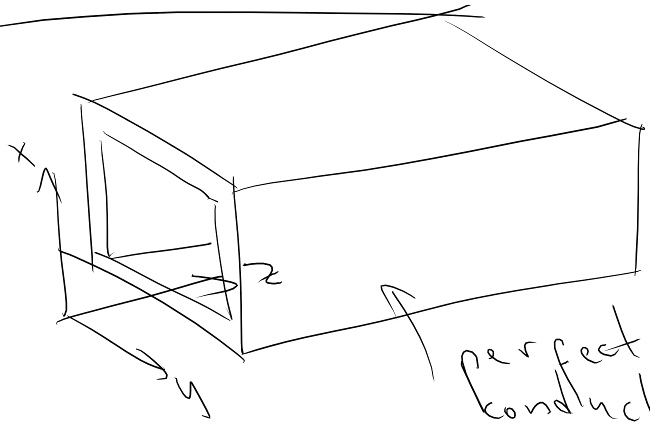
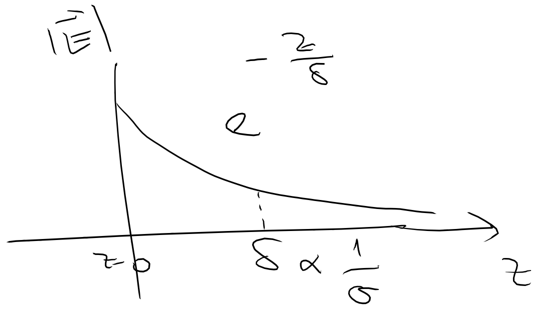
<u>Hand in your HW!</u>

Wave Guides



perfect conductor $\vec{E}=0$, $\vec{B}=0$ } inside the conductor.

conductor



$z<0$ $\quad z=0 \quad$ $z>0$

$$\vec{E} = \vec{E}_0(x,y)\, e^{i(kz-\omega t)} \qquad \vec{E}_0 = E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z}$$

$$\vec{B} = \vec{B}_0(x,y)\, e^{i(kz-\omega t)} \qquad \vec{B}_0 = B_{0x}\hat{x} + B_{0y}\hat{y} + B_{0z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu\vec{J} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}$$

$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{B}_\perp$ is continuous $\Rightarrow B_\perp = 0$ on the surface

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow \vec{E}_\parallel$ is continuous $\Rightarrow \vec{E}_\parallel = 0$ on the surface.

$$\vec{\nabla}\cdot\vec{E} = 0 \Longrightarrow \frac{\partial E_{0x}}{\partial x} + \frac{\partial E_{0y}}{\partial y} + ikE_{0z} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0 \Longrightarrow \frac{\partial B_{0x}}{\partial x} + \frac{\partial B_{0y}}{\partial y} + ikB_{0z} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = i\omega\vec{B}$$

$$i\omega B_{0x} = \frac{\partial}{\partial y}E_{0z} - ikE_{0y}$$

$$i\omega B_{0y} = -\frac{\partial}{\partial x}E_{0z} + ikE_{0x}$$

$$i\omega B_{0z} = \frac{\partial}{\partial x}E_{0y} - \frac{\partial}{\partial y}E_{0x}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} = -\frac{i\omega}{c^2}\vec{E}$$

$$-\frac{i\omega}{c^2}E_{0x} = \frac{\partial B_{0z}}{\partial y} - ikB_{0y}$$

$$-\frac{i\omega}{c^2}E_{0y} = -\frac{\partial B_{0z}}{\partial x} + ikB_{0x}$$

$$-\frac{i\omega}{c^2}E_{0z} = \frac{\partial B_{0y}}{\partial x} - \frac{\partial B_{0x}}{\partial y}$$

$$\left.\begin{aligned}
E_{0x} &= \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_{0z}}{\partial x} + \omega\frac{\partial B_{0z}}{\partial y}\right)\\
E_{0y} &= \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_{0z}}{\partial y} - \omega\frac{\partial B_{0z}}{\partial x}\right)\\
B_{0x} &= \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_{0z}}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_{0z}}{\partial y}\right)\\
B_{0y} &= \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_{0z}}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_{0z}}{\partial x}\right)
\end{aligned}\right\} \text{180 (pg 431)}$$

$$i\omega B_{0z} = -\frac{\partial}{\partial y}\left[\frac{i}{(\omega/c)^2-k^2}\left(\cancel{k\frac{\partial E_{0z}}{\partial x}} + \omega\frac{\partial B_{0z}}{\partial y}\right)\right]$$

$$+\frac{\partial}{\partial x}\left[\frac{i}{(\omega/c)^2-k^2}\left(\cancel{k\frac{\partial E_{0z}}{\partial y}} - \omega\frac{\partial B_{0z}}{\partial x}\right)\right]$$

$$\cancel{i\omega}B_{0z} = \frac{-\cancel{i\omega}}{(\omega/c)^2-k^2}\frac{\partial^2 B_{0z}}{\partial y^2} - \frac{\cancel{i\omega}}{(\omega/c)^2-k^2}\frac{\partial^2 B_{0z}}{\partial x^2}$$

$$\boxed{\left\{\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} + \left(\left(\frac{\omega}{c}\right)^2 - k^2\right)\right\}\left\{\begin{matrix} B_{0z} \\ E_{0z}\end{matrix}\right\} = 0}$$

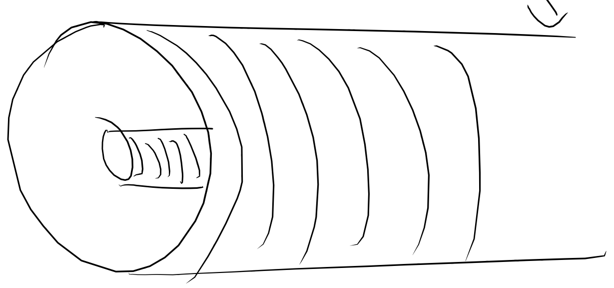
if $E_{0z} = 0$: TE (transverse electric) wave

if $B_{0z} = 0$ TM (" magnetic) wave

if $E_{0z} = 0$ & $B_{0z} = 0$ TEM wave

TEM waves do not exist in an empty wave guide.

coaxial wave guide



Assume $E_{0z} = 0$
$B_{0z} = 0$

$$\frac{\partial E_{0x}}{\partial x} + \frac{\partial E_{0y}}{\partial y} = 0 \qquad \underbrace{\frac{\partial E_{0y}}{\partial x} - \frac{\partial E_{0x}}{\partial y} = 0}$$

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\times\vec{E} = 0 \quad \text{(in 2D)}$$

$\Rightarrow \vec{E} = -\vec{\nabla}\cancel{\phi}$ (in 2D)

or $E_x = -\dfrac{\partial \phi}{\partial x}$ $E_y = -\dfrac{\partial \phi}{\partial y} \Leftarrow$

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0} \Rightarrow \phi = \text{const}$$

$$E_x = E_y = 0$$

$$\Delta\cancel{\phi} = -\int \vec{E}_0 \cdot \vec{dl}$$

TE Waves $(E_z = 0)$ $B_z \neq 0$

$$E_{0x} = \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \, \omega \frac{\partial B_{0z}}{\partial y}$$

$$E_{0y} = \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \left(-\omega \frac{\partial B_{0z}}{\partial x}\right)$$

$$B_{0x} = \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \left(k \frac{\partial B_{0z}}{\partial x}\right) = -\frac{k}{\omega} E_{0y}$$

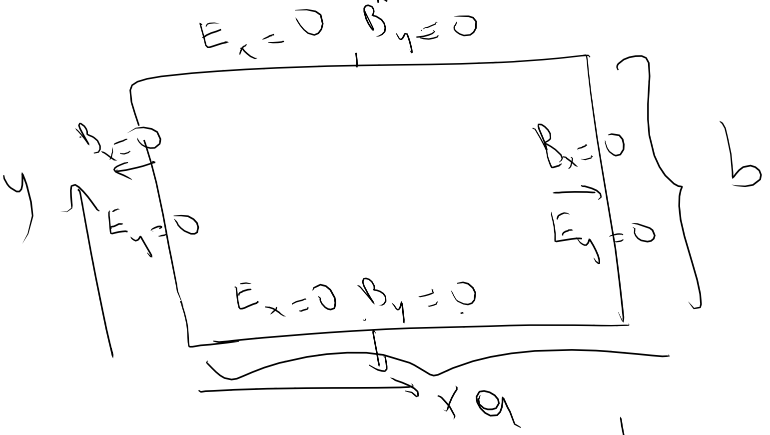
$$B_{0y} = \frac{i}{\left(\frac{\omega}{c}\right)^2 - k^2} \, k \frac{\partial B_{0z}}{\partial y} = \frac{k}{\omega} E_{0x}$$

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} + \left(\left(\frac{\omega}{c}\right)^2 - k^2\right)\right] B_{0z}(x,y) = 0$$

$$B_{0z} = X(x)\, Y(y)$$

$$\underbrace{\frac{X''}{X}}_{-k_x^2} + \underbrace{\frac{Y''}{Y}}_{-k_y^2} + \left(\frac{\omega}{c}\right)^2 - k^2 = 0 \quad k_x^2 + k_y^2 + k^2 = \left(\frac{\omega}{c}\right)^2$$

$$\begin{cases} X(x) = A \cos(k_x x + \delta_x) \\ Y(y) = B \cos(k_y y + \delta_y) \end{cases}$$



$$\left.\frac{\partial B_{0z}}{\partial y}\right|_{\substack{y=0\\ \text{or}\\ y=b}} = 0 \Longrightarrow Y'(y=0) = Y'(y=b) = 0$$

$$\delta_y = 0 \qquad k_y b = n\pi$$

$$\boxed{k_y = \frac{n\pi}{b}} \qquad n = 0, 1, 2, \dots$$

$$\left.\frac{\partial B_{0z}}{\partial x}\right|_{\substack{x=0\\ \text{or}\\ x=a}} = 0 \Longrightarrow X'(x=0) = 0 \quad X'(x=a) = 0$$

$$\delta_x = 0 \qquad k_x a = m\pi$$

$$\boxed{k_x = \frac{m\pi}{a}}$$

$$B_{0z} = C_{mn} \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right)$$

$$k^2 + k_x^2 + k_y^2 = \left(\frac{\omega}{c}\right)^2$$

$$k_{mn} = \sqrt{\left(\frac{\omega}{c}\right)^2 \underbrace{- \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}_{\frac{\omega_{mn}^2}{c^2}}}$$

$$k_{mn} = \left(\sqrt{\omega^2 - \omega_{mn}^2}\right) \frac{1}{c}$$

$$\frac{\omega}{k_{mn}} = c \frac{\omega}{\sqrt{\omega^2 - \omega_{mn}^2}} = \frac{c}{\sqrt{1 - \left(\frac{\omega_{mn}}{\omega}\right)^2}} > c$$

$$B_{0z} = C_{mn} \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right)$$

$$B_{0z} = \frac{C_{mn}}{4} \left(e^{i\frac{m\pi}{a}x} + e^{-i\frac{m\pi}{a}x}\right)\left(e^{i\frac{n\pi}{b}y} + e^{-i\frac{n\pi}{b}y}\right)$$

$$B_z = B_{0z} e^{i(kz - \omega t)}$$

$$\vec{k} = \pm \frac{m\pi}{a} \hat{x} \mp \frac{n\pi}{b} \hat{y} + k \hat{z}$$

$$\vec{k}^2 = \left(\frac{\omega}{c}\right)^2$$