of life, regardless of their content and internal consistance or coherence, are autonomous systems with their own sense and their own ways of distinguishing between truth and falsity. He also believes that the true and the false depend upon one's Weltbild or form of life. He says "a style gives us satisfaction, but one style is not more rational than another". P. Winch argues that "the criteria of logic are not a direct gift of God, but arise out of, and are only intelligible in the context of ways of living or modes of social life". Winch bases this conclusion on Wittgenstein's claim that agreement in judgements is derived from a pre-cognitive agreement in forms of life. Winch states that learning to infer is not just a matter of being thought about explicit logical relations between propositions; it is learning to do something within the social context of forms of life [Gier-81: 188-191]. Spengler distinguishes between an 'organic' logic, which is instinctive, temporal, and true to life, and an 'inorganic' logic, which is cognitive, atemporal, and based on the concept of lifeless extention. G. Simmel concurs in this life-philosophical view of logic by insisting that truth is not independent from life and by offering the following test: "if it is true it will preserve and support our life; if it is an error, it will lead us into ruin" [Gier-81: 200].

Wittgenstein's notion of language-games, which imagines a plurality of games generating just as many logical forms, "destroys grammatical prejudices and makes it possible for us to see the use of a word as it really is". During the early modern period Aristotle' s theory of judgement was merged with another methodological bias, viz., the priority of epistemology. In order, for humans to make sense and judge assertions to be true, they first have to know, clearly and distinctly, the concepts are using and the rules of judgement. Wittgenstein's On Certainty is a radical critique of this philosophical method. Judging is not based on explicit knowledge, but on cultural conditioning and upbringing : "we do not learn the practice of making emprical judgment by learning rules: we are taught judgments and their connection with" other people, with those whom the person shares a Weltbild. If people are to use language to communicate, there must already be a pre-epistemological agreement in judgments [Gier-81: 193].

Brouwer says that the rejection of the law of the excluded middle would have no effect on the operation of other laws of logic. Wittgenstein says that if there happens to be a contradiction in a system that has served us well, we should not reject the system, we modify strict reason and our concept of certainty. As the word clearly implies, a contradiction is a 'talking-against' not necessarily a 'being-against'. I.M. Bochenski recognizes that "there are systems in which the principle of contradiction does not hold, but these systems must themselves be constructed without contradictions" [Gier-81: 201]. H. LeRoy states that making sense is not based upon knowing, but rather knowing itself must make sense. In other words, knowledge and logic can never be seperated from living in a 'world' in a certain way, putting it together according to specific language-games and forms of life [Gier-81: 199]. $(\sim p \rightarrow p)$ is simply empty if it can play no role in significant discourse. J.N. Findlay concurs "the logic of strict entailment is a limited and impoverished paradigm of the ways in which thought typically functions. According to Wittgenstein, there is nothing iron-clad about this particular law that two negations yield an affirmation. To establish the law as a logical product of the concept of negation and the law of contradiction is simply to choose one way of our thoughts and speech [Gier-81: 192-193].

Notes

(1) By a sentence is meant "any grammatically correct and complete string of expressions of a natural language" [Haack-78: 75].

(2) By a statement is meant a declarative sentence uttered for the purpose of asserting something [Haack-78: 76].

(3) If an idea is in essence an instrument to a practical end, its success lies in reaching that end, and such success is truth. Similarly failure is falsity [Lewis-64: 347].

(4) The hardest critics against James, Dewey, and generally pragmatism were made by B. Russell. An interesting question he asks is that "the pragmatist view of truth is itself true?"

Most of the objections against James's doctrine of truth (thus Dewey's) can be summed up in one argument, which, historically, represents the paradigm case against pragmatism. Are ideas or beliefs true or not depending on whether or not they 'work' or their consequences are 'successful' or 'satisfactory'? They answer :

a. we can never know whether an idea is true or false, since we can never know all of the consequences or far-reaching effects;

b. the same idea can be both true and false, since it may prove satisfactory at one time, unsatisfactory at others, or may satisfy some persons while dissatisfying others;

c. we can never know whether the pragmatic definition of truth is justifiable or useful.

On the larger social and ethical consequences of pragmatism, the paradigm case alarms us to these dangers : truth and falsehood will be determined by the desires and interests of men with power; the state, police, or politicians will decide what consequences of what ideas are 'successful' or 'satisfactory'. Pragmatically, then, pragmatism is a socially disastrous philosophy [Scheffler-74: 202].

(5) I join Rorty to some degree, or from some respect, but not completely. It is evident that engineers solve lots of problems with pragmatic approaches. (In my thesis, I am interested especially in their solutions locally, that's without taking care of their post and side effects to other things.) However, from a global point of view, sometimes, we are paying much for their solutions to local problems; because, they are causing greater problems to appear, in a wider sense, as a result of their work. To see examples, look at your environment, weather, seas, soil. Consider an ultimate criteria, by forcing the limits of pragmatism, to have truth, and let this be the happiness of all human-beings. Are we happier than our ancestors who lived, say, 100 years before us? Can we say that people who will live after 100 years will be happier than us? Will engineers, with other technicians and scientists, be able to prepare a better future for us? Unfortunately, I am not sure about the answers of these questions.

(6) There are some philosophers who have suggested that many-valued systems shouldn't count as logics. It is true that some many-valued systems were devised and investigated out of purely formal interest, or for purposes of computer technology; but it is also true, and important that such pioneers as Lukasiewicz and Bouchvar quite clearly regarded themselves as presenting logical systems as alternatives to classical apparatus [Haack-78: 8].

(7) If particular weight is placed on the role of logic as a guide to reasoning, as in my thesis, as means of informal arguments, one

might see some point in requiring that logical systems be decidable that there be a mechanical procedure for settling whether or not a formula is a theorem. From some point of views this might restrict the scope of logic indeed, for though sentence calculus is decidable, predicate calculus is not [Haack-78: 7].

(8) Self-evidency itself is a great problem.

(9) World-view.

(10) Form of life, or Life-form.

(11) Meaning and Unmeaning.

CHAPTER 6
FUZZY REASONING

6.1. Complexity Of Human Reasoning

It may be argued that most of the concepts encountered in various domains of human knowledge are, in reality, much too complex to admit of simple or precise definition. This is true, for example of the concepts of recession and utility in economics; schizophrenia and arthritis in medicine; stability and adaptivity in system theory; sparseness and stiffness in numerical analysis; grammaticality and meaning in linguistics; performance measurement and correctness in computer science; truth and casuality in philosophy; intelligence and creativity in psychology; and obscenity and insanity in law [Zadeh-76: 249]. Much of human reasoning is approximate rather than precise in nature. As a case point, we reason in approximate terms when we decide on how to cross a traffic intersection, which route to take to a desired destination, how much to bet in poker, and what approach to use in proving a theorem. Indeed it could be argued that only a small fraction of our thinking could be categorised as precise in either logical or quantitative terms [Zadeh-75a]. Much of the logic behind human reasoning is not the traditional two-valued or even multivalued logic, but a logic with fuzzy truths, fuzzy connectives and fuzzy rules of inference [Zadeh-65, 73, 75a].

Especially in humanistic systems, as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which complexity, precision, and significance can no longer coexist. Using a linguistic approach makes it possible to analyse in an approximate manner those humanistic as well as mechanistic systems which are too complex for the application of classical techniques [Zadeh-76: 250]. Hence, any attempts to model, or emulate, humanistic systems by formal methods of increasing precision will lead to decreasing validity [Gaines-76: 628].

But, in addition to these, it should be noted that, it would clearly be unfruitful to deliberately 'fuzzify' a situation unnecessarily [Gaines-76: 632].

6.2. Approximate Reasoning

Informally, by approximate or, equivalently fuzzy reasoning (1) it's meant the process or processes by which a possibly imprecise conclusion is deduced from a collection of imprecise premises. Such reasoning is, for the most part, qualitative rather than quantitative in nature and almost all of it falls outside the domain of applicability of classical logic [Dubois, Prade-80: 173].

Approximate reasoning may be viewed as the determination of an approximate solution of a system of relational assignment equations in which the assigned relations are generally fuzzy rather than non-fuzzy subsets of a universe of discourse [Zadeh-65: 409]. A characteristic feature of approximate reasoning is the fuzziness and non-uniqueness of consequents of fuzzy premises. Simple examples of fuzzy reasoning are [Zadeh-75a: 407] :

a. Most men are vain;
Socrates is a man;
Therefore it is very likely that Socrates is vain.

b. x is small;
x and y are approximately equal;
Then it is likely that y is small .

6.2.1. Inference

The words 'infer', 'inferring', 'inference' refer to the mental act of drawing and asserting a conclusion and since this is a personal act the mind may be gravitated toward the concluding assertion by a variety of forces. Certainly in real life reasoning is not simply structured in full and conscious conformity with logical rules of evidence.

Implication is an important concept used in inference. The word 'implication' does not refer to at all to the context of personal reasoning. It denotes a relationship between propositions such that the meaning of one proposition (the conclusion) is folded up (implicated: 'plex', a fold) in the meaning of another proposition or in the combined meaning of other propositions (evidence). The implication is objective; it is there whether or not any person is subjectively conscious of the fact. Thus "All S is P" implies that "Some S is P"; ideally, of course, my inference that "Some S is P" should be governed by the perception of the implication. On the other hand "Some S is P" does not imply that "All S is P"; notwithstanding the lack of implication, I may in real life make that kind of inference [Bastable-75: 23].

For fuzzy logic, it is obvious that inference also is a fuzzy concept. If we restrict ourselves to propositional variables with only values true and false, "p -> q" (material implication) is indistinguishable from "~p V q". But when one considers the range of intermediate values between true and false, "p -> q" becomes very different from "~p V q " [Lakoff-73: 465].

6.2.3. Rules Of Inference

In order to perform approximate reasoning with statements having imprecise truth values, we need some rules so as to assign truth values to them (2). Specification of these rules are needed to make precise applications of approximate reasoning, hence of approximate truth -also to understand and to express them [Weston-87: 225]. These rules can be summarised as follows :

1. Composition rules. They pertain to the translation of a proposition p that is a composition of propositions q and r, such as conjunction (also its extended versions, namely conjunction introduction, and conjunction elimination -for example from "John is tall and fat" we can infer "John is tall"), disjunction, implication [Zadeh-65, 71].

2. Quantification rules. These rules work on propositions of the form p = "FX are A" where F is a fuzzy quantifier [Niskanen-93][Yager-91]. The classical logic allows for the inclusion of only two quantifiers, for all and there exists (not none) called the universal (∀) and existential (∃) quantifiers. Fuzzy logic uses an extended set of quantifiers (e.g. 'most', 'many', 'few', 'some', etc.) to be able to represent linguistic quantification. Here, also X may be a fuzzy concept. Truth values for classical logic's quantifiers can simply be obtained by using $v(\forall x\ p(x)) = \min \{v(p(x))\}$, and $v(\exists x\ p(x)) = \max \{v(p(x))\}$ [Lakoff-73: 467].

3. Qualification rules. Among pertinent qualifications for propositions Zadeh considered three of them in particular [Zadeh-65, 73, 75-b,c,d]:

a. Linguistic truth qualification. A truth-qualified version of a proposition such as "X is A" is a proposition expressed as "X is A is k" where k is a linguistic truth value.

b. Linguistic probability qualification. A probability-qualified version of a proposition such as "X is A" is a proposition expressed as "X is A is y" where y is a linguistic probability value such as 'likely', 'very likely'.

c. Linguistic possibility qualification. A possibility-qualified version of proposition such as "X is A" is a proposition expressed "X is A is w" where w is a linguistic possibility value such as 'possible', 'very possible', 'almost possible'.

In fact, linguistic truth qualification can easily be transformed into numerical truth qualification, as used in most of the works in the literature [Mamdani-75, 77] [Zadeh-73, 75a, b, c, d].

4. Modifier rules for propositions. Given the propositions p1 = "X is A" such that v(p1) = v1, these rules are needed to find v(p2) = v2 related to p2 = "X is mA", where m is a modifier such as 'not', 'very', 'more or less', etc. [Lakoff-73: 465]. These can be categorised as follows :

i. for simple propositions, i.e. p = "X is mA".

ii. for composed propositions, i.e. m("X is A and/or Y is B").

iii. for quantified propositions, i.e. "(mF)X are A".

iv. for qualified propositions, i.e. "X is A is md", where d is a linguistic qualifier.

This kind of propositions, generally, causes semantic entailment. A proposition p1 semantically entails another proposition p2, that's p1 => p2, iff v(p1) > v(p2). Let p1 = "John is very tall", and p2 = "John is tall", in such a case p1 semantically entails p2.

6.2.4. Generalisation Of Modus Ponens

An important special case of the compositional rule of inference is obtained when p and q are of the form p = "X is A", q = "If X is A, then Y is B". The modus ponens rule allows r = "Y is B" to be inferred from p and q in propositional logic. Since "p -> q" becomes different from "~p V q ", on the contrary the case in classical logics, in fuzzy logics, the treatment of modus ponens is not so straightforward [Dubois, Prade-80: 167-187]. We need some modifications. Modus ponens is not only a valid form of inference, but it can be generalised so that it preserves degrees of truth [Lakoff-73: 465]. These modified forms of modus ponens which are called as generalised modus ponens may be viewed as a generalised extrapolation [Dubois, Prade-80: 174]. The generalised modus ponens (GMP) is an inference rule in fuzzy logic, with the following scheme :

if s1 is p1 then s2 is p2,	(= a -> b)
but s1 is q1,	(= a1)
______________________	
hence s2 is q2.	(= b1)

where s1 and s2 are subjects, p1 and p2 are objects or properties - usually a vague or fuzzy linguistic notion. The objective is to determine q2, given p1, p2, and q1. If we know that a is true at least to degree x, and we know that a -> b is true, then we can expect that b is true at least to some degree [Lakoff-73: 465]. The predicate p1 is fuzzy, hence the condition s1 is p1 to derive s2 is p2 is not strict. Therefore, it should be possible to render a reasonable approximation of q2 when q1 diverges slightly from p1.

In the literature there are three basic assumptions for dealing with GMP in fuzzy logic [Hellendoorn-92: 29] [Reddy, et al.-92]:

1. the if-then rule is implemented by a linguistic fuzzy relation (3).

2. the premise p1 can be strengthen or weakened to obtain q1 by taking a power of the membership function of p1,

3. the conclusion q2 is derived by means of the max-min composition rule.

These are some examples of reasoning with GMP :

If John is tall, then he is heavy.
John is very tall.

∴ John is considerably heavy.

If a tomato is red, then it is ripe.
This tomato is light red.

∴ This tomato is not very ripe.

The only criteria to test the validity of the methods used for GMP is practical applicability. To be able to in accordance with the practical conditions these are some criteria which GMP ought to satisfy (it is assumed that p1 and p2 are increasing notions) [Hellendoorn-92: 31] :

1. when q1 is equal to p1, then it is evident that q2 should be equal to p2.

2. when q1 is somewhat weaker than p1, then q2 will also be somewhat weaker than p2, although it is not known to what degree.

3. when q1 is somewhat stronger than p1, then with the same arguments as above, q2 will be somewhat stronger than p2.

4. "if s1 is p1 then s2 is p2" loses its power when then antecedent deviates from p1.

5. A small change in the antecedent should not cause a great change in the consequent.

Another important reasoning method, namely modus tollens (that's, we know a -> b, but this time we know ~b, then we can infer ~a. For our case, we know ~b1 which is slightly different than ~b), can be handled, that's generalised for approximate reasoning, by using methods similar to those used for generalisation of modus ponens [Sugeno, Takagi-83].

Notes

(1) Possibilistic reasoning is another term used in parallel with fuzzy reasoning [Zadeh-78] [Ruspini-96: 13-17].

(2) This process is called as 'fuzzification' in the technical literature. Similar to this, the process of transforming an arbitrary truth value, resulting from inference, into a meaningful value (a real number, etc.; for example, to use for valve setting, temperature, weight, etc.). Basically four methods are used for defuzzification, which are called as 'maximum of maxima', 'mean of maximum', 'center of maxima', 'centroid of largest area' [Zimmermann-96: 84-88] [Kara-Zaitri, Fleming-96: 15].

(3) A relation, quite generally, is defined as a set of ordered pairs. Then to say that an object x bears relation r to object y will simply

mean that the ordered pair <x, y> is a member of the relation r [Lemmon-68: 64].

Roughly, a pair, {x, y}, is a set of sets x and y [Lemmon-68: 34]. Then the ordered pair of x and y, in symbols <x, y>, is distinguished from the pair {x, y}, such that {x, y} = {y, x} in general fails for ordered pair. <x, y> represents the entities in that order, and so is not in general identical with <y, x>. So ordered pairs <x, y> and <u, v> will be identical only if they have identical first elements and also identical second elements [Lemmon-68: 57].

CHAPTER 7
CONCLUSION

Classical (bivalent) logics have inefficiencies in applications to everyday life. Because our normal language habits, to represent the imprecise situations of real life, seem so different. Fortunately, we have an alternative to fill the gaps of classical logics' treatment of daily life situations.

In fact, there still are some, as me, who sees fuzzy logics as families of bivaluated logics [Fox-81] [Klement, Schwyhla-82] [Castro-94]. I think that whatever you do with fuzzy logic, you can do it without it, that's only with bivaluated logics; but paying much effort. Most of the time, reinterpretation of the problem in bivaluated terms should be required for doing this. For example, notion of possible worlds -where it is not a particular kind of thing or place, but it is what truth is relative to, what it is the point of rational activities such that deliberation, communication, and inquiry to distinguish between [Stalnaker-86: 117]- may be a good starting point for such a case.

Haack ends her critiques on fuzzy logic by saying that we don't need it [Haack-79]. It is impossible to share her idea. Because, first of all, it provides, at least, a method to handle lots of problem in a simpler way than before, and different than other multi-valued logics (see "Randomness Versus Fuzziness").

To sum up what I said up to here, first of all, in natural language, truth is a matter of degree, not an absolute. So, it requires special treatment. Fuzzy logic promises, even already gives, a better way, than those of bivalent ones, to perform this. Fuzzy concepts occurring in it have internal structure. Basic internal distinction is that they are either 'true to some degree', or 'true from some respect'. Their semantics is not independent of pragmatic considerations. For detailed handling, most of the time, algebraic functions play a role in the semantics of certain hedges. Fuzzy logics including hedges requires serious semantic analysis for all predicates, because of special characteristics of hedges. Hedges show that formal semantics is a convenient approach to the logic of natural language [Zadeh-71] [Giles-79, 82] [Klein-80].

As regards fuzzy reasoning, today applicability is the ultimate criterion, and the examination thus mainly pivots on the semantic and pragmatic problems [Niskanen-93: 1]. Is all-out pragmatisation a desirable and/or viable project? This is yet a question, which is extending the aim of this thesis, to be answered, given philosophers' unquenchable desire for questioning and pushing the cognitive limits of the animal we are and for staying somewhat outside the turmoil of daily flux. Under the influence of pragmatism, philosophy is now viewed by some as having lost its normative character to a great extent. Fuzzy logic, with explanations it brings into human reasoning, and with its working applications especially in scientific and technologic area, constitutes a strong example for pragmatism. Truth is one of the basic concepts for a fuzzy logician, or any user of it. For a fuzzy logician, truth, simply, is only a common name for propositions which are true, at least to some degree. What makes a proposition 'true' is the events; that's it is made true in the course of experience. For him, since it is one of the basic concepts for reasoning, first requirement is that the concept of truth, whatever it is, should be, as for other logicians, a valuable one. The second one, going one step further from the previous one, is that is should be measurable; because what is of importance, for him, instead is the ordering relation between the truth values of various propositions. With this approach, he solves lots of complex problems in an easier way. He also adapts, for this purpose, classical reasoning mechanisms to what he has in hand, instead of waiting what he has in hand to satisfy the requirements of them.

Is it really a logic? Sure. It deals with what classical logics deals with. It uses the same language; but some different axioms. It is a result of reaction against idealism and universalism. It takes care of friction and resistance, unlike others, to feel the real world.

In fact, it can be thought, at least for now, that we should not expect more on fuzzy logic than that it facilitates the development and study of models of systems which are ambiguous or inexact. I think this a sufficient pro to continue to study on logic of fuzzy concepts seriously.

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