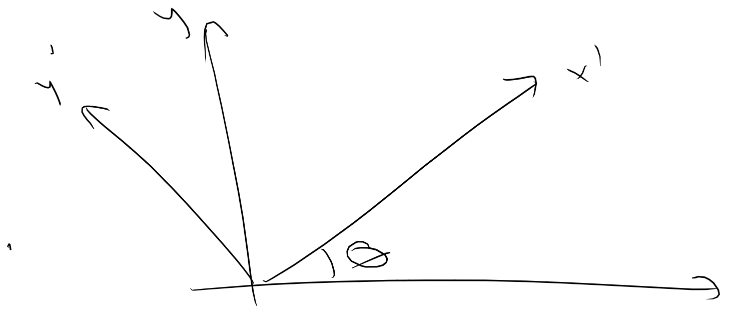
Lorentz Transformations

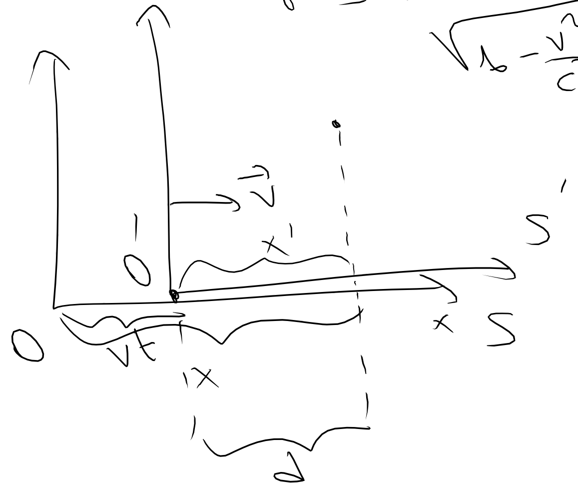


$$\vec{V} = V_x \hat{x} + V_y \hat{y}$$
$$= V_{x'} \hat{x}' + V_{y'} \hat{y}'$$

$$\begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$\vec{V} = \{V_i\}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



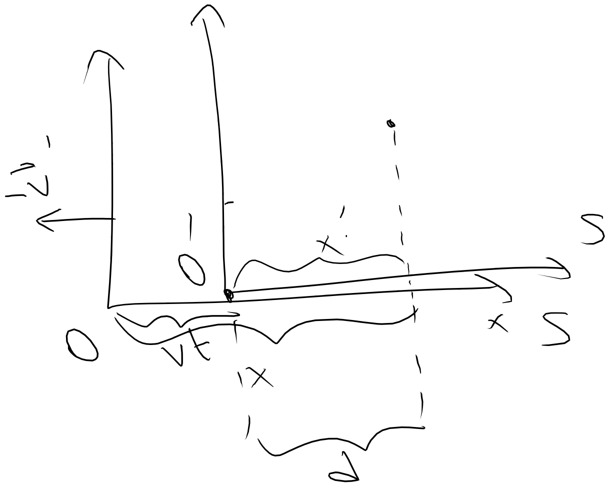
$$(t', x', y', z')$$
$$(t, x, y, z)$$

$$x = vt + d$$

$$d = \sqrt{1 - \frac{v^2}{c^2}}\, x' = \frac{x'}{\gamma}$$

$$x = vt + \frac{x'}{\gamma} \Rightarrow \boxed{x' = \gamma(x - vt)}$$

$$x' = -vt' + \frac{x}{\gamma} \Rightarrow \boxed{x = \gamma(x' + vt')}$$



$$\frac{x}{\gamma} = \gamma(x - vt) + vt'$$

$$t' = \frac{1}{v}\left[\frac{x}{\gamma} - \gamma x + \gamma v t\right]$$

$$= \frac{1}{v}\gamma x\left[\frac{1}{\gamma^2} - 1\right] + \gamma t$$

$$= \frac{1}{v}\gamma x\left[\left(\cancel{1} - \frac{v^2}{c^2}\right) - \cancel{1}\right] + \gamma t$$

$$\boxed{t' = \gamma\left[t - \frac{v}{c^2}x\right]}$$

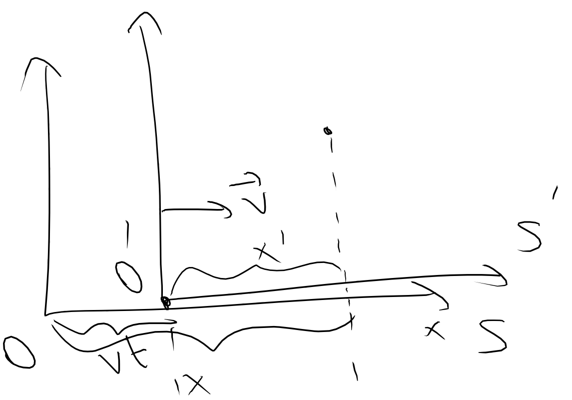
$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \\ y' &= y \\ z' &= z \end{aligned}}$$

Lorentz Transformations

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta x' = \gamma(\Delta x - v\Delta t) \xRightarrow{\Delta t = 0} \boxed{\Delta x' = \gamma \Delta x}$$

$$\boxed{\Delta x = \frac{1}{\gamma}\Delta x'}$$



$\Delta x'$: Length of the object in the S' system.

Δx: Length of the object in the S system. $\Rightarrow \Delta t = 0$

$$x' = \gamma(x - vt)$$

$$x'_e = \gamma(x_e - vt_1)$$

$$x'_f = \gamma(x_f - vt_1)$$

$$x'^2 - c^2 t'^2 = [\gamma(x - vt)]^2 - c^2\left[\gamma\left(t - \frac{v}{c^2}x\right)\right]^2$$

$$= \gamma^2\left\{\left(x^2 + v^2t^2 - \cancel{2xvt}\right) - c^2\left(t^2 + \frac{v^2}{c^4}x^2 - \cancel{2\frac{v}{c^2}xt}\right)\right\}$$

$$= \frac{1}{\cancel{1 - \frac{v^2}{c^2}}}\left\{x^2\cancel{\left(1 - \frac{v^2}{c^2}\right)} - c^2 t^2\cancel{\left(1 - \frac{v^2}{c^2}\right)}\right\}$$

$$\boxed{x'^2 - c^2 t'^2 = x^2 - c^2 t^2}$$

$$-c^2t^2 + x^2 + y^2 + z^2 = \text{interval}$$

invariant under Lorentz Transformations

$$\boxed{-c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 = (\Delta \tau)^2}$$

$$x' = \gamma\left(x - \frac{v}{c}ct\right)$$
$$ct' = \gamma\left(ct - \frac{v}{c}x\right)$$
$$y' = y$$
$$z' = z$$

$$\beta \equiv \frac{v}{c}$$
$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\Lambda} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$x^\mu = (ct, x, y, z)$$
$$x^0 \equiv ct \; ; \; x^1 = x \; ; \; x^2 = y \; ; \; x^3 = z$$

$$\boxed{x'^\mu = \Lambda^\mu{}_\nu x^\nu}$$

μ: mu
ν: nu

μ, ν : 0, 1, 2, 3

rotation
$$\Lambda_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & +\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Delta x^\mu = x^\mu_A - x^\mu_B$$

$$(\Delta\tau)^2 = (\Delta x^\mu)^2 \equiv -(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2$$

$(\Delta\tau)^2 < 0$: the separation of two events is timelike
$(\Delta\tau)^2 = 0$: " " " " " is lightlike
$(\Delta\tau)^2 > 0$: " " " " " is space like

$(\Delta\tau)^2 < 0 \Rightarrow$ it is always possible to find a reference frame in which
$\Delta x^\mu = (\Delta x^0, \vec{0})$

$(\Delta\tau)^2 > 0 \Rightarrow$ it is always possible to find a reference frame in which
$\Delta x^\mu = (0, \Delta\vec{r})$

in minkowsky space, V^2 need not be positive!

$(\Delta s)^2 = (x^0)^2 - (x^1)^2$

$(\Delta s)^2 = 0 \Rightarrow x^0 = \pm x^1$