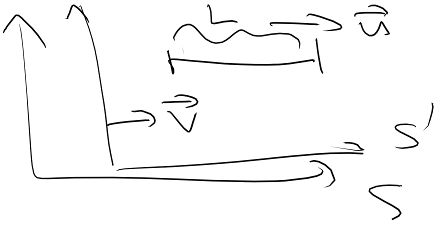
$$L' = \frac{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{\left(1 - \frac{uv}{c^2}\right)} L$$



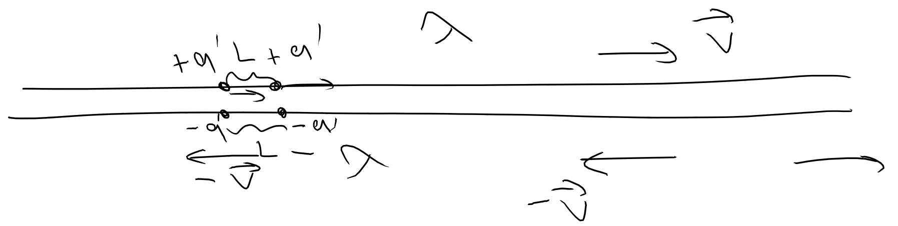

$$(\Delta\tau)^2 = -(c\Delta t)^2 + (\Delta\vec{r})^2$$

$$x^4 = ict$$

$$(\Delta\tau)^2 = (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2 + (\Delta x^4)^2$$

## Magnetism as a relativistic phenomena

$$\lambda = \frac{q'}{L}$$




$$I = 2\lambda v$$

$q \longrightarrow \vec{u}$ $\qquad \lambda_{tot} = 0$

Go to a reference frame in which $q$ is at rest.


$$\lambda'_{tot} = \lambda_+ + \lambda_-$$

$$\vec{E}' = \frac{1}{2\pi\varepsilon_0} \frac{\lambda'_{tot}}{d} \hat{\rho}$$

$$\vec{F}'_{\perp} = \frac{1}{2\pi\varepsilon_0} \frac{\lambda_{tot}\, q}{d} \hat{\rho}$$

$$\vec{F}_{\perp} =$$

$$\vec{F}_\perp' = \frac{d\vec{p}_\perp'}{dt'} \longrightarrow \frac{d\vec{p}_\perp}{dt/\gamma} = \cancel{\gamma}\vec{F}_\perp$$

$$\boxed{\vec{F}_\perp = \sqrt{1 - u^2/c^2}\;\vec{F}_\perp'}$$

$$\Rightarrow F_\perp = \sqrt{1 - u^2/c^2}\;\frac{q}{2\pi\varepsilon_0 d}\,\lambda_{tot}'$$

$$\lambda_+ = \frac{q}{L_+} \;;\; \lambda_- = \frac{-q}{L_-}$$

$$\lambda_+ = \frac{q}{\dfrac{\left(1 - \frac{u^2}{c^2}\right)^{1/2} L}{1 - \frac{uv}{c^2}}} \;;\; \lambda_- = \frac{-q}{\dfrac{\left(1 - \frac{u^2}{c^2}\right)^{1/2} L}{1 + \frac{uv}{c^2}}}$$

$$\lambda_{tot} = \lambda\frac{1 - \frac{uv}{c^2}}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} - \lambda\frac{1 + \frac{uv}{c^2}}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} = -\frac{2uv}{c^2}\gamma\lambda$$

$$\boxed{\lambda_{tot} = -\frac{2uv}{c^2}\gamma\lambda}$$

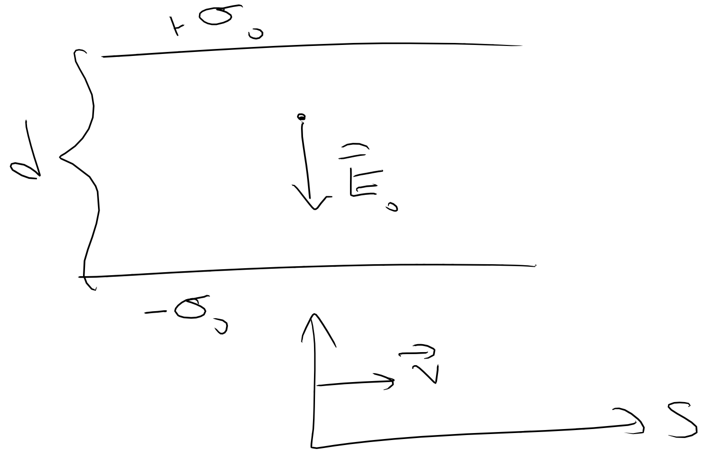
$$F_\perp = \cancel{\sqrt{1 - \frac{u^2}{c^2}}}\;\frac{q}{2\pi\varepsilon_0 d}\;\frac{-2uv}{c^2}\;\frac{1}{\cancel{\sqrt{1 - \frac{u^2}{c^2}}}}\lambda$$

$$F_\perp = -\frac{2uv\,\varepsilon_0\mu_0}{2\pi\varepsilon_0 d}\lambda$$

$$= -\frac{\mu_0}{2\pi}u\frac{(2v\lambda)}{d} = -\frac{\mu_0}{2\pi}\frac{uI}{d}$$

$$= -u\,\frac{\mu_0}{2\pi}\frac{I}{d}$$

# Transformation Law for the electric & magnetic fields



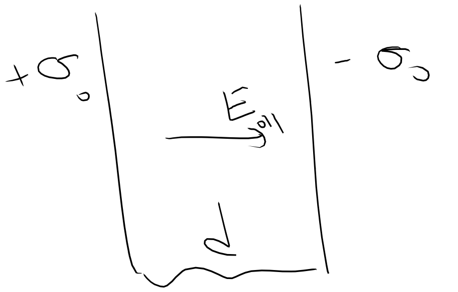

$$E_0 = \frac{\sigma_0}{\varepsilon_0}$$

$$\sigma = \gamma \sigma_0$$

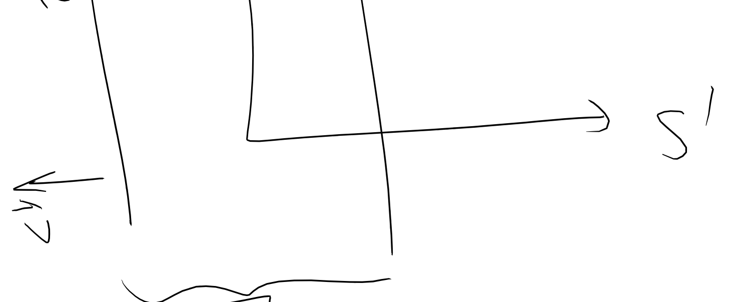

$$E' = \frac{\sigma}{\varepsilon_0} = \gamma \frac{\sigma_0}{\varepsilon_0} = \gamma E_0$$

$$\boxed{\vec{E}'_{\perp} = \gamma \vec{E}_{0\perp}}$$

What about $\vec{E}_{\parallel}$?




$$E_{0\parallel} = \frac{\sigma_0}{\varepsilon_0}$$

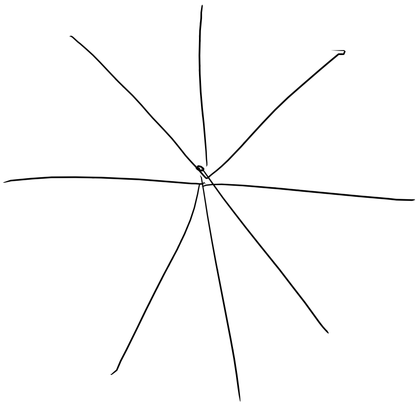



$\sqrt{1-\frac{v^2}{c^2}}\, d$

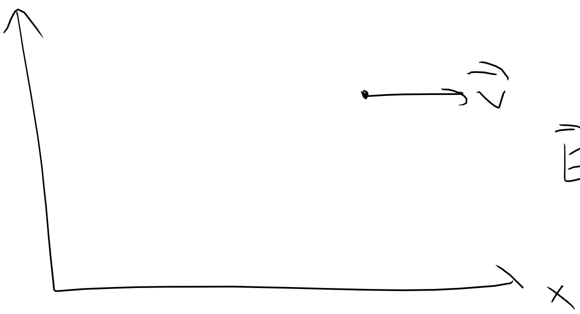
$$\sigma = \sigma_0$$

$$\boxed{\vec{E}_{0\parallel} = \vec{E}'_{\parallel}}$$

Example



$$\vec{E}^0 = \frac{1}{4\pi\varepsilon_0}\, q\, \frac{x_0\hat{x} + y_0\hat{y} + z_0\hat{z}}{(x_0^2 + y_0^2 + z_0^2)^{3/2}}$$



$$E'_{\parallel} = E^0_{\parallel} = \frac{q}{4\pi\varepsilon_0}\, \frac{x_0}{(x_0^2 + y_0^2 + z_0^2)^{3/2}}$$

$$\vec{E}'_{\perp} = \gamma \vec{E}_{\perp} = \frac{q}{4\pi\varepsilon_0}\, \gamma\, \frac{y_0\hat{y} + z_0\hat{z}}{(x_0^2 + y_0^2 + z_0^2)^{3/2}}$$

$$x = \sqrt{1 - v^2/c^2}\, x_0 = \frac{x_0}{\gamma} \Rightarrow x_0 = \gamma x$$

$$y = 1\, y_0 \qquad y_0 = y$$

$$z = 1\, z_0 \qquad z_0 = z$$

$$\delta \equiv \sqrt{x_0^2 + y_0^2 + z_0^2} = \sqrt{\gamma^2 x^2 + y^2 + z^2}$$

$$\vec{E} = E_{\parallel}\hat{x} + \vec{E}_{\perp} = \frac{q}{4\pi\varepsilon_0}\, \frac{1}{\delta^3}\left(\gamma x\hat{x} + \gamma y\hat{y} + \gamma z\hat{z}\right)$$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0}\, \frac{\gamma}{\delta^3}\left(x\hat{x} + y\hat{y} + z\hat{z}\right)}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\, \frac{\gamma}{\delta^3}\left((x - x(t))\hat{x} + (y - y(t))\hat{y} + (z - z(t))\hat{z}\right)$$

$x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z} = \vec{r}(t)$ : is the position of the particle at the time t.

$$\frac{1}{\delta^3} = \frac{1}{(\gamma^2 x^2 + y^2 + z^2)^{3/2}} = \frac{1}{\left[(\gamma^2 - 1)x^2 + y^2 + z^2 + x^2\right]^{3/2}}$$

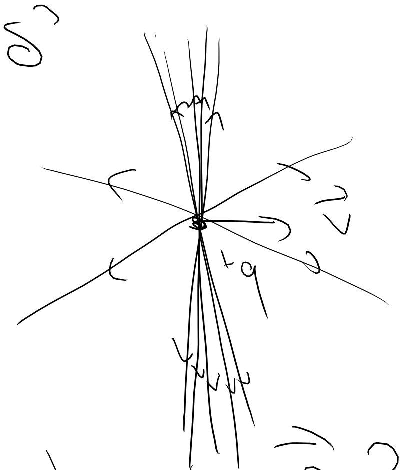
$\vec r \cdot \hat x = r\cos\theta$

$$= \frac{1}{\left[(\gamma^2-1)\, r^2\cos^2\theta + r^2\right]^{3/2}}$$

$$= \frac{1}{r^3\left[(\gamma^2-1)\cos^2\theta + 1\right]^{3/2}}$$

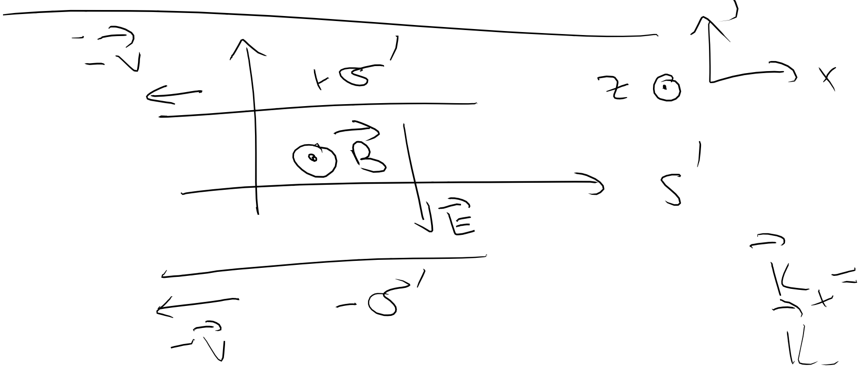
$$= \frac{1}{r^3\left[\gamma^2\cos^2\theta + \sin^2\theta\right]^{3/2}}$$

$$\frac{\gamma}{\gamma^3} \xrightarrow{\gamma\to\infty} 0 \quad \text{unless} \quad \cos\theta = 0$$




$\frac{v}{c} \lesssim 1$

what about $\vec B$?

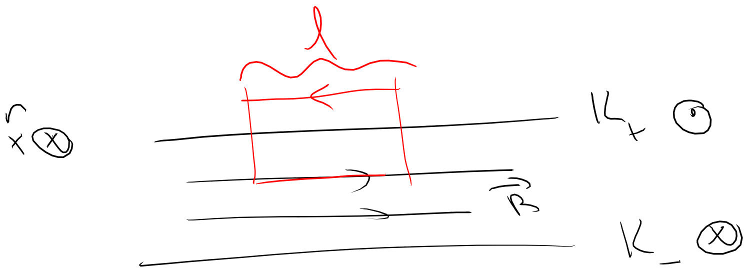



$E'_\perp = \gamma E_\perp$

$\vec B' = B'_z$

$\vec K'_+ = \sigma' v(-\hat x)$

$\vec K'_- = \sigma' v(+\hat x)$




$\oint \vec B \cdot d\vec l = \mu_0 I_{enc}$

$B\cancel{l} = \mu_0 K \cancel{l}$

$$\boxed{B = \mu_0 \sigma' v}$$

$$\boxed{\begin{aligned}\vec E &= \frac{\sigma'}{\epsilon_0}(-\hat y)\\ \vec B &= \mu_0\sigma' v\,\hat z\end{aligned}}$$


$S'$ moves with velocity $\vec{v} = v\hat{x}$ relative to $S_0$

$S$ moves with velocity $\vec{u}' = u'\hat{x}$ relative to $S'$
and with velocity $\vec{u} = u\hat{x}$ relative to $S_0$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}(-\hat{y})$$

$$\sigma = \frac{\sigma_0}{\sqrt{1 - u^2/c^2}}$$

$$\vec{B} = \mu_0 \sigma u \hat{z}$$

$(\sigma_0, u) \longrightarrow$ express interms of $(\sigma', u')$

$(\sigma', u') \longrightarrow$ express in terms of $B'$ & $E'$