

$$\vec{r} = \vec{w}(t)$$

$$t_r = t - \frac{|\vec{r} - \vec{r}'|}{c}$$

$$\vec{r}' = \vec{w}(t_r)$$

$$|\vec{r} - \vec{r}'| = |\vec{r} - \vec{w}(t_r)|$$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\phi(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{w}(t_r) - (\vec{r} - \vec{w}(t_r)) \frac{\vec{v}(t_r)}{c}|}$$

$$\vec{A}(\vec{r}, t) = \frac{\vec{v}(t_r)}{c^2} \phi$$

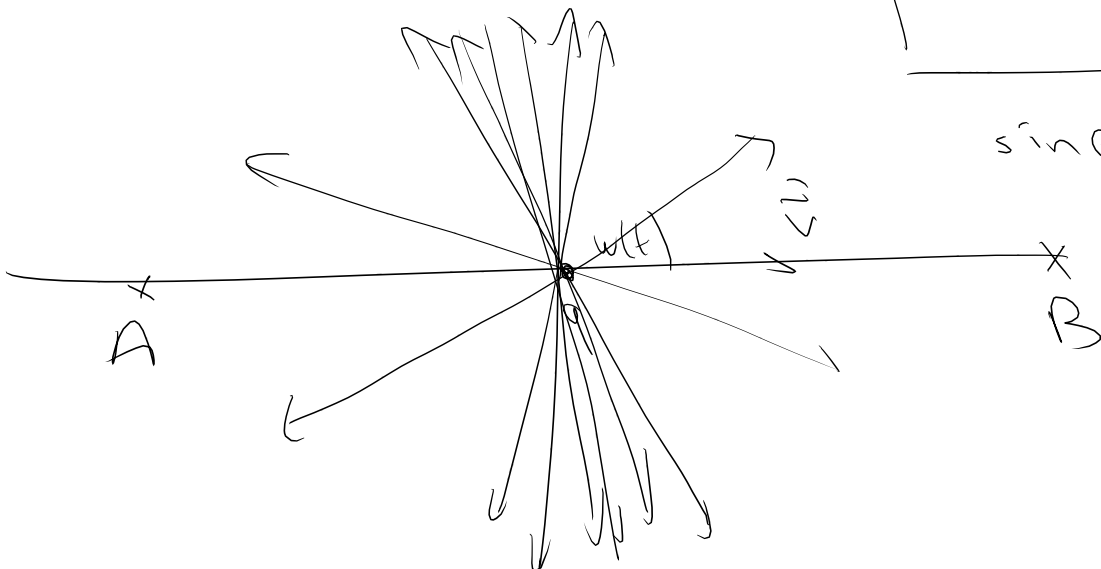
Particle moving at constant velocity

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{R}{R^2} \frac{1 - \beta^2/c^2}{1 - \beta^2/c^2 \sin^2 \theta}$$

$$\vec{R} = \vec{r} - \vec{w}(t)$$

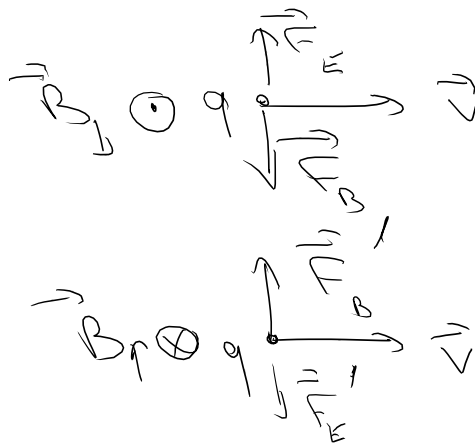
$$\vec{B} = \frac{1}{c^2} \vec{v} \times \vec{E}$$

$$\sin \theta = \sin(\alpha - \theta)$$





reference
frame 1



reference
frame 2

$$\frac{d\vec{p}}{dt} = \vec{F} = q \left[\vec{E}(\vec{r}(t), t) + \vec{v}(t) \times \vec{B}(\vec{r}(t), t) \right]$$

$$\frac{d}{dt} f(\vec{r}(t), t) = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i} \frac{dx_i}{dt}$$

$$\boxed{\frac{d}{dt} f = \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f}$$

$$\frac{d\vec{p}}{dt} = q \left[\underbrace{-\vec{\nabla} \phi}_{\vec{E}} - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times \underbrace{(\vec{\nabla} \times \vec{A})}_{\vec{B}} \right]$$

$$\begin{aligned} \left[\vec{v} \times (\vec{\nabla} \times \vec{A}) \right]_i &= \epsilon_{ijk} v_j (\vec{\nabla} \times \vec{A})_k \\ &= \epsilon_{ijk} v_j \epsilon_{klm} \partial_l A_m \\ &= (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) v_j \partial_l A_m \\ &= v_j \partial_i A_j - v_j \partial_j A_i \end{aligned}$$

$$\vec{v} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{v} \cdot \vec{A}) - (\vec{v} \cdot \vec{\nabla}) \vec{A}$$

$$\begin{aligned} \frac{d\vec{p}}{dt} &= q \left[-\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times (\vec{\nabla} \times \vec{A}) \right] \\ &= q \left[-\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t} - (\vec{v} \cdot \vec{\nabla}) \vec{A} + \vec{\nabla} (\vec{v} \cdot \vec{A}) \right] \\ &\quad \underbrace{\hspace{10em}}_{\frac{d\vec{A}}{dt}} \end{aligned}$$

$$\frac{d}{dt} (\vec{p} + q\vec{A}) = q \left[-\vec{\nabla} \phi + \vec{\nabla} (\vec{v} \cdot \vec{A}) \right]$$

$$\frac{d}{dt} (\vec{p} + q\vec{A}) = -\vec{\nabla} \left(q\phi - q\vec{v} \cdot \vec{A} \right)$$

$\underbrace{\hspace{5em}}_{\vec{p}}$
 $\underbrace{\hspace{10em}}_{U_{vel}}$

$$\frac{d\vec{p}}{dt} = -\vec{\nabla} U_{vel}$$