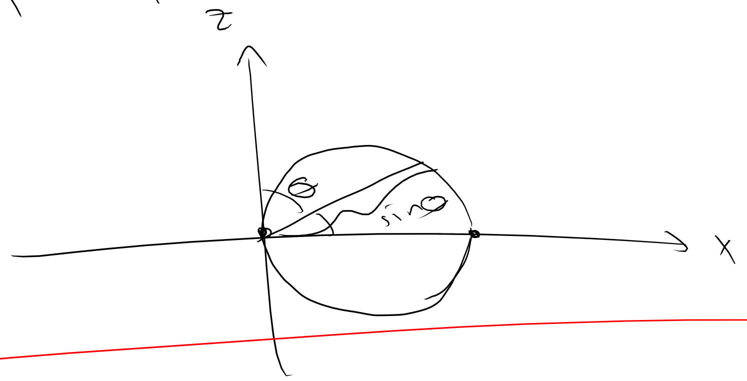
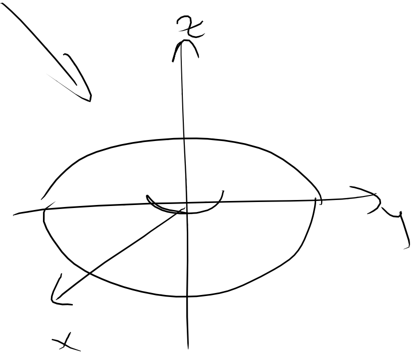
Midterm 2, 13th p422

$$P \propto \left(\frac{\sin\theta}{r}\right)^2$$

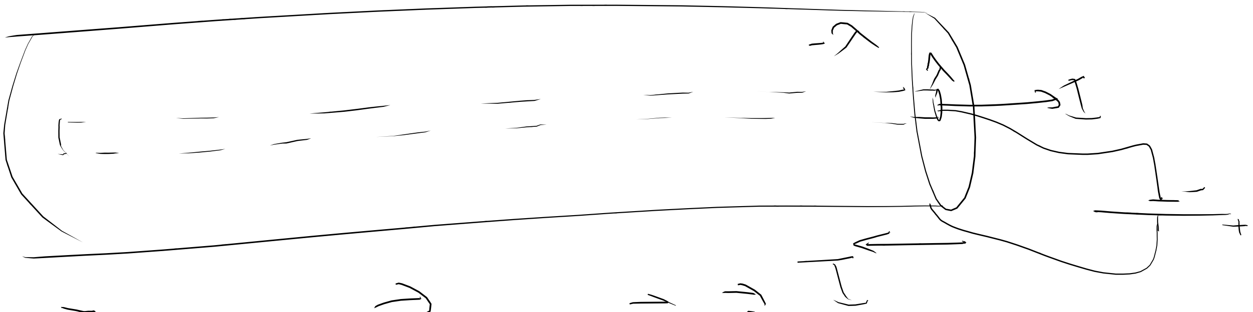
shows the magnitude

Surface eqn $r = \sin\theta$



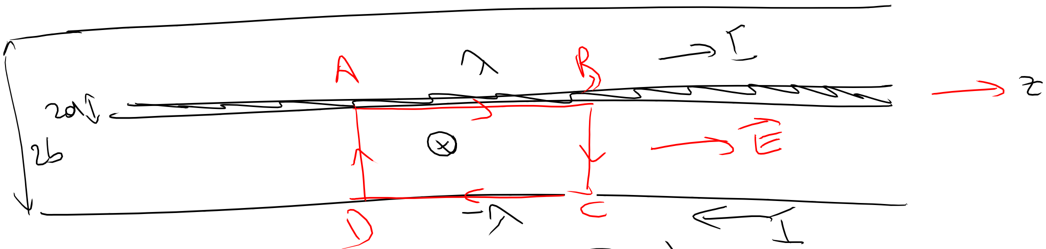


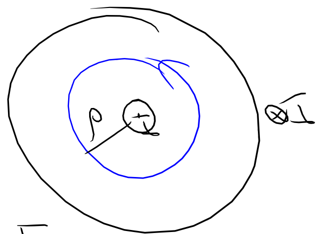
cheat sheet add the eqns for radiation



$$\vec{P} = \frac{1}{c^2}\vec{S} = \epsilon_0 \vec{E}\times\vec{B}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$





$$\Rightarrow \frac{F}{l} = \lambda E(a) - \lambda E(b)$$

$$\Phi_B = \int \vec{B}\cdot d\vec{S} = \int_0^l \int_a^b B(\rho)\, d\rho\, dz = \int_0^l dz \int_a^b \frac{\mu_0}{2\pi}\frac{I}{\rho}\, d\rho$$

$$\boxed{\Phi_B = \frac{\mu_0}{2\pi}\, l\, I \ln\left(\frac{b}{a}\right)}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$2\pi\rho B(\rho) = \mu_0 I$$

$$\boxed{B(\rho) = \frac{\mu_0}{2\pi}\frac{I}{\rho}}$$

$$\oint \vec{E}\cdot d\vec{l} = \int_A^B \vec{E}\cdot d\vec{l} + \int_C^D \vec{E}\cdot d\vec{l} = E(a)\,l - E(b)\,l$$

$$(E(a)-E(b))\,l = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 l}{2\pi}\frac{dI}{dt}\ln\left(\frac{b}{a}\right)$$

$$\boxed{E(a)-E(b) = -\frac{\mu_0}{2\pi}\frac{dI}{dt}\ln\left(\frac{b}{a}\right)}$$

$$\frac{d(P/l)}{dt} = \frac{F}{l} = \lambda\left(E(a)-E(b)\right) = -\frac{\mu_0}{2\pi}\lambda\frac{dI}{dt}\ln\left(\frac{b}{a}\right)$$

$$\int_{t_0}^{t_f} dt\,\frac{d}{dt}\left(\frac{P}{l}\right) = -\frac{\mu_0}{2\pi}\lambda\ln\left(\frac{b}{a}\right)\int_{t_0}^{t_f} dt\,\frac{dI}{dt}$$

$$\frac{P_f}{l} = -\frac{\mu_0}{2\pi}\lambda\ln\left(\frac{b}{a}\right)(0-I)$$

$$\boxed{\frac{P_f}{l} = \frac{\mu_0}{2\pi}\lambda I\ln\left(\frac{b}{a}\right)} \qquad P = \frac{S}{c^2}$$

$$\frac{P_{em}}{l} = \frac{1}{2\pi\varepsilon_0 c^2}\lambda I\ln\left(\frac{b}{a}\right) = \frac{\mu_0}{2\pi}\lambda I\ln\left(\frac{b}{a}\right)$$

Example

$$S\begin{cases} A = (x_A = 5\text{m} \;\; y_A = 3\text{m} \;\; z_A = 0\text{m}) & t_A \quad ct_A = 15\text{m} \\ B \quad (10, 8, 0)\,\text{m} & ct_B = 5\text{m}\end{cases}$$

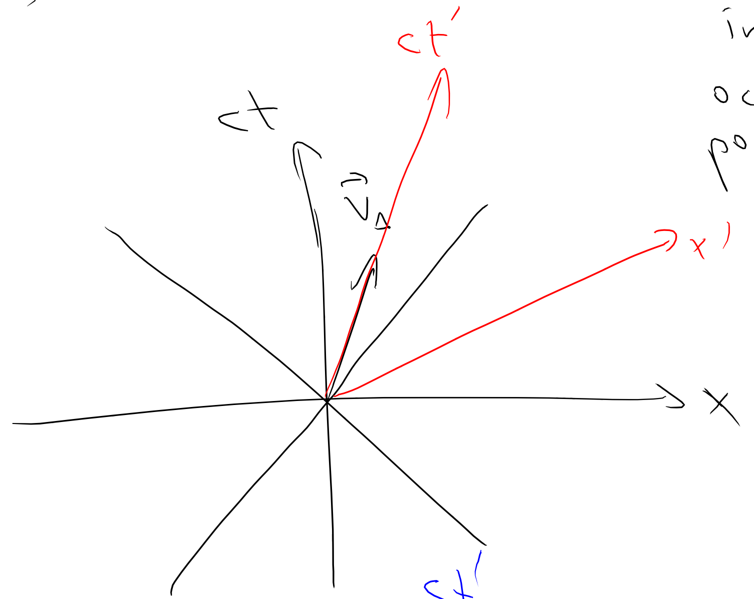
i) $(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$

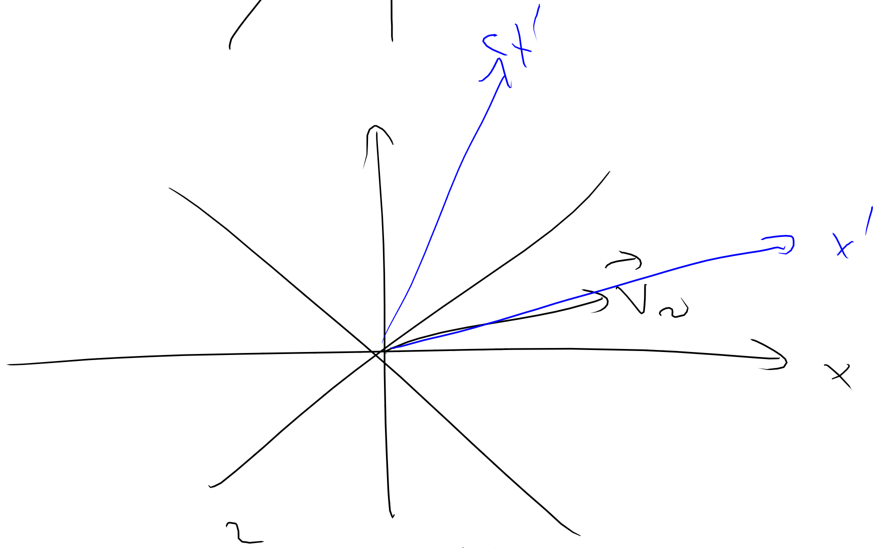
$$= -(10\text{m})^2 + (-5\text{m})^2 + (-5\text{m})^2 + (0\text{m})^2$$

$$= -100\text{m}^2 + 50\text{m}^2 = \boxed{-50\text{m}^2 = (\Delta s)^2}$$

ii) $(\Delta s)^2 > 0 \Rightarrow$ spacelike $\Rightarrow \exists$ a reference frame in which the events are simultaneous

$(\Delta s)^2 < 0 \Rightarrow$ timelike $\Rightarrow \exists$ a reference frame in which the events occur at the same point.





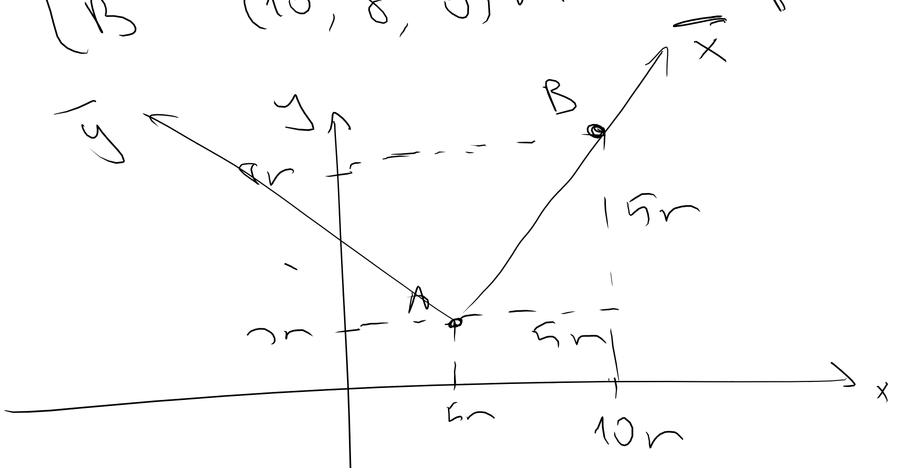
ii) Since $(\Delta s)^2 < 0$, No

iii) Since $(\Delta s)^2 < 0$, Yes

$$(\Delta t)' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 0$$

$$\frac{v}{c^2} = \frac{\Delta t}{\Delta x} =$$

$$S \begin{cases} A = (x_A = 5m \quad y_A = 3m \quad z_A = 0m) & t_A \quad ct_A = 15m \\ B \quad (10, 8, 0)\,m & ct_B = 5m \end{cases}$$



$A \quad (0, 0, 0) \quad ct_A = 15m$

$B \quad (5\sqrt{2}\,m, 0, 0) \quad ct_B = 5m$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta\bar{x}\right)$$

$$\Delta\bar{x}' = \gamma(\Delta\bar{x} - v\Delta t)$$

$$\Delta\bar{y}' = \Delta\bar{y} = 0$$

$$\Delta\bar{z}' = \Delta\bar{z} = 0$$

$$\Delta\bar{x}' = 0 \Rightarrow v = \frac{\Delta\bar{x}}{\Delta t} = \frac{5\sqrt{2}\,m}{10\,m/c} = \frac{c}{\sqrt{2}}$$

$$\boxed{v = \frac{c}{\sqrt{2}}}$$

$$\vec{v} = \frac{c}{\sqrt{2}}\left(\frac{\hat{x} + \hat{y}}{\sqrt{2}}\right) = \boxed{\frac{c}{2}(\hat{x} + \hat{y}) = \vec{v}}$$

Q A $(x_A, 0, 0)$ t_A

B $(x_B, 0, 0)$ t_B

$$(\Delta s)^2 > 0 \Rightarrow \text{space like}$$

$$(\Delta t') = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 0$$

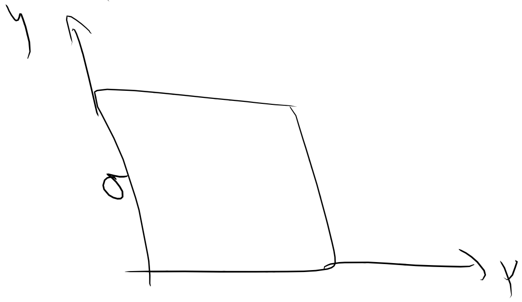
$$\frac{v}{c^2} = \frac{\Delta t}{\Delta x} \Rightarrow \frac{v}{c} = \frac{c\Delta t}{\Delta x}$$

$$\frac{v}{c} = \frac{c(t_B - t_A)}{(x_B - x_A)}$$

$$\frac{c\Delta t}{\Delta x} \overset{?}{<} 1 \qquad (\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 > 0$$

$$|\Delta x| > |c\Delta t|$$

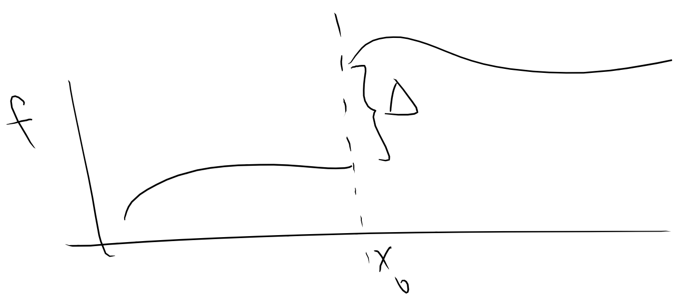
$$\frac{|c\Delta t|}{|\Delta x|} < 1$$



$$\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0$$

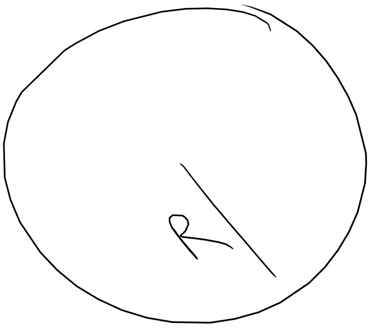
$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

$$\rho = \sigma\delta(x) \ldots$$



$$\left.\frac{df}{dx}\right|_{x = x_0} = \Delta\,\delta(x - x_0)$$

Q



⊗ $\vec{\omega}$

$$\frac{\text{circumference}}{R} = ?$$

$$\tilde{v} = \frac{dx^1}{dx^4}$$

Q4 HW7

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$x^4 = ict$$

$$ict' = \gamma\left(ict - i\frac{v}{c}x\right)$$

$$x^{4'} = \gamma(x^4 + \tilde{\beta}x^1)$$

$$\boxed{\tilde{\beta} = -i\frac{v}{c}} = \frac{\tilde{v}}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 + \tilde{\beta}^2}}$$

if $\tilde{\beta} \in \mathbb{R}$

$$x^{4'} = \gamma(x^4 + \tilde{\beta}x^1)$$

$$= \gamma x^4 + \gamma\tilde{\beta}x^1$$

$$(\gamma)^2 + (\gamma\tilde{\beta})^2 = 1 \Rightarrow \gamma = \cos\theta$$

$$\gamma\tilde{\beta} = \sin\theta$$

$$x^{4'} = \cos\theta\, x^4 + \sin\theta\, x^1$$

$$x^{1'} = \gamma\left(x^1 - \frac{v}{ic}ict\right) = \gamma\left(x^1 + i\frac{v}{c}x^4\right)$$

$$= \gamma(x^1 - \tilde{\beta}x^4)$$

$$x^{1'} = \cos\theta\, x^1 - \sin\theta\, x^4$$

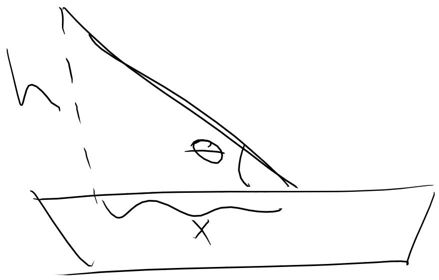
$$\tilde{\beta} = \tan\theta = -i\frac{v}{c}$$

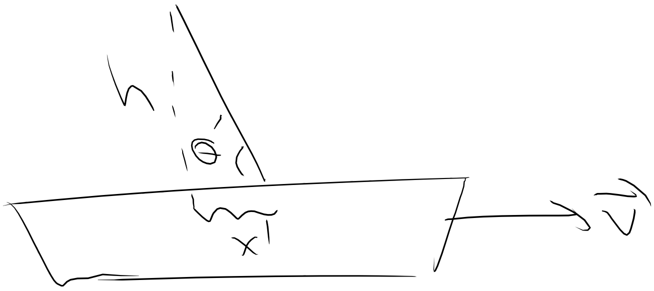
$$\tan(i\bar{\theta}) = \frac{\sin(i\bar{\theta})}{\cos(i\bar{\theta})} = \frac{\dfrac{e^{i(i\bar{\theta})} - e^{-i(i\bar{\theta})}}{2i}}{\dfrac{e^{i(i\bar{\theta})} + e^{-i(i\bar{\theta})}}{2}}$$

$$= -i\,\frac{e^{-\bar{\theta}} - e^{\bar{\theta}}}{e^{-\bar{\theta}} + e^{\bar{\theta}}} = i\tanh\bar{\theta} = -i\frac{v}{c}$$

$$\boxed{\tanh\bar{\theta} = -\frac{v}{c}}$$

Q





$$x' = \sqrt{1-\frac{v^2}{c^2}}\,x$$

$$\tan\theta' = \frac{h}{x'} = \frac{h}{\sqrt{1-\frac{v^2}{c^2}}\,x} = \gamma\tan\theta$$

$$\boxed{\tan\theta' = \gamma\tan\theta}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$M^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} x^{\alpha} p^{\beta}$$

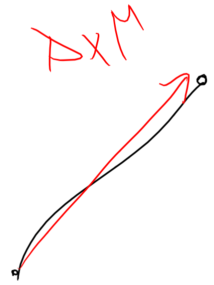
$$\omega^{\mu\nu}$$

$$\boxed{\eta^{\mu} = \frac{dx^{\mu}}{d\tau}}$$

$$(c\,d\tau)^2 = (c\,dt)^2 - (dx)^2$$

$$\varepsilon^{\mu\nu\alpha\beta}$$

$$\varepsilon^{0123} = 1$$



$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}\, dt$$