Ex

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{p} = \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\vec{F} = \text{const} \qquad \boxed{\vec{p}(t=0) = 0}$$

$$\vec{p}(t) = \vec{F}t \Rightarrow \frac{m\vec{u}}{\sqrt{1-u^2/c^2}} = \vec{F}t$$

$$\vec{F} = F\hat{x} \qquad \boxed{\frac{u}{\sqrt{1-u^2/c^2}} = \frac{F}{m}t}$$

$$\vec{u} = u\hat{x}$$

$$u^2 = \left(\frac{Ft}{m}\right)^2\left(1-\frac{u^2}{c^2}\right)$$

$$u^2\left(1+\left(\frac{Ft}{mc}\right)^2\right) = \left(\frac{Ft}{m}\right)^2$$

$$\frac{dx}{dt} = \boxed{u(t) = \frac{\frac{Ft}{m}}{\sqrt{1+\left(\frac{Ft}{mc}\right)^2}}} = \frac{c^2 m}{F}\frac{d}{dt}\sqrt{1+\left(\frac{Ft}{mc}\right)^2}$$

$$\frac{dx}{dt} = \frac{d}{dt}\left[\left(\frac{mc^2}{F}\right)\sqrt{1+\left(\frac{Ft}{mc}\right)^2}\right] \qquad \lim_{t\to\infty} u(t) = c$$

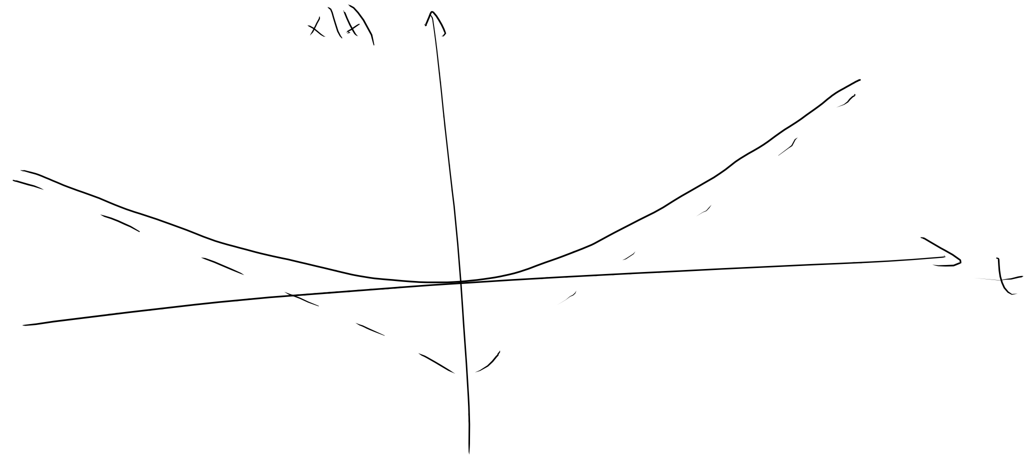
$$x(t=0) = 0$$

$$\boxed{x(t) = \frac{mc^2}{F}\left[\sqrt{1+\left(\frac{Ft}{mc}\right)^2} - 1\right]}$$

$$\lim_{t\to\infty} x(t) = \frac{mc^2}{F}\left[\frac{Ft}{mc} - 1\right] = ct - \frac{mc^2}{F}$$

$$\left(x(t) + \frac{mc^2}{F}\right)^2 = 1 + \left(\frac{Ft}{mc}\right)^2$$

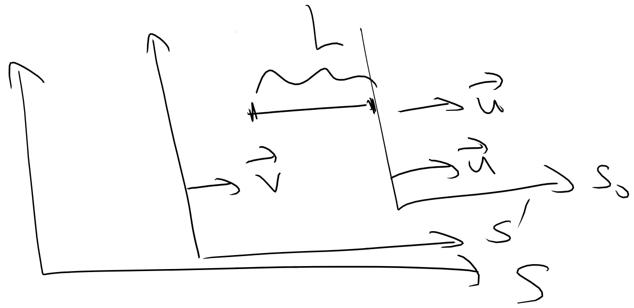
$$\Rightarrow \left(x(t) + \frac{mc^2}{F}\right)^2 - \left(\frac{Ft}{mc}\right)^2 = 1 \Leftarrow \text{eqn of a hyperbola}$$



$$\lim_{t\to 0} x(t) = \lim_{t\to 0} \frac{mc^2}{F}\left[\sqrt{1+\left(\frac{Ft}{mc}\right)^2} - 1\right] = \frac{mc^2}{F}\left[\cancel{1} + \frac{1}{2}\left(\frac{Ft}{mc}\right)^2 \cancel{-1}\right]$$

$$= \frac{1}{2}\left(\frac{F}{m}\right)t^2$$

Example



$$u' = \frac{-u+v}{1-\frac{uv}{c^2}}$$

$$L = \sqrt{1-\frac{u^2}{c^2}}\, L_0$$

$$L' = \sqrt{1-\frac{u'^2}{c^2}}\, L_0$$

$$L' = \sqrt{1-u'^2/c^2}\, \frac{L}{\sqrt{1-\frac{u^2}{c^2}}}$$

L_0: length in the reference frame in which the object is at rest

L: length in the RF S

L': length in the RF S'

$$\boxed{L' = \frac{\sqrt{1-u'^2/c^2}}{\sqrt{1-u^2/c^2}}\, L}$$

$$\boxed{L' = \frac{\left[1 - \frac{1}{c^2}\left(\frac{(v-u)}{1-\frac{uv}{c^2}}\right)^2\right]^{1/2}}{\sqrt{1-u^2/c^2}}\, L}$$

$$L' = \frac{1}{c\left(1 - \frac{uv}{c^2}\right)} \frac{\left[c^2\left(1 - \frac{uv}{c^2}\right)^2 - (v-u)^2\right]^{1/2}}{\sqrt{1 - u^2/c^2}} L$$

$$L' = \frac{1}{c\left(1 - \frac{uv}{c^2}\right)} \frac{\left[c^2 \cancel{- 2uv} + \frac{u^2v^2}{c^2} - (v^2 \cancel{- 2uv} + u^2)\right]^{1/2}}{\sqrt{1 - u^2/c^2}} L$$

$$L' = \frac{1}{c\left(1 - \frac{uv}{c^2}\right)} \frac{\left[c^2 + \frac{u^2v^2}{c^2} - v^2 - u^2\right]^{1/2}}{\sqrt{1 - u^2/c^2}} L$$

$$= \frac{1}{\left(1 - \frac{uv}{c^2}\right)} \frac{\left[1 + \left(\frac{u^2}{c^2}\right)\left(\frac{v^2}{c^2}\right) - \frac{u^2}{c^2} - \frac{v^2}{c^2}\right]^{1/2}}{\sqrt{1 - u^2/c^2}} L$$

$$L' = \frac{1}{\left(1 - \frac{uv}{c^2}\right)} \frac{\left[\cancel{\left(1 - \frac{u^2}{c^2}\right)}\left(1 - \frac{v^2}{c^2}\right)\right]^{1/2}}{\cancel{\sqrt{1 - u^2/c^2}}} L$$

$$\boxed{L' = \frac{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{\left(1 - \frac{uv}{c^2}\right)} L}$$

<u>Limiting cases</u> i) $\vec{v} = 0 \Rightarrow S' = S$

$L' = L$ $\checkmark$

ii) $\vec{v} = \vec{u} \Rightarrow S' = S_0 \Rightarrow L' = L_0$

$$L' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} L = L_0 \checkmark$$

iii) $\vec{u} = 0 \Rightarrow S = S_0 \Rightarrow L' = \sqrt{1 - \frac{v^2}{c^2}}\, L$ $\checkmark$

$$E = \sqrt{m^2c^4 + \vec{p}^{\,2}c^2} \qquad KE = E - E_0$$

$$\vec{v} = \frac{\vec{p}c^2}{E} \qquad \vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \qquad E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{\vec{p}}{E} = \frac{\vec{v}}{c^2} \Rightarrow \vec{v} = \frac{\vec{p}c^2}{E} = \left(\frac{\vec{p}c}{E}\right)c$$

$$\vec{v} = \frac{\vec{p}c}{\sqrt{m^2c^4 + (\vec{p}c)^2}} \cdot c$$

$$v = \frac{p}{\sqrt{m^2c^2 + p^2}}\, c$$

if $m = 0$ $\quad v = c$

if $m \neq 0$ $\quad v < c$