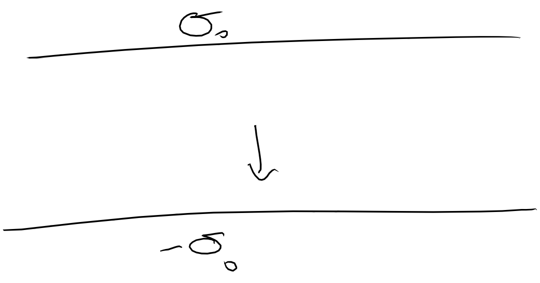
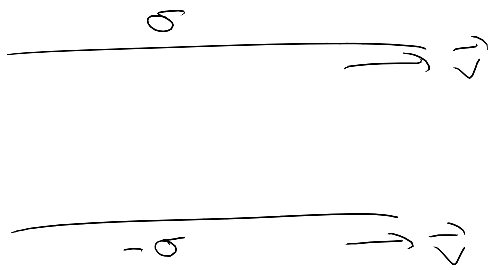


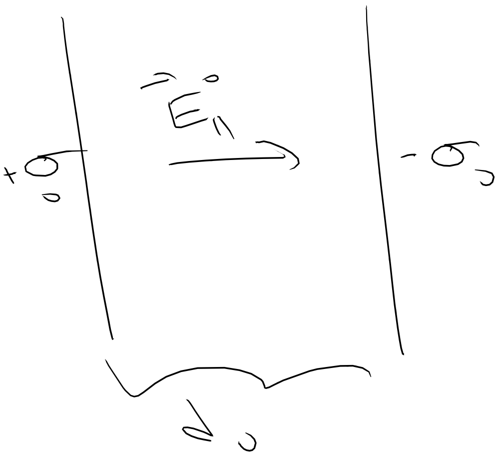


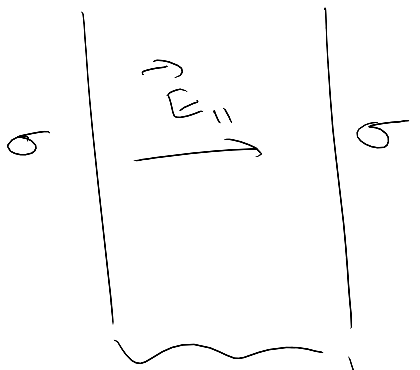
$$\sigma = \gamma \sigma_0$$

$$\vec{E}_\perp^{(0)} = \frac{\sigma_0}{\varepsilon_0}(-\hat{z})$$

$$\vec{E}_\perp = \frac{\sigma}{\varepsilon_0}(-\hat{z}) = \gamma \vec{E}_\perp^{(0)}$$

↳ since $\vec{E}$ is perpendicular to $\vec{v}$.





$$d = \frac{d_0}{\gamma}$$

$$\sigma = \sigma_0$$

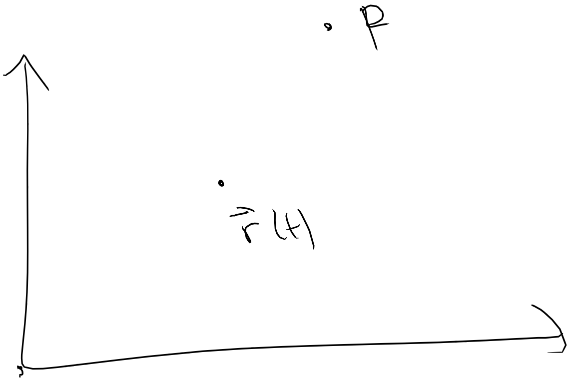
$$\boxed{\vec{E}_\parallel = \vec{E}_\parallel^{(0)}}$$

$$x' = \gamma(x - \cancel{v}t)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Example of a point charge

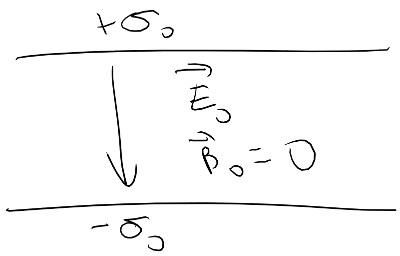
$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\gamma}{\delta^3}\vec{r}$$



$$\vec{E}(P) = \frac{q}{4\pi\varepsilon_0}\frac{\gamma}{\delta^3}\left(\vec{r}-\vec{r}(t)\right)$$

$$\frac{d\vec{r}(t)}{dt} = \vec{v} = \text{const.}$$

First reference frame S_0



$\vec{E}_0$

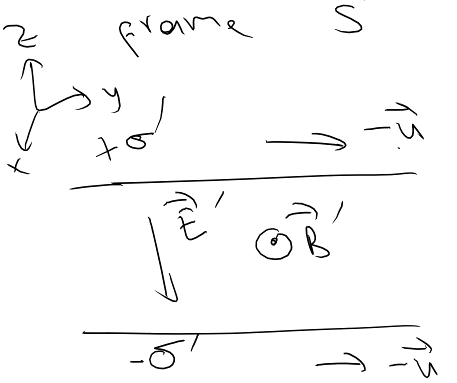
$\vec{B}_0$

$$\boxed{v = \frac{v_0 - u}{1 - \frac{v_0 u}{c^2}}}$$

$$v_0 = \frac{v+u}{1+\frac{vu}{c^2}}$$

$$E_z = -\frac{1}{\sqrt{1-v_0^2/c^2}}\frac{\sigma_0}{\varepsilon_0}$$

Second reference frame S'



$\vec{u}$: speed of S' relative to S_0

$$\vec{E}' = \frac{\sigma'}{\varepsilon_0}(-\hat{z})$$

$$\vec{B}' = \mu_0\sigma' u(\hat{x})$$

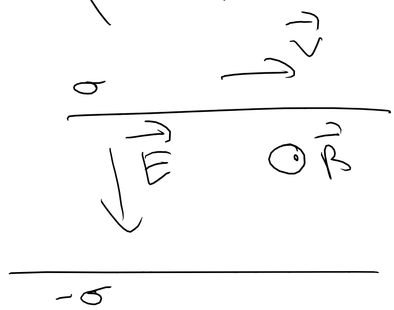
$$\vec{E}' = \gamma_u \vec{E}^{(0)}$$

$$\vec{B}' = -\mu_0\varepsilon_0 \vec{E}' \times \vec{u}'$$

$$\vec{B}' = \frac{1}{c^2}(+\vec{u}) \times \vec{E}'$$

$$E_z' = -\frac{1}{\sqrt{1-u^2/c^2}}\frac{\sigma_0}{\varepsilon_0}$$

third reference frame S



$\vec{v}$: velocity of S relative to S'

$\vec{v}_0$: velocity of S relative to S_0

$$\vec{E} = \gamma_{v_0}\vec{E}^{(0)}$$

$$\vec{B} = \frac{1}{c^2}\vec{v}_0 \times \vec{E}$$

$$E_z = \frac{\sqrt{1-u^2/c^2}}{\sqrt{1-v_0^2/c^2}}\, E_z' \qquad v_0 = \frac{v+u}{1+\frac{vu}{c^2}}$$

$$1-\frac{v_0^2}{c^2} = 1 - \frac{(v+u)^2}{c^2}\frac{1}{\left(1+\frac{vu}{c^2}\right)^2} =$$

$$= \frac{1}{\left(1+\frac{vu}{c^2}\right)^2}\left[\left(1+\frac{vu}{c^2}\right)^2 - \frac{(v+u)^2}{c^2}\right]$$

$$= \frac{1}{\left(1+\frac{vu}{c^2}\right)^2}\left[1+\frac{v^2u^2}{c^4}+\cancel{2\frac{vu}{c^2}}-\frac{v^2}{c^2}-\frac{u^2}{c^2}-\cancel{\frac{2uv}{c^2}}\right]$$

$$\boxed{1-\frac{v_0^2}{c^2} = \frac{1}{\left(1+\frac{vu}{c^2}\right)^2}\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)}$$

$$E_z = \frac{\cancel{\sqrt{1-u^2/c^2}}\left(1+\frac{vu}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}\,\cancel{\sqrt{1-u^2/c^2}}}\, E_z'$$

$$E_z = \gamma\left[E_z' + \frac{v}{c^2}\left(uE_z'\right)\right] \qquad uE_z' = -\frac{B_x'}{\mu_0\varepsilon_0}$$

$$E_z = \gamma\left[E_z' + \frac{v}{\cancel{c^2}}\left(-\cancel{c^2}B_x'\right)\right]$$

$$\boxed{E_z = \gamma\left[E_z' - vB_x'\right]} \qquad x' = \gamma(x-vt)$$

$$B_x = \mu_0 \sigma v_0 = \mu_0 \gamma_{v_0} \sigma_0 v_0$$

$$= \mu_0 \frac{1}{\sqrt{1 - v_0^2/c^2}} v_0 \sigma_0$$

$$B_x = \mu_0 \left(\frac{v+u}{\cancel{1+\frac{vu}{c^2}}}\right) \sigma_0 \left(\cancel{1+\frac{uv}{c^2}}\right) \frac{1}{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}$$

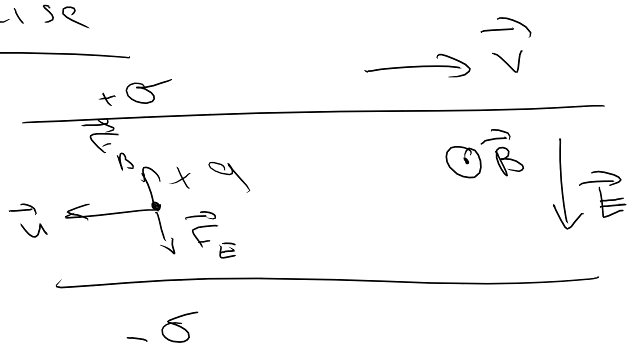
$$B_x = \mu_0 \frac{v+u}{\sqrt{1-\frac{v^2}{c^2}}\cancel{\sqrt{1-\frac{u^2}{c^2}}}} \varepsilon_0 \cancel{\sqrt{1-u^2/c^2}}\left(-E_z'\right)$$

$$B_x = -\mu_0 \varepsilon_0 \frac{(v+u)E_z'}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{1}{c^2}\gamma_v\left[vE_z' + uE_z'\right]$$

$$B_x = -\frac{1}{c^2}\gamma_v\left[vE_z' - c^2 B_x'\right]$$

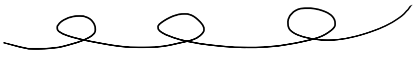
$$B_x = \gamma_v\left[B_x' - \frac{v}{c^2}E_z'\right] \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Exercise



$$\vec{F} = q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

?



Transformation properties of the $\vec{E}$ & $\vec{B}$ fields $\vec{v} = v\hat{x}'$

$$E_x = E_x' \qquad E_y = \gamma(E_y' - vB_z') \qquad E_z = \gamma(E_z' + vB_y')$$

$$B_x = B_x' \qquad B_y = \gamma\left(B_y' + \frac{v}{c^2}E_z'\right) \qquad B_z = \gamma\left(B_z' - \frac{v}{c^2}E_y'\right)$$

$$\vec{E}\cdot\vec{B} = E_xB_x + E_yB_y + E_zB_z$$

$$= E_x'B_x' + \gamma^2(E_y' - vB_z')\left(B_y' + \frac{v}{c^2}E_z'\right)$$

$$+ \gamma^2(E_z' + vB_y')\left(B_z' - \frac{v}{c^2}E_y'\right)$$

$$= E_x'B_x' + \cancel{\gamma^2} E_y'B_y'\cancel{\left(1 - \frac{v^2}{c^2}\right)}$$

$$+ \cancel{\gamma^2} E_z'B_z'\cancel{\left(-\frac{v^2}{c^2} + 1\right)}$$

$$+ \gamma^2 E_y'E_z'\cancel{\left(\frac{v}{c^2} - \frac{v}{c^2}\right)}$$

$$+ \gamma^2 B_y'B_z'\cancel{(-v + v)}$$

$$E_xB_x + E_yB_y + E_zB_z = E_x'B_x' + E_y'B_y' + E_z'B_z'$$

$$\vec{E}\cdot\vec{B} = \vec{E}'\cdot\vec{B}'$$

$$\boxed{\vec{E}^2 - c^2\vec{B}^2 = \vec{E}'^2 - c^2\vec{B}'^2}$$ Exercise

if $cB < E \Rightarrow cB' < E'$