$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho(\vec{r}\,',t_r)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') + \frac{\dot{\rho}(\vec{r}\,',t_r)}{c|\vec{r}-\vec{r}\,'|^2}(\vec{r}-\vec{r}\,') + \frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c^2|\vec{r}-\vec{r}\,'|}\right]d^3r'$$

$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(\vec{r}\,',t_r)}{|\vec{r}-\vec{r}\,'|^3} + \frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c|\vec{r}-\vec{r}\,'|^2}\right]\times(\vec{r}-\vec{r}\,')\,d^3r'$$

$$\vec{B}(\vec{r},t) = \vec{\nabla}\times\vec{A} = \vec{\nabla}_r\times\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)}{|\vec{r}-\vec{r}\,'|}\,d^3r'$$

Radiation

$$\frac{d(ME)_V}{dt} = -\oint_{\partial V}\vec{S}\cdot d\vec{A}$$

$$\lim_{R\to\infty}\oint\vec{S}\cdot d\vec{A} = P_{radiated}$$

sphere with
radius R
centered at my
charges

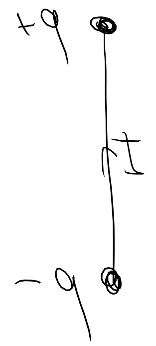
Electrostatics: $\oint\vec{E}\cdot d\vec{A} = 0 \Rightarrow |\vec{E}| \overset{R\to\infty}{\propto} \frac{1}{R^2}$

magnetostatics: $\oint\vec{B}\cdot d\vec{l} = I_{enc} \Rightarrow |\vec{B}| \overset{R\to\infty}{\propto} \frac{1}{R}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \propto \frac{1}{R^3}$$

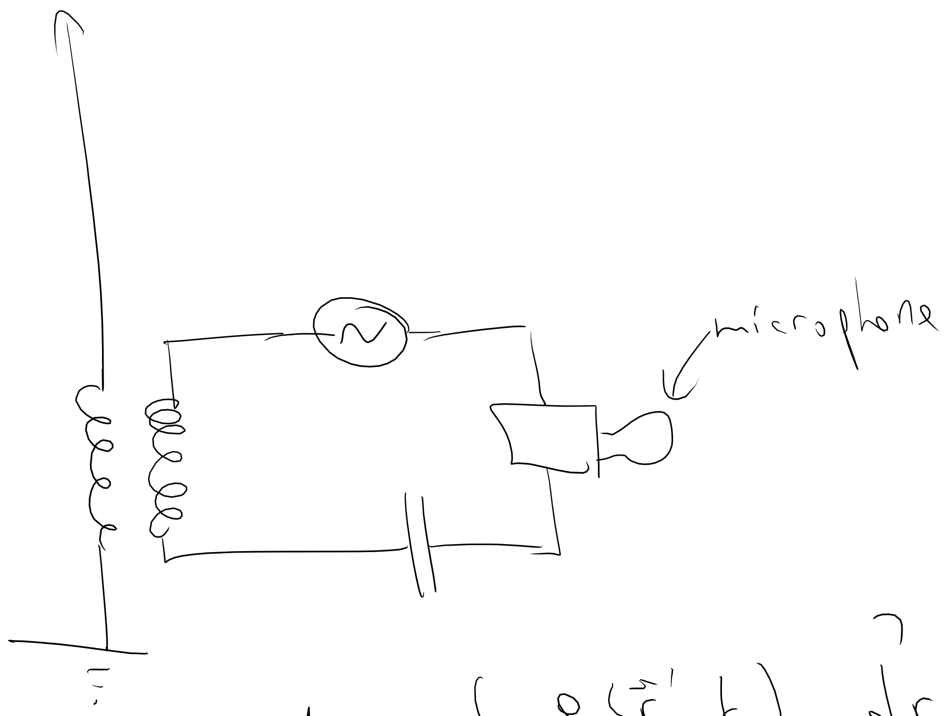
$$\int \vec{S} \cdot d\vec{A} \propto \frac{1}{R^3} R^2 = \frac{1}{R}$$

Electric Dipole Radiation

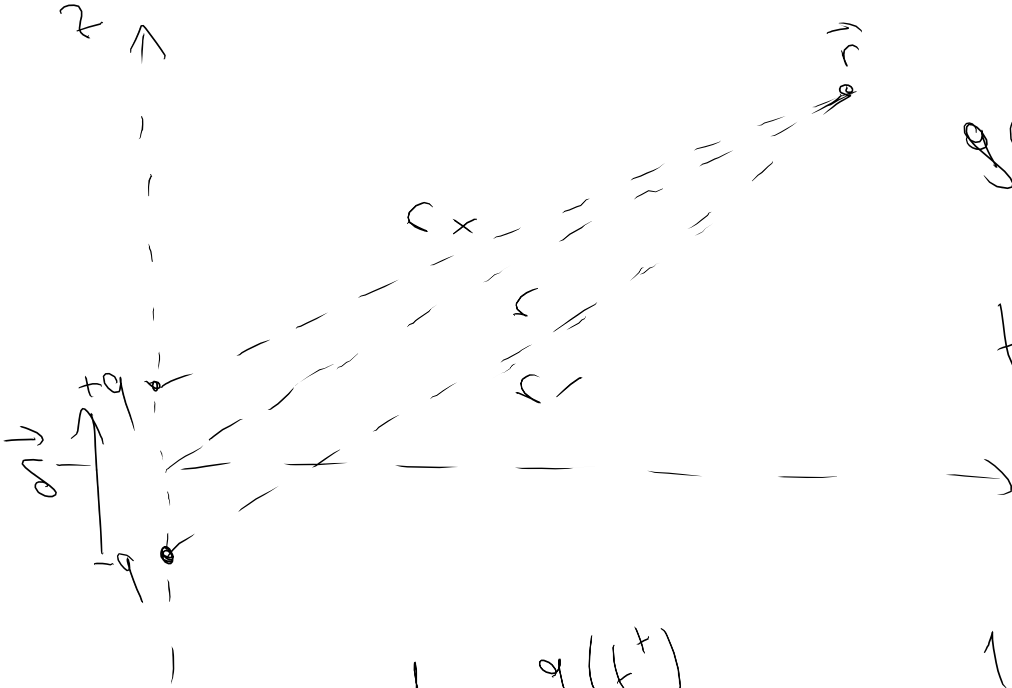


$$q(t) = q_0 \cos(\omega t)$$

$$I(t) = \frac{dq}{dt} = -\omega q_0 \sin(\omega t)$$



$$\Phi(\vec{r}, t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}', t_r)}{|\vec{r} - \vec{r}'|} d\vec{r}'$$



$$\rho(\vec{r},t) = q(t)\,\delta(\vec{r}-\vec{\delta}/2) - q(t)\,\delta\left(\vec{r}+\frac{\vec{\delta}}{2}\right)$$

$$t_r = t - \frac{|\vec{r}-\vec{r}\,'|}{c}$$

$$\Phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{q(t_r^+)}{|\vec{r}-\vec{\delta}/2|} - \frac{1}{4\pi\varepsilon_0}\frac{q(t_r^-)}{|\vec{r}+\vec{\delta}/2|}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_0\cos\left(\omega t - \omega|\vec{r}-\vec{\delta}/2|/c\right)}{|\vec{r}-\vec{\delta}/2|}$$

$$- \frac{1}{4\pi\varepsilon_0}\, q_0\,\frac{\cos\left(\omega t - \omega|\vec{r}+\vec{\delta}/2|/c\right)}{|\vec{r}+\vec{\delta}/2|}$$

$$|\vec{r}| \gg |\vec{\delta}|$$

$$|\vec{r}\mp\vec{\delta}/2| = \left[\left(\vec{r}\mp\frac{\vec{\delta}}{2}\right)^2\right]^{1/2} = \left[r^2 + \frac{\delta^2}{4} \mp \vec{r}\cdot\vec{\delta}\right]^{1/2}$$

$$= r\left[1 + \frac{\delta^2}{4r^2} \mp \frac{\vec{r}\cdot\vec{\delta}}{r^2}\right]^{1/2}$$

$$\simeq r\left[1 \mp \frac{\vec{r}\cdot\vec{\delta}}{2r^2}\right]$$

$$|\vec{r}\mp\vec{\delta}/2| \simeq r \mp \frac{\hat{r}\cdot\vec{\delta}}{2}$$

$$\frac{1}{|\vec{r}\mp\vec{\delta}/2|} \simeq \frac{1}{r}\left[1 \pm \frac{\vec{r}\cdot\vec{\delta}}{2r^2}\right]$$

$$\Phi = \frac{1}{4\pi\varepsilon_0} \frac{q_0 \cos\left(\omega t - \omega |\vec{r} - \vec{\delta}/2|/c\right)}{|\vec{r} - \vec{\delta}/2|}$$

$$- \frac{1}{4\pi\varepsilon_0} q_0 \frac{\cos\left(\omega t - \omega |\vec{r} + \vec{\delta}/2|/c\right)}{|\vec{r} + \vec{\delta}/2|}$$

$$\simeq \frac{q_0}{4\pi\varepsilon_0 r} \left[\cos\left(\omega t - \frac{\omega}{c} r + \omega \frac{\vec{\delta}\cdot\hat{r}}{2c}\right) - \cos\left(\omega t - \frac{\omega}{c} r - \omega \frac{\vec{\delta}\cdot\hat{r}}{2c}\right) \right.$$

$$\Phi(\vec{r},t) \simeq -\frac{q_0}{2\pi\varepsilon_0 r} \sin\left(\omega t - \frac{\omega}{c} r\right) \sin\left(\frac{\omega \vec{\delta}\cdot\hat{r}}{2c}\right)$$

$$\cos(A \mp B) = \cos A \cos B \pm \sin A \sin B$$

Assumption 2: $\frac{\omega \delta}{c} \ll 1$

$$\lambda = \frac{c}{\omega} \gg \delta$$

$$\Phi(\vec{r},t) \simeq -\frac{q_0}{2\pi\varepsilon_0 r} \sin\left(\omega t - \frac{\omega}{c} r\right) \frac{\omega \vec{\delta}\cdot\vec{r}}{2c}$$

$$\Phi(\vec{r},t) = -\frac{(q_0\vec{\delta})\cdot\hat{r}}{4\pi\varepsilon_0 c r} \omega \sin\left(\omega t - \frac{\omega}{c} r\right)$$

$$= \left.\frac{d}{dt}(q\vec{\delta})\right|_{t = t - \frac{1}{c} r} \cdot \frac{\hat{r}}{4\pi\varepsilon_0 c r} \qquad q = q_0 \cos(\omega t)$$

$$\boxed{\phi(\vec{r},t) = \frac{\dot{\vec{p}}\left(t-\frac{r}{c}\right)\cdot\hat{r}}{4\pi\varepsilon_0 c r}} + O\left(\frac{1}{r^2}\right)$$

$$\vec{A}(\vec{r},t) = \int \frac{\vec{J}(\vec{r}', t_r)}{|\vec{r}-\vec{r}'|} d^3r'$$

$$= \int \frac{I(t_r)\hat{z}}{|\vec{r}-\vec{r}'|} dl$$

$$A_z(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{-\delta/2}^{\delta/2} \frac{I(t_r)}{|\vec{r}-\vec{r}'|} dz$$

$$I(t) = -\omega q_0 \sin(\omega t)$$

$$A_z(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{-\delta/2}^{\delta/2} \frac{(-\omega q_0)\sin\left(\omega t - \frac{\omega|\vec{r}-\vec{r}'|}{c}\right)}{|\vec{r}-\vec{r}'|} dz$$

$$= \frac{\mu_0}{4\pi} \frac{(-\omega q_0)\sin\left(\omega t - \frac{\omega r}{c}\right)}{|\vec{r}|}\int_{-\delta/2}^{\delta/2} dz$$

$$= \frac{\mu_0}{4\pi} \frac{(-\omega q_0)\sin\left(\omega t - \frac{\omega r}{c}\right)\delta}{r}$$

$$\boxed{A_z(\vec{r},t) = \frac{\mu_0}{4\pi} \frac{\dot{p}_z\left(t-\frac{r}{c}\right)}{r}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \left(A_z \hat{z}\right)$$

$$= -\hat{y}\left(\frac{\partial}{\partial x} A_z\right) + \hat{x}\left(\frac{\partial}{\partial y} A_z\right)$$

$$= -\hat{y}\left(\frac{\partial A_z}{\partial r}\right)\frac{x}{r} + \hat{x}\left(\frac{\partial A_z}{\partial r}\right)\frac{y}{r}$$

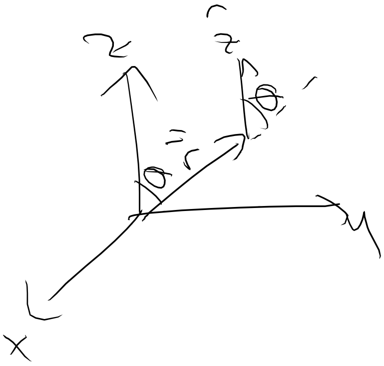
$$= \left(\frac{\partial A_z}{\partial r}\right)\frac{1}{r}\left(y\hat{x} - x\hat{y}\right)$$

$$\vec{r} \times \hat{z} = \left(x\hat{x} + y\hat{y} + z\hat{z}\right) \times \hat{z} = \left(x(-\hat{y}) + y\hat{x}\right)$$

$$= \left(y\hat{x} - x\hat{y}\right)$$

$$\vec{B} = \frac{\partial A_z}{\partial r}\frac{1}{r}\left(\vec{r} \times \hat{z}\right) = \frac{\partial A_z}{\partial r}\left(\hat{r} \times \hat{z}\right)$$

$$= \frac{\partial A_z}{\partial r}\left(-\hat{\phi}\right)\sin\theta$$



$$A_z = \frac{\mu_0}{4\pi}\frac{\dot{p}_z\left(t - \frac{r}{c}\right)}{r}$$

$$\vec{B}_{rad} = \frac{\mu_0}{4\pi}\frac{\ddot{p}_z\left(t - \frac{r}{c}\right)\left(-\frac{1}{c}\right)}{r}\left(-\hat{\phi}\right)\sin\theta$$

$$\boxed{\vec{B}_{rad} = \frac{\mu_0}{4\pi c}\frac{\ddot{p}_z\,\hat{\phi}\,\sin\theta}{r}} + \mathcal{O}\left(\frac{1}{r}\right)$$

$$\vec{E}_{rad} = \frac{+\mu_0}{4\pi}\ddot{p}_z\left(\frac{\sin\theta}{r}\right)\hat{\theta}$$

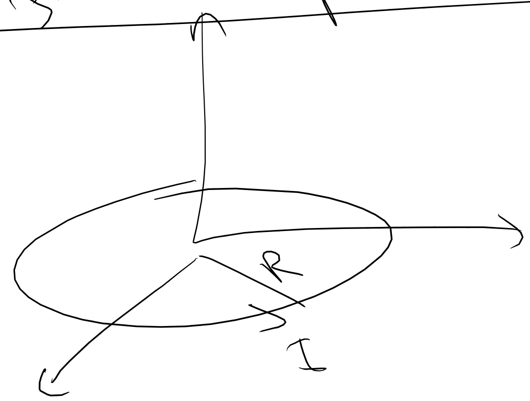
$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{\mu_0}{c}\frac{1}{(4\pi)^2}\left(\frac{\sin\theta}{r}\right)^2\left(\ddot{p}_z\right)^2\hat{r}$$

$$\langle P\rangle = \int\langle\vec{S}\rangle\cdot d\vec{A} = \frac{\mu_0 p_0^2\omega^4}{12\pi c}$$

$$\vec{p}(t) = q\delta_0\cos(\omega t) \equiv \vec{p}_0\cos(\omega t)$$

$$\ddot{\vec{p}} = -\omega^2\vec{p}$$

Magnetic Dipole Radiation



$$\boxed{\vec{\mu}(t) = I(t)\pi R^2\hat{z}}$$

$$\vec{\mu}(t) = \mu(t)\hat{z}$$

$$\vec{E}_{rad} = -\frac{\mu_0\ddot{\mu}(t)}{4\pi c}\left(\frac{\sin\theta}{r}\right)\hat{\phi}$$

$$\vec{B}_{rad} = \frac{\mu_0}{4\pi c^2}\ddot{\mu}(t)\left(\frac{\sin\theta}{r}\right)\hat{\theta}$$